Resonant tunneling and long-range proximity effect

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The critical current of SIS tunnel structures in which the primary current-transport mechanism is a resonant tunneling through a single localized state is calculated from the Gor'kov equations. The critical current is governed by a competition between tunneling and thermal mechanisms for the decay of a state into a localized state. The results found here agree satisfactorily with experimental data. They explain the anomalous proximity effect and also the properties of grain-boundary Josephson junctions made of high-T_c superconductors.

Experimental studies of high-T_c Josephson junctions with layers of semiconducting oxides have revealed a long-range proximity effect.[1–7] This effect is the existence of a significant critical current J_c in structures with layer thicknesses $d \geqslant$ 10–100 nm. It follows from the experimental $J_c(d)$ dependence that the decay lengths ξ_n^* of these materials are temperature-independent and an order of magnitude longer (10–50 nm) than in superconducting oxides (1–3 nm). Furthermore, it was shown clearly in Ref. 6 that this effect occurs in a region of oxide compositions in which a metal–insulator phase transition actually occurs.

The latter circumstance suggests that the systems which were studied in Refs. 1–7 were actually Josephson junctions with layers of narrow-gap semiconductors. There has been no experimental or theoretical study of the processes which occur in low-temperature analogs of such structures.

Previous experimental studies (e.g., Refs. 8 and 9) were devoted primarily to structures with silicon layers. The width of the band gap in Si is on the order of 0.3 eV. This figure leads to the following estimate of ξ_n^* (or of the radius of localized states, α)($h=1$):

$$\xi_n^* = \alpha = \frac{1}{\sqrt{2m(V-\mu)}} \approx 1 \text{ nm}. \tag{1}$$

Here m is the effective mass, μ is the chemical potential, and V is the potential of the bottom of the conduction band. At such small values of α, the situation which prevailed experimentally was essentially always such that the relation $d/\alpha \gg 1$ held. For this reason, the theoretical models[10,11] have also been limited for the most part to this particular case.[1)]

In narrow-gap semiconductors, the height of the potential barrier $(V-\mu)$ can take on much smaller values. The parameters d and α are comparable, and the transport of quasiparticles through the layer occurs primarily by a tunneling through one

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or two localized states. Only the normal properties of such structures have been studied previously.[15] It has been shown, for example, that their resistance R_n is given by

$$R_n^{-1}=\sum_m \sigma_m,\quad \sigma_m \propto T^{\left(m-\frac{2}{m+1}\right)}\exp\left(-\frac{2d}{(m+1)\alpha}\right), \tag{2}$$

where σ_m is the conductivity of the channel formed by m localized states.

Our purpose in the present letter is to study the Josephson effect in such junctions for the case in which the effective interaction of electrons in localized states can be ignored.

We assume that the density of localized states in the layer is small (the interaction of quasiparticles belonging to different centers is inconsequential) and that the distribution of these states is uniform over volume and also with respect to energy (at least in an interval on the order of T_c in width near the Fermi energy E_f). We further assume that the potential barrier for quasiparticles in the layer is rectangular and that its height and thickness satisfy the conditions

$$(V-\mu)\ll\mu,\quad dT_c/(V-\mu)\leqslant\alpha\leqslant d, \tag{3}$$

where T_c is the transition temperature of the electrodes.

Under the assumptions made above, it is convenient to use the known expression[16] for the supercurrent J_s to calculate the temperature dependence of J_c:

$$J_s=-\frac{ieT}{(2\pi)^4}\sum_\omega \int dz_1 dz_2 d^2p_1 d^2p_2 \Delta(z_1)\Delta^*(z_2)$$
$$\times G_\omega^n(p_1,p_2,z_1,z_2)G_{-\omega}(p_2,p_1,z_2,z_1)\left[\mathrm{sign}(z_1)-\mathrm{sign}(z_2)\right]. \tag{4}$$

Here p_1 and p_2 are transverse momenta, z_1 and z_2 are coordinates reckoned from the middle of the layer in the direction perpendicular to the boundaries, $\Delta(z_1)=\Delta\exp(i\varphi/2)$ and $\Delta(z_2)=\Delta\exp(-i\varphi/2)$ are the order parameters in the electrodes, $\omega=\pi T(2n+1)$ are the Matsubara frequencies, and G_ω^n and G_ω are Fourier components of the normal and superconducting Green's functions, which incorporate the existence of localized states in the I layer. These components are related to the unperturbed Green's functions by the following relations[10,16] (in the absence of localized states):

$$G_\omega(p,p',z,z')=(2\pi)^2\delta(p-p')G_\omega^0(p,z,z')+L_\omega e^{-i(p-p')\rho_0}G_\omega^0(p,z,z_0)G_\omega^0(p',z_0,z'), \tag{5}$$

$$L_\omega=\left(\int d^3rV(r)\right)\left[1-\int d^3rV(r)G_\omega^0(r_0,r)\right]^{-1}.$$

Here r_0 is the coordinate of the localized state, $V(r)$ is the localized potential of this state, and $G_\omega^0(r_0,r)$ is the unperturbed Green's function in the r representation.

