Sub-Doppler cooling of Λ atoms in the field of two standing waves with a spatial phase shift

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A new mechanism is proposed for sub-Doppler cooling of three-level Λ atoms in the field of standing waves with a spatial phase shift. The dynamics of the Λ atom in the field of the standing waves is analyzed on the basis of exact solutions of the steady-state equations for elements of a density matrix. At certain values of this phase shift, the effective temperature of the Λ atoms can reach values much lower than the Doppler limit.

Various mechanisms for cooling neutral atoms below the Doppler limit $T_D = \hbar\gamma/k_B \approx 10^{-4}$ K, which is set by the intrinsic width, 2γ, of the atomic transition line, have recently been the subject of active research. At this point, the list of these mechanisms runs as follows: cooling by virtue of a polarization gradient,[1] cooling in a magnetic field,[2] and cooling of atoms through the use of coherent population trapping.[3]

In this letter we are reporting a new mechanism for sub-Doppler laser cooling, which operates in an interaction of three-level Λ atoms (Fig. 1) with two standing waves, between which there is a spatial phase shift φ. We show below that a nonzero shift φ causes a qualitative change in the dynamics of the Λ atoms in the field of the standing waves, and that it has a dramatic effect on the evolution of the atomic distribution and on the effective temperature of the atomic ensemble. At certain values of the parameters of the laser light, the temperature can reach values much lower than T_D.

Physically, this behavior of the Λ atoms is explained as a manifestation in this interaction scheme of the "Sisyphus" slowing mechanism,[1] which arises when spatially nonuniform optical pumping is combined with optical shifts of the atomic energy levels. While they are in one of the lower states, the atoms move in the potential set up by the periodic optical shift of this level (the period is $\lambda/2$, where λ is the length of the exciting waves). When they are put into another unexcited state, with a smaller optical shift, by the optical pumping, the atoms lose their kinetic energy. This picture of the motion of atoms is created in other schemes for sub-Doppler laser cooling by means of a polarization gradient of the exciting waves[1] or a magnetic field.[2] In the scheme which we are proposing here, two standing waves are sufficient for the purpose. Since the waves interacting with the Λ atoms are standing waves, their optical shifts change from a node to an antinode, giving the potentials the necessary periodicity. Optical pumping arises when there is a nonzero spatial shift φ.

To solve this problem, we write the field of the standing waves in the form

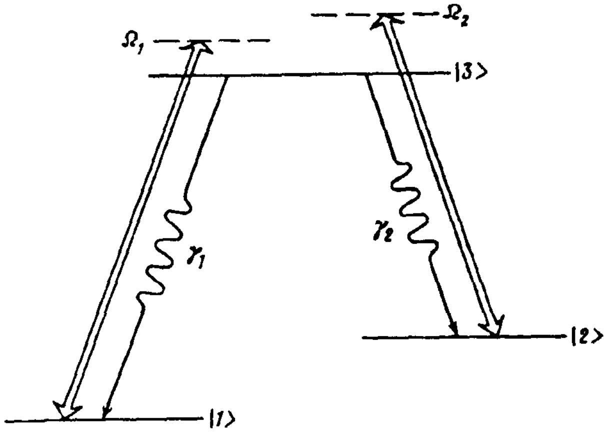

FIG. 1. The Λ interaction scheme. Here $\Omega_m=\omega_m-\omega_{3m}$ are the frequency detunings of the exciting waves (ω_{3m} are the frequencies of the $|m\rangle-|3\rangle$ transitions), and G_m are the probabilities for spontaneous decays by the $|3\rangle-|m\rangle$ channels.

$$E(z,t)=2E_0[\cos(\omega_1 t)\cos(kz)+\cos(\omega_2 t)\cos(kz+\varphi)], \tag{1}$$

where ω_m are the frequencies of the optical waves of the resonant transitions $|m\rangle-|3\rangle$ of the Λ atom (Fig. 1, $m=1, 2$), φ is their spatial shift, and it is assumed that the amplitudes and wave vectors of the two standing waves are identical.

The radiation force averaged over the light wavelength λ can be found, according to Ref. 5, as

$$F_z=\hbar kg\sum_{n=\pm 1}(-n)[Q(n)+P(n)\exp(-in\varphi)], \tag{2}$$

where $g=dE_0/2\hbar$ is the Rabi frequency, which is the same for the two transitions of the Λ atom, and $Q(n)=\rho_{31}(n)+\rho_{13}(n)$ and $P(n)=\rho_{32}(n)+\rho_{23}(n)$ are the sums of the Fourier components of the off-diagonal elements ρ_{jm}, found after an expansion of the density matrix of the three-level atom in an infinite spatial series:

$$\rho_{jm}=\sum_{n=-\infty}^{\infty}\rho_{jm}(n)\exp(inkz), \quad j, m=1, 2, 3. \tag{3}$$

