Nonlocal effects and surface impedance of a plate of a hard superconductor

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A model is proposed for describing the electrodynamics of hard superconductors with dimensions comparable to the London depth. The nonlinear response of the superconductor to a change in the external magnetic field is examined for the case in which there is an irreversible motion of Abrikosov vortices. Certain aspects of the low-frequency surface impedance of a superconducting plate are explained successfully by this model.

1. Bean's critical-state model is widely used to describe the low-frequency electromagnetic properties of hard superconductors. Since it makes use of only a single phenomenological parameter—the critical current density J_c—this model is a rather crude approximation. In particular, it cannot describe many phenomena which occur in systems with typical dimensions comparable to λ, the London field penetration depth. In this letter we give an example of one of these phenomena.

The surface resistance $\mathscr{R}$ of a superconducting plate is known to have a maximum when the penetration depth δ of the alternating magnetic field is on the order of the sample thickness d (Ref. 1, for example). According to the critical-state model, the height of this peak, normalized to the surface reactance of the sample in its normal state, $\chi_n = 2\pi\omega d/c^2$, does not depend on any physical parameters:

$$R'_{\max} = \mathscr{R}_{\max}/\chi_n = 3/4\pi. \tag{1}$$

The result in (1) does not depend on how the penetration depth of the alternating field changes; the impedance can be studied as a function of a static field, the temperature, the amplitude of an alternating field, etc. We have pointed out that in certain cases $R'_{\max}$ is much smaller than $3/4\pi$. As an example, Fig. 1 shows the temperature dependence of the dimensionless surface resistance $R' = \mathscr{R}/\chi_n$ for various amplitudes of an alternating field. We studied a sample of a textured Y–Ba–Cu–O ceramic with a thickness ~ 1 mm. The impedance was measured at a frequency of 130 Hz by the standard induction method. All the curves exhibit these peaks. At low temperatures, the heights of these peaks agree with the predictions of the critical-state model, but the peaks shrink as the temperature approaches the transition temperature. We have suggested that this behavior of the impedance stems from an increase in the penetra-

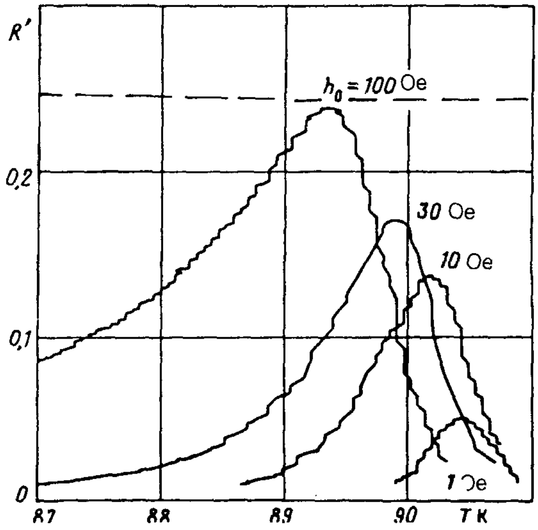

FIG. 1. Temperature dependence of the surface resistance R' at various amplitudes of the alternating field, h_0, at $\omega/2\pi=130$ Hz for a textured Y–Ba–Cu–O ceramic with $d=1.4$ mm. The dashed line is the maximum peak height, $3/4\pi$, corresponding to the standard critical-state model.

tion depth λ, which may turn out to be comparable to the characteristic geometric dimensions of the system (e.g., the characteristic thickness of the grains making up the sample).

2. To explain this effect we need to construct a new model, which incorporates the variation of the fields over length scales on the order of λ. We will discuss here the case of a fairly strong magnetic field, $H_{c1} \ll H \ll H_{c2}$, for which the characteristic distance between Abrikosov vortices, a, is much smaller than λ. We introduce the functions $B(x)$ and $n(x)$, which are the results of an averaging of the microscopic magnetic field and the number of Abrikosov vortices over a length scale much larger than a but small in comparison with λ. The magnetic field $B(x)$ is the sum of two terms, one of which is the Meissner field, while the other is the contribution of vortices (Ref. 2, for example). For a plate occupying the region $-d/2<x<d/2$ in an external field H we have

$$B(x)=H\frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)}+B_v(x). \tag{2}$$

The average vortex field B_v satisfies the equation

$$B_v-\lambda^2\frac{\partial^2 B_v}{\partial x^2}=n\Phi_0 \tag{3}$$

with the boundary condition $B_v=0$ at $x=\pm\, d/2$.