The first term in (5) is responsible for direct tunneling through the barrier. When substituted into the expression for the supercurrent, it leads to a term which is exponentially small in comparison with the terms describing resonant tunneling. Ignoring it and assuming that we can set $\rho_0=0$ in (5) without any loss of generality, we find

$$G_\omega(p,p',z,z')=L_\omega G^0_\omega(p,z,z_0)G^0_\omega(p',z_0,z'). \quad (6)$$

Going through calculations like those described in the Appendix in Ref. 17, using the known expressions for the unperturbed Green's functions of an NIN contact with a rectangular barrier,[18] and incorporating limitations (3), we find the following expression for the amplitude for resonant scattering, L_ω:

$$L_\omega^{-1}=\frac{m}{2\pi}\left\{\frac{1}{\alpha_0}-\frac{1}{\alpha}-f(z_0)+i\left[\frac{\omega}{2\alpha(V-\mu)}+\frac{2\omega}{\sqrt{\omega^2+\Delta^2}}\sqrt{\frac{V-\mu}{\mu}}f(z_0)\right]\right\},$$

$$f(z)=\frac{\exp(-d/\alpha)}{2}\left\{\frac{\exp(2z/\alpha)}{(d/2)-z}+\frac{\exp(-2z/\alpha)}{(d/2)+z}\right\},\quad \alpha_0^{-1}=\sqrt{2m(V-E_0)}, \quad (7)$$

where E_0 is the resonant value of the energy of the localized state. The resonant-scattering amplitude L^n_ω, which determines the normal Green's function, is found from (7) by setting $\Delta=0$.

The function $f(z_0)$ in the real part of (7) determines the renormalization of the energy level of the localized state. This renormalization is of no fundamental importance to the discussion below, because of the averaging over the energy of the localized state in the final step of the J_c calculation. The imaginary part of L_ω^{-1} is responsible for the decay of the resonant state because of thermal excitations (the first term) and tunneling into the electrodes (the second term).

Substituting (6) and (7) into expression (4) for the supercurrent, we find a sinusoidal relationship between $J(\varphi)$ and the critical current:

$$J_c=-\frac{4eT\Delta^2}{(2\pi)^4}\sum_\omega L^n_\omega L_{-\omega}\left\{\int d^2p_1\int_{d/2}^{\infty}G^n_\omega(p_1,z_1,z_0)G_{-\omega}(p_1,z_1,z_0)dz_1\right\}$$

$$\times\left\{\int d^2p_2\int_{-\infty}^{-d/2}G^n_\omega(p_2,z_0,z_2)G_{-\omega}(p_2,z_0,z_2)dz_2\right\}. \quad (8)$$

Equation (8) must be averaged over the energy and the coordinates of the localized state. The first of these operations gives us

$$\langle L^n_\omega L_{-\omega}\rangle_{E_0}=\frac{(2\pi)^2}{\alpha m^3}n(E_0)\left\{\frac{\omega}{\alpha(V-\mu)}+2f(z_0)\sqrt{\frac{V-\mu}{\mu}}\left(\frac{\omega}{\sqrt{(\omega^2+\Delta^2)}}+\text{sign}\,\omega\right)\right\}^{-1}, \quad (9)$$

where $n(E_0)$ is the density of states of the localized state. Substituting (9) into (8), and taking an average over the coordinates of the localized state, we finally find

$$\langle J_c\rangle=\frac{8T\Delta^2}{e\alpha\rho_n}\sum_{\omega>0}\frac{1}{\omega^2+\Delta^2}\int_{-d/2}^{d/2}\frac{dz_0}{(\omega\Gamma_{\text{LS}}/2\pi T_c)+B\cosh(2z_0/\alpha)}, \quad (10)$$

$$\rho_n^{-1}=\frac{2e^2\pi n(E_0)n(z_0)}{md}\sqrt{\frac{V-\mu}{\mu}}\exp\left\{-\frac{d}{\alpha}\right\}, \quad (11)$$

$$B=1+\frac{\omega}{\sqrt{\omega^2+\Delta^2}}, \quad \Gamma_{\rm LS}=\frac{\pi T_c}{V-\mu}\sqrt{\frac{\mu}{V-\mu}}\frac{d}{2\alpha}\exp\left\{\frac{d}{\alpha}\right\}.$$

Here ρ_n is the part of the resistivity of the junction which is determined by tunneling processes, and $n(z_0)$ is the density of localized states. We see that the temperature dependence of the critical current depends on the dimensionless suppression parameter $\Gamma_{\rm LS}$, which is proportional to the ratio of the time scale for the decay of a state into localized states because of thermal activation and tunneling into the banks. At small values $\Gamma_{\rm LS} \ll 1$, the last of these processes is the governing factor, and expression (10) simplifies substantially:

$$\langle J_c \rangle = \frac{4\pi T\Delta^2}{e\rho_n} \sum_{\omega>0} \frac{1}{\sqrt{\omega^2+\Delta^2}(\omega+\sqrt{\omega^2+\Delta^2})}. \tag{12}$$