After (3) is substituted into the system of equations for the elements of the density matrix, one finds,[6] in the steady-state case, an infinite system of algebraic recurrence relations for the $\rho_{jm}(n)$. A solution of this system of equations can be sought as a matrix chain fraction. For this purpose we write this system of equations in the form

$$A_n\mathbf{x}_{n+2}+B_n\mathbf{x}_n+C_n\mathbf{x}_{n-2}=\delta_{n0}\mathbf{s}, \quad n=0,\pm 2,\pm 4, \ldots, \tag{4}$$

where $\mathbf{x}_n$ is the vector $[\rho_{11}(n)-\rho_{33}(n), \rho_{22}(n)-\rho_{33}(n), \rho_{12}(n), \rho_{21}(n)]$, and $\mathbf{s}$ is a vector constructed from the values of the probabilities for spontaneous decays:

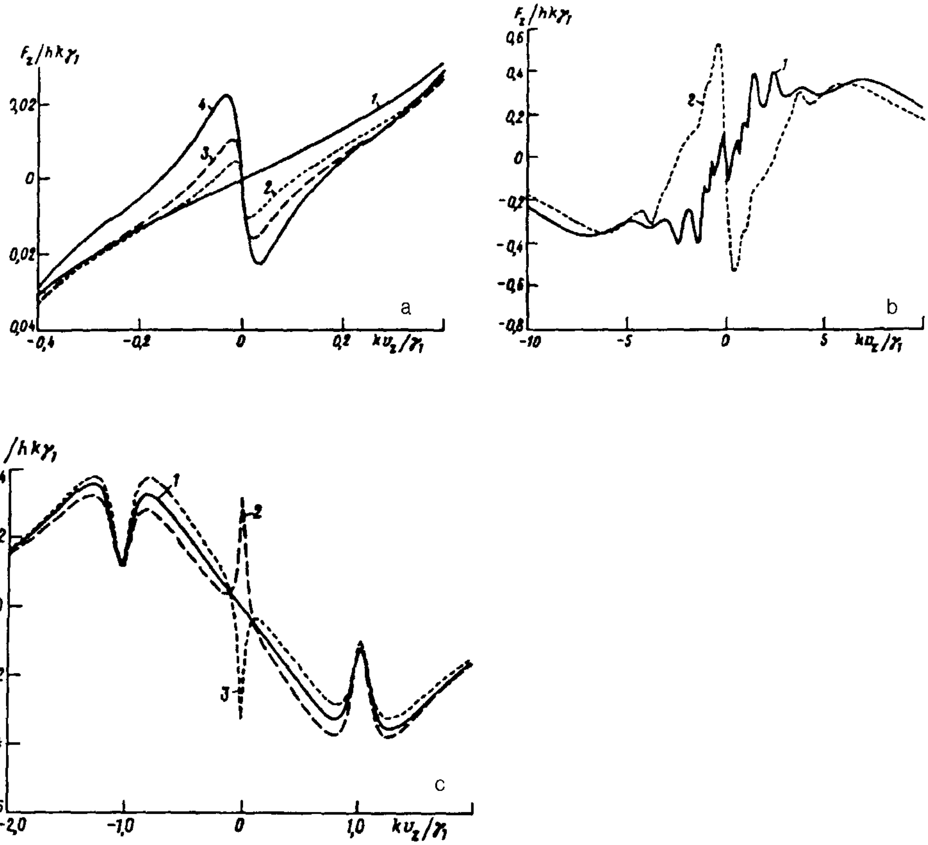

FIG. 2. Average radiation-pressure force F_z versus the velocity (v_z) of the Λ atom for various values of the phase shift φ. a: $\Omega_1=\gamma$, $\Omega_2=2\gamma$, $g=0.5\gamma$, $\gamma_1=\gamma_2=\gamma$. *1*—$\varphi=0$; *2*—$\pi/6$; *3*—$\pi/4$; *4*—$\pi/2$. b: $\Omega_1=3\gamma$, $\Omega_2=6\gamma$, $g=5\gamma$, $\gamma=\gamma_{1,2}$. *1*—$\varphi=0$; *2*—$\pi/2$. c: $\Omega_1=-\gamma_1$, $\Omega_2=\gamma_1$, $g=0.5\gamma_1$, $\gamma_2=0.5\gamma_1$. *1*—$\varphi=0$; *2*—$\pi/4$; *3*—$-\pi/4$.