A second equation of our model, which we need in order to determine the functions $B(x)$ and $n(x)$, is found from a consideration of the balance of forces. This equation is

$$\partial B/dx=4\pi f/\Phi_0, \tag{4}$$

where f is the force of the interaction of a vortex with a pinning center, and Φ_0 is the magnetic flux quantum. In deriving this equation we took account of (in addition to pinning) the interaction of vortices with each other, with the Meissner field, and with antivortices. Equation (4) is similar in form to the basic equation of the standard critical-state model. In our case, however, the pinning force can take on not only the extreme values $f=\pm F$, where $F=\Phi_0 J_c/c$, but any value in the interval $-F<f<+F$.

In superconductors in which the pinning is not very strong, and in which the magnetic induction varies over distances larger than λ, Eq. (3) leads to the known local relation $B_v=n\Phi_0$. This relation is used in the standard critical-state model. In systems in which the critical current density is fairly high, on the other hand, we need to use Eq. (3), which is more general. In contrast with the standard model, the state of the superconductor is described by two equations, for the two unknown functions $B(x)$ and $n(x)$. As a result, there are some unusual features in the behavior of $B(x)$ and $n(x)$ in the plate as the external magnetic field varies.

We would first like to point out that in all cases—regardless of the spatial distribution of the vortex density—there exists an interval ΔH of the external magnetic field in which the function $n(x)$ is constant everywhere in the sample. Specifically, as long as the force exerted on the vortex system by the magnetic field varies between $-F$ and F and does not exceed the maximum pinning force, the vortex system in the sample will remain frozen. The magnetic field $B(x)$, in contrast, changes, because of the Meissner component in (2). The value of ΔH can be determined easily by working from Eqs. (2) and (4) with $f=F=\pm\Phi_0 J_c/c$:

$$\Delta H=(8\pi/c)J_c\lambda\coth(d/2\lambda). \qquad (5)$$

This result means that the system contains a barrier which prevents changes in $n(x)$, and that the height of this barrier is proportional to J_c.

As the external force varies over a range broader than (5), the pinning force and thus the current density reach their maximum values in a surface layer $x_0<|x|<d/2$. Vortices either penetrate into this region or escape from it. The field $B(x)$ here can be found from (4) with $f=\pm\Phi_0 J_c/c$. The vortex density can be calculated from the result found for $B(x)$ with the help of (3). We see from Eqs. (3) and (4) that in the region in which $n(x)$ varies there is always a single-valued relationship between $n(x)$ and $B(x)$ because of the condition $J=J_c=\text{const}$:

$$B(x)=n(x)\Phi_0. \qquad (6)$$

Elsewhere in the plate, at $|x|<x_0$, the vortex density $n(x)$ is frozen, as before. The field in this region is found from Eq. (3) with the known function $n(x)$. The current density, which is below the critical value here, is found from one of Maxwell's equations. The relation $B=n\Phi_0$ does not hold in this region. The final step of the procedure of determining the functions $B(x)$ and $n(x)$ is to join the resulting solutions at the point x_0, through the use of the continuity conditions on $B(x)$ and $\partial B/\partial x$. These conditions follow from an analysis of Eq. (3).

We would like to stress that the density of vortices, $n(x)$—in contrast with $B(x)$—has discontinuities at the points $|x|=x_0$. The reason for these discontinuities

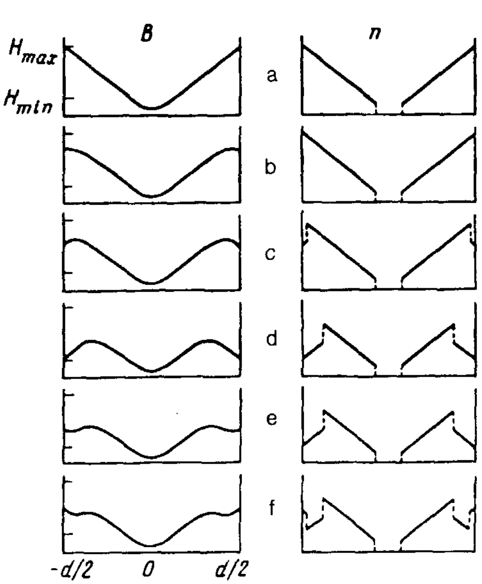

FIG. 2. Evolution (a–f) of the spatial distribution of the magnetic induction (at the left) and that of the vortex density (at the right) during a periodic variation of the external magnetic field.

