Possible excitation of nonlinear plasma waves by a relativistic electron beam

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The problem of the generation of plasma waves with a phase velocity close to the velocity of light by a relativistic electron bunch of finite length is analyzed in the 1D approximation. For a given bunch density, there exist optimum values of the bunch length and of the plasma density, which maximize the amplitude of the wakefield oscillations of the electron density. Even at comparatively low current densities (~ 10 kA/cm^2), huge accelerating electric fields, ~ 1 GV/m, can be excited.

The possibility of exciting large-amplitude plasma waves with a phase velocity close to the velocity of light is attracting considerable interest in connection with the search for new methods for accelerating charged particles (see, for example, the review by Bulanov *et al.*[1]). On the experimental side, the approach which has been studied most thoroughly is a resonant excitation which makes use of the beats of two laser waves of approximately the same frequency.[2] Clayton *et al.*[3] have demonstrated the generation of accelerating fields as high as 700 MV/m.

In the present letter we wish to analyze an alternative approach: the excitation of nonlinear plasma waves (Langmuir waves) by a dense relativistic beam in a plasma (this approach was originally proposed and studied in the linear approximation by Chen *et al.*[4]). Our goal here is to estimate the amplitude of the accelerating fields which would arise in the process and to identify the conditions which would maximize these fields. As we will show below, for a given density of the electron beam there exist certain optimum values of the plasma density and of the length of the electron bunch, which maximize the amplitude of the wakefield oscillations of the electron density after the passage of the electron pulse. The plasma waves which are generated near this optimum point turn out to be highly nonlinear.[5]

To derive some simple estimates, we restrict the discussion to the 1D case,[2)] and we assume that the density of the bunch remains constant over time. We ignore the slowing of the beam electrons and the possible onset of beam-plasma instabilities. We assume that the plasma is cold and collisionless. In this approximation the excitation of plasma waves is described by the following system of equations:

$$\frac{\partial E}{\partial z}=4\pi e(\tilde{n}+n_b),\quad \frac{\partial E}{\partial t}=-4\pi e(n_0+\tilde{n})v-4\pi e n_b v_0,$$
$$\frac{dp}{dt}=eE, \tag{1}$$

where $E=E_z(z,t)$ is the longitudinal electric field, n_b and v_0 are respectively the density and velocity of the beam electrons $(1-v_0 \ll 1)$, n_0 is the unperturbed value of the plasma density, $\tilde{n}$ is the variable part of the plasma density, $v(z,t)$ is the velocity of the plasma electrons, and $p=mv/\sqrt{1-v^2}$ is their momentum. The first two equations of system (1) are Maxwell's equations for the electric field; the last equation describes the dynamics of the plasma electrons.

We are interested below in steady-state solutions which are "traveling" along the z axis at a velocity v_0. It is thus convenient to switch to the new variable $\zeta=z-v_0 t$. In the simplest case, the beam density n_b can be represented by a step function:

$$n_b(\zeta)=\begin{cases} 0, & \zeta>0, \\ N, & -\tau<\zeta<0, \\ 0, & \zeta<-\tau, \end{cases} \tag{2}$$

where τ is the length of the bunch. In this case, Eqs. (1) become

$$\frac{dE}{d\zeta}=4\pi e(\tilde{n}+N),$$

$$v_0\frac{dE}{d\zeta}=4\pi e(n_0+\tilde{n})v+4\pi eNv_0, \tag{3}$$

$$(v-v_0)\frac{dp}{d\zeta}=eE.$$

We are assuming that the initial state of the plasma is unperturbed:

$$E(\zeta=0)=0, \quad \tilde{n}(\zeta=0)=0, \quad p(\zeta=0)=0.$$

We switch to the new variable α in accordance with

$$d\alpha=d\zeta/[v_0-v(\zeta)], \tag{4}$$

and we express $\tilde{n}$ and E in terms of p. We then find the equation

$$\frac{d^2p}{d\alpha^2}+4\pi e^2(n_0-N)\frac{2}{\sqrt{p^2+m^2}}+4\pi e^2Nv_0=0. \tag{5}$$