The normal resistance of the junction is governed exclusively by the tunneling of quasiparticles. Consequently, the channels for the normal and superconducting currents coincide, and the result of the Aslamazov–Larkin theory[19] follows from (12) as the temperature T approaches T_c. At low temperatures $T \ll T_c$, going over from a summation to an integration over ω in (12), we find

$$\langle J_c(0) \rangle = \frac{2\Delta(0)}{e\rho_n} = \frac{4}{\pi} J_c^{AB}(0), \tag{13}$$

where $J_c^{AB}(0)$ is the critical current of SIS contacts according to the Ambegaokar–Baratoff theory.[20]

As the parameter $\Gamma_{\rm LS}$ increases, the critical current is suppressed, and in the limit $\Gamma_{\rm LS} \gg \exp\{d/\alpha\}$, i.e., at

$$T_c \gg (V-\mu)\frac{\alpha}{d}\sqrt{\frac{V-\mu}{\mu}}, \tag{14}$$

the critical current is exponentially small:

$$\langle J_c \rangle = \frac{32T\Delta^2}{e\rho_n}(V-\mu)\sqrt{\frac{V-\mu}{\mu}} \sum_{\omega>0} \frac{1}{(\omega^2+\Delta^2)\omega} \exp\left\{-\frac{d}{\alpha}\right\}. \tag{15}$$

The temperature dependence of the critical current has been calculated numerically for arbitrary values of the suppression parameter $\Gamma_{\rm LS}$; the results are shown in Fig. 1. As $\Gamma_{\rm LS}$ increases, there is indeed a suppression of J_c. In contrast with suppression mechanisms which operate in structures with a proximity effect[21] (SNS, SNINS, and SNIS junctions), the $J_c(T)$ dependence remains smooth even at large values of $\Gamma_{\rm LS}$ at temperatures $T \leqslant 0.3T_c$. This $J_c(T)$ behavior is in reasonable agreement with the large body of experimental evidence which has been obtained on high-T_c junctions both at grain boundaries and in structures with interlayers of semiconducting oxides.[21] Also shown in Fig. 1 are experimental data from Ref. 7, found on $YBa_2Cu_3O_{7-\delta}/Y_{0.3}Pr_{0.7}Ba_2Cu_3O_{7-\delta}/YBa_2Cu_3O_{7-\delta}$ contacts for various layer thicknesses (25, 50, and 75 nm). Although there are substantial differences in the layer thicknesses, the differences in the experimental values of the product $J_c\rho_n$ are small,

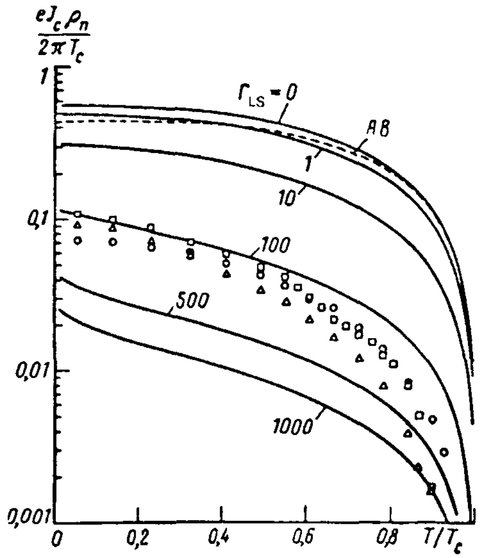

FIG. 1. Temperature dependence of the critical current of a Josephson junction with a localized state in an insulating layer according to calculations for various values of the suppression parameter Γ_{LS}. The points are experimental.

and the temperature dependence of this product agrees satisfactorily with the theoretical curves. It should also be noted that experimentally J_c and R_n are exponential functions of d,

$$J_c \propto \exp\{-d/\xi_n^*\}, \quad R_n \propto \exp\{-d/a\}, \quad \xi_n^* \approx a \approx 20 \text{ nm},$$

with approximately equal length scales in the exponential functions.

The model proposed in this paper is so far the only one capable of explaining the entire set of results observed in Refs. 1–7, and it gives a reasonable explanation of the long-range proximity effect.

A resonant tunneling through localized states in barriers is apparently also responsible for the properties of grain-boundary high-T_c junctions. Evidence for this assertion comes from not only the smooth $J_c(T)$ dependence predicted by this model but also Eqs. (11) and (15), which yield an explanation of the "scaling law," *i.e.*, the experimentally observed proportionality $J_c \propto \rho_n^{-2}$ between the critical current and the normal resistance of structures.

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[1]An expression for the critical current of SIS junctions incorporating a tunneling through single noninteracting localized states in a barrier has been derived only by the tunneling-Hamiltonian method, in fourth order in the transmission and under the assumption that a strong Coulomb repulsion of the electrons at

the localized state[12] suppresses the mechanism of resonant supercurrent flow in the structures. In the case under consideration here, this assumption contradicts the significant J_c values observed experimentally for high-Tc-superconductor structures. It should also be noted that the nature of the interaction of the electrons at a localized state depends strongly on the parameters of the layer material. In several cases it may even correspond to an effective attraction between electrons.[13,14] U. Kabasawa *et al.*, Jpn. J. Appl. Phys. **30**, 1670 (1991).

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