can be explained as follows: The vortex density and the field are related by Eq. (6) in the regions $x_0 < |x| < d/2$, as we have already mentioned, since in these regions the function $n(x)$ changes. On the other side of x_0, in contrast, the function $n(x)$ is frozen, and it is related by the same equation, (6), with the external field, i.e., with $B(x)$, at which the density of vortices in this region was formed. The height of these jumps is proportional to the height of the barrier, ΔH: $\Delta n = \Delta H/\Phi_0$. The appearance of discontinuities in our model means that the vortex density changes over distances much smaller than λ near the points x_0. A corresponding effect is known in soft superconductors:[3] In contrast with the magnetic induction $B(x)$, the density of vortices falls off not over distances λ but over a length scale a toward the boundary of the sample.

It follows from our model that, regardless of the value of the external magnetic field, there is always a region at the center of the plate at which the vortex density is $n=0$. This is a consequence of the mutual repulsion of Abrikosov vortices at the center of the plate. The size of this region, l_0, is given implicitly by

$$\delta(H) = (d - l_0)/2 + \lambda \coth(l_0/2\lambda), \tag{7}$$

where $\delta(H) = cH/4\pi J_c$. This conclusion is qualitatively at odds with the predictions of the standard critical-state model, in which a region with $n=0$ would exist only in a field weaker than the penetration field $H_p = (2\pi/c) J_c d$.

Figure 2 shows profiles $B(x)$ and $n(x)$ in the plate for a periodic variation of the external field in the interval $H_{\min} \leqslant H \leqslant H_{\max}$. The height of the discontinuity in the vortex density, Δn, the barrier ΔH, and the size of the region with $n=0$ at $H > H_p$ are proportional to λ, and they vanish in the local limit $\lambda \to 0$.

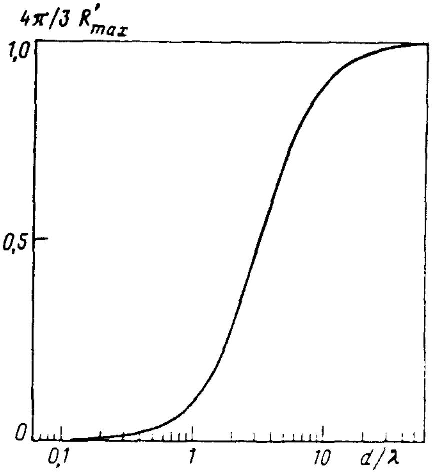

FIG. 3. Theoretical height of the peak, $R'_{\max}$, in the surface impedance as a function of the parameter d/λ.

3. To interpret the experimental results in Fig. 1, we calculated the surface resistance of a plate in an alternating external field $H(t)=H_0+h_0\cos(\omega t)$ on the basis of the nonlocal model proposed here. The result can be written in the form

$$R'=\frac{4}{3\pi}\left[\frac{l}{\delta(h_0)}\right]^2\frac{3\delta(h_0)-2l}{d},\quad l=\frac{d-l_0(h_0)}{2}. \tag{8}$$

The quantity R' goes through a maximum as a function of δ; the height of this maximum, $R'_{\max}$, depends on the ratio d/λ. Figure 3 shows that the height $R'_{\max}$ at $\lambda>d$ is considerably smaller than $3/4\pi$.

Let us analyze the experimental and theoretical results which have been found. According to the curve in Fig. 3, the relative decrease in the peak height R' observed experimentally, at $h_0=1$ Oe and $T_c-T\sim0.5$ K (Fig. 1), for example, can occur at a value $d/\lambda\sim 2$. The penetration depth λ for Y–Ba–Cu–O at this temperature is a few microns, while the thickness of the sample is more than 1 mm. This system must therefore contain another length scale, on the order of 10 μm, which manifests itself as an observable size effect. We know that fused high-T_c superconducting samples have a platelike structure with a typical grain thickness of about 10 μm. In our sample, according to microstructural analysis, the typical grain size d is 8–10 μm. It can thus be suggested that the thickness of the grains in our experiment is the parameter d of the nonlocal theory.

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