$\mathbf{s}=[i\Gamma_1, i\Gamma_2, 0, 0]$, where $\Gamma_1=(2\gamma_1+\gamma_2)/3$, $\Gamma_2=(\gamma_1+2\gamma_2)/3$, and δ_{n0} is the Kronecker delta. The quantities A_n, B_n, and C_n in (4) are defined as 4×4 matrices whose elements are linear combinations of the quantities f_{mnl}:

$$f_{mnl}=\frac{g^2\exp(il\varphi)}{i\gamma\mp(\Omega_{m3}\pm(n\pm 1)kv_z)},\quad m=1,2;\quad l=0,\pm 1,\pm 2.$$

Here Ω_{m3} are the frequency detunings, $\gamma=(\gamma_1+\gamma_2)/2$, and v_z is the z component of the velocity of the atom. The solution of Eq. (4) is

$$\mathbf{x}_0=(A_0+B_0T_-+C_0T_+)^{-1}\mathbf{s},\quad \mathbf{x}_2=T_+\mathbf{x}_0,\quad \mathbf{x}_{-2}=T_-\mathbf{x}_0,$$

$$T_+=-\{A_2-C_2[A_4-C_4(A_6-C_6(\dots)^{-1}B_6)^{-1}B_4]^{-1}B_2\},$$

$$T_-=-\{A_{-2}-B_{-2}[A_{-4}-B_{-4}(A_{-6}-B_{-6}(\dots)^{-1}C_{-6})^{-1}C_{-4}]^{-1}C_{-2}\}.\qquad (5)$$

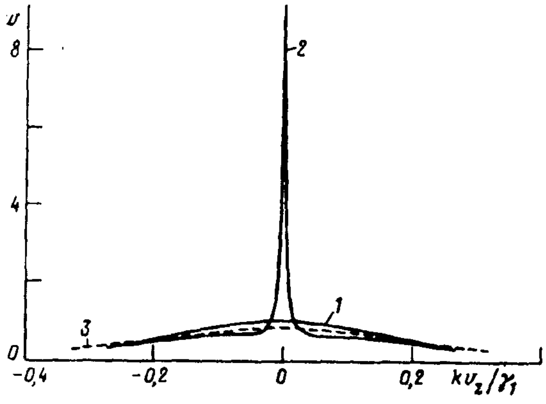

FIG. 3. Evolution of the velocity distribution of the Λ atoms under the influence of the force F_z. *1*—Initial distribution, with a width $\Delta v_z(t=0)=0.5\gamma/k$, corresponding to the temperature T_D (for Na); *2*—final velocity distribution for the case $\varphi=\pi/2$ and for an interaction time $t=1.5\omega_R^{-1}=5\times10^{-6}$ s ($\omega_R=\hbar k^2/M$, where M is the mass of the atom); *3*—final velocity distribution for the case $\varphi=0$ and for an interaction time $t=3\omega_R^{-1}=10^{-5}$ s. The parameter values are otherwise the same as in Fig. 2a.

Summing the necessary number of terms of fraction (5), we can find a solution with any prespecified accuracy, and we can find the force F_z.

Figure 2a shows the radiation-pressure force F_z versus the velocity of the atom for various values of the spatial shift φ. We see that at nonzero values of φ there is a sharp increase in the dynamic friction coefficient $\beta=(-\partial F_z/\partial v_z)$ at very low velocities, and the $F_z(v_z)$ dependence assumes its characteristic dispersive form,[1] which is most pronounced in the case $\varphi=\pi/2$. We note that the friction is negative in the case $\varphi=0$; i.e., a heating occurs.

Figure 2b shows F_z as a function of v_z for high intensities of the standing waves. We can clearly see nonlinear resonances of the multiphoton absorption ($\varphi=0$), even at very low velocities.[5] The existence of a spatial shift φ again in this case leads to a significant increase in the friction coefficient for $v_z=0$.

Finally, Fig. 2c demonstrates the change in the form of the radiation-pressure force upon a change in the sign of one of the detunings of the standing waves. In this case we observe a peak in the radiation-pressure force at very low velocities of the Λ atoms. Not only the amplitude but also the sign of the force F_z depend on the value of φ near $v_z\approx0$.

Since momentum diffusion is insignificant over short interaction times, the evolution of the velocity distribution can be found by solving a Liouville equation with the force in (2):

$$\frac{\partial w}{\partial t}=-\frac{\partial(F_z w)}{\partial p_z}. \tag{6}$$

Figure 3 shows the results of a numerical solution of Eq. (6) for various values of the shift φ. With $\varphi=0$, for example, there is a very slight increase in the width of the velocity distribution. At $\varphi=\pi/2$, in contrast, the velocity distribution shrinks dramatically, and the width of the peak of collimated atoms which forms is $\Delta v_z \approx 0.01\gamma/k$. The corresponding effective temperature is $T \approx 2\times 10^{-2} T_D$ (for sodium atoms).

The possibility of sub-Doppler cooling of Λ atoms in the field of two standing waves with a spatial phase shift was recently demonstrated experimentally.[7]

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