It is easy to see that this equation describes the oscillations of a mass point in a potential field

$$U(p)=4\pi e^2(n_0-N)\sqrt{p^2+m^2}+4\pi e^2Nv_0p. \tag{6}$$

The change of variables in (4) is legitimate only under the condition $v<v_0$; i.e., the velocity of the plasma electrons must not exceed the phase velocity of the wave. Otherwise, a "breaking" of the wave will occur, and there can be no solutions which are stationary in terms of ζ.

Equation (5) describes the excitation of a nonlinear plasma wave moving at a phase velocity v_0. Working from this equation, we can determine how the amplitude of the excited wave varies with the density N and length τ of the electron beam. It turns out that the excitation of the plasma waves occurs in two stages. In the first stage, the electric field generated by the bunch accelerates plasma electrons in the direction opposite the direction in which the bunch itself is moving. After a time τ, the electron bunch then moves off in the forward direction. The result is an abrupt change in the potential $U(p)$ [see Eq. (6)], by an amount

$$U_0(p)=4\pi e^2 n_0 \sqrt{p^2+m^2}. \tag{7}$$

The plasma electrons then execute free oscillations, which are described by Eq. (5) with $N=0$, while the initial conditions are determined by the values which the variables p and $dp/d\alpha$ have reached by the time τ. It is easy to see that the amplitude of these wakefield oscillations is at a maximum if the "coordinate" p is at a maximum at the time τ, i.e., if $dp/d\alpha=0$.

By virtue of the obvious conservation law

$$\frac{1}{2}\left(\frac{dp}{d\alpha}\right)^2+4\pi e^2\,[v_0 Np+(n_0-N)\sqrt{p^2+m^2}]=4\pi e^2(n_0-N)m, \tag{8}$$

the maximum amplitude which we are seeking, p_*, is determined by the equation

$$v_0 Np_*+(n_0-N)\sqrt{p_*^2+m^2}=(n_0-N)m. \tag{9}$$

We find

$$p_*\approx\frac{-2(n_0-N)mN}{n_0(n_0-2N)}. \tag{10}$$

It is easy to see that the N dependence of p_* has a pole: $|p_*|\to\infty$ as $N\to n_0/2$. The reason is that at $N>n_0/2$ the potential $U(p)$ loses its minimum, and the solution ceases to be oscillatory. Since only solutions with $v<v_0$ are physically meaningful, this circumstance means that "traveling" solutions are generated only by relatively short pulses under the condition $n_0-2N\leqslant(n_0/2)\sqrt{1-v_0^2}$.

Going back to (3), we note that the electric field amplitude, thought of as a function of the plasma density, goes through a relatively sharp maximum at $n_0\sim 2N$. Specifically, at $n_0>2N$ we have

$$E_{\max}\simeq 2\sqrt{4\pi Nm}\sqrt{N/(n_0-2N)}. \tag{11}$$

The fraction in the second radical can reach the value $\gamma_0=(1-v_0^2)^{-1/2}$ at the maximum. At $n_0<2N$, the electric field amplitude is determined by the breaking:

$$E_{\max}\approx\sqrt{4\pi Nm}\sqrt{2\gamma_0 n_0/N}. \tag{12}$$

The corresponding length of the bunch is

$$\tau_0=\int_{p_*}^{0}\frac{dp[v_0-p/(p^2+m^2)^{1/2}]}{\sqrt{8ne^2\{(n_0-N)[m-(p^2+m^2)^{1/2}]-v_0 Np\}}}. \tag{13}$$

This length also has a sharp maximum at $n_0\sim 2N$.

Figures 1 and 2 show curves plotted from Eqs. (11)–(13) for the case $\gamma_0=100$ (the energy of the accelerated electrons is $\mathscr{E}\approx 50$ MeV). We again note that reaching the maximum amplitude of the wakefield oscillations of a charge at a given beam

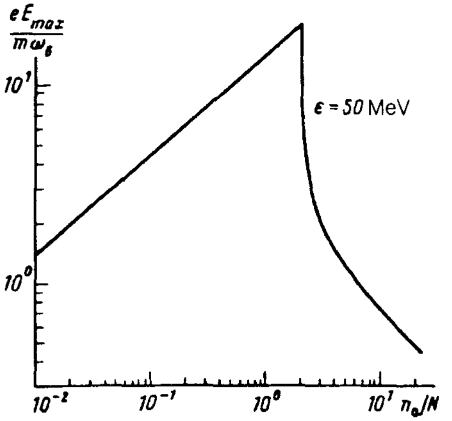

FIG. 1. Amplitude of the accelerating electric field in the wake of an electron bunch as a function of the plasma density. Here and in Fig. 2, $\omega_b = \sqrt{4\pi e^2 N/m}$.

density requires that the length of the bunch be strictly correlated with the plasma density. The amplitude of the wakefield oscillations falls off as the bunch becomes either shorter or longer.

We would now like to estimate the amplitude of the accelerating fields which are reached at realistic values of the density and energy of the electron beam. Let us assume for definiteness that the bunch density is $N = 10^{12}$ e^-/cm^3; the corresponding current density is $j \sim 5$ kA/cm^2. At a plasma density $n = 5 \times 10^{12}$ e^-/cm^3 the optimum bunch length is then $\tau_0 \approx 1$ cm, and the amplitude of the accelerating field is $E_{max} \approx 120$ MV/m. As n_0 approaches the value $2N$, the amplitude of the accelerating field increases sharply. At $n_0 = 2.04 \times 10^{12}$ e^-/cm^3, for example, the amplitude of the accelerating field reaches the gigantic value $E_{max} \approx 1$ GV/m (!). The maximum value which E_{max} can reach at a given beam density increases with increasing electron energy, in proportion to $\gamma_0^{1/2}$.

The stability of the electron bunch itself (the bunch which generates the wake-

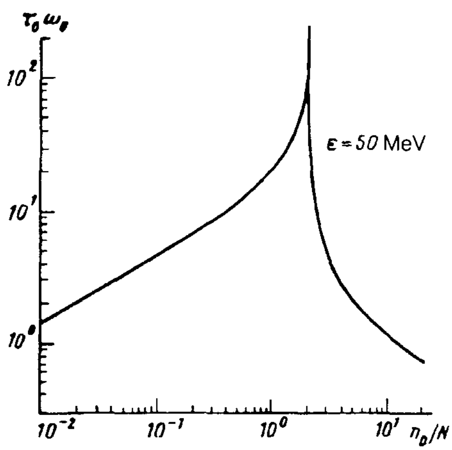

FIG. 2. Optimum bunch length as a function of the plasma density.

field) is a more involved question, requiring further study, which goes beyond the scope of the present letter. A numerical analysis shows that the bunch would remain stable over distances considerably greater than its own dimension τ_0.

A plasma modulated deeply ($\delta n/n \geqslant 1$) by a nonlinear wave would be an extremely suitable medium for exciting transverse electromagnetic waves. The reason is that a beam of relativistic electrons moving opposite a nonlinear wave and passing in succession through layers of plasma with a periodically varying electron density would be unstable with respect to electromagnetic radiation and with respect to the amplification of this radiation at a frequency well above the modulation period.[6] The practical possibilities in this case depend on the development of compact "undulators" (wigglers) for free-electron lasers and on moving into the short-wave region, since the technical capabilities in terms of shortening the period of magnetic wigglers are presently limited to values ~ 1 cm.

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[2)]See Ref. 7 regarding a numerical analysis of a similar problem in the 2D case.

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