Spectrum and splitting of *p*-wave mesons

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A relativistic potential model is used to analyze the spectrum and fine structure. A mixing of light and heavy *p*-wave mesons is taken into account. A limiting transition to a Dirac equation with an infinitely heavy quark is analyzed. The role played by chiral effects for light mesons is discussed.

1. Simonov has proposed[1] a relativistic potential model based on a new nonperturbative approach which starts from the first principles of QCD. That approach uses a technique in which all the information on both the perturbative and nonperturbative dynamics of the model is embodied in an expectation value of a Wilson loop, $\langle W(c)\rangle$. The short-range contribution is written in the form of perturbative corrections. In lowest order in g^2, it yields the Coulomb potential. Long ranges enter the problem through irreducible correlation functions of gluon fields, $\langle\langle F(x_1)...F(x_n)\rangle\rangle$, which contribute a linear confinement potential $\sigma|\mathbf{r}|$ in any order in g. This approach has been used to derive a spin-zero Hamiltonian for a quark–antiquark system. An eigenvalue of this Hamiltonian yields the corresponding mass of the system:[2]

$$M_n(\mu_1,\mu_2)=\frac{m_1^2}{2\mu_1}+\frac{m_2^2}{2\mu_2}+\frac{\mu_1+\mu_2}{2}+\epsilon_n(\tilde{\mu})-c_0, \tag{1}$$

where the new parameters μ_1 and μ_2 play the role of dynamic masses. These parameters are found by minimizing the expression $M_n(\mu_1,\mu_2)$ with respect to the parameters μ_1 and μ_2 (Ref. 2). Here $\tilde{\mu}=\mu_1\mu_2/(\mu_1+\mu_2)$ is the reduced mass of the system, $\epsilon_n(\tilde{\mu})$ is the energy of the relative motion of the system, given by

$$\epsilon_n(\tilde{\mu})=\frac{p^2}{2\tilde{\mu}}+\sigma|\mathbf{r}|-\frac{4}{3}\frac{\alpha_s}{|\mathbf{r}|}, \tag{2}$$

where p is the momentum of the quark (or antiquark) in the c.m. frame of reference.

The condition that (1) be minimized with respect to μ_1 and μ_2 yields the following results for the dynamic masses:

$$\mu_1=\sqrt{p^2+m_1^2},\quad \mu_2=\sqrt{p^2+m_2^2}. \tag{3}$$

The mass of the system is

$$M_n=\sqrt{p^2+m_1^2}+\sqrt{p^2+m_2^2}+V(r), \tag{4}$$

where $V(r)$ is the quark–antiquark interaction, which generally includes a spin-dependent potential.

Expression (4) for the mass was derived by a nonperturbative approach[3] without an expansion of the root in the area law. [The error of the expansion in Ref. 2 was

$\sim 10\%$; it leads to the interaction $\sigma|\mathbf{r}|$ in (2).] However, expression (4) is not used in the present study because of some complications which arise in the numerical calculations. The problem is accordingly solved by the simplified scheme of Ref. 2, which essentially makes use of the mean-field approximation for $\mu_i(\tau)$, i.e., $\mu_i(\tau) \approx \mu_i$ $(i=1, 2)$, where τ is the time which has elapsed along the trajectory of the quark (or antiquark). The error of this approximation is[3] $\sim 5\%$. Accordingly, the eigenvalues $M_n(\mu_1, \mu_2)$ are first found as a function of μ_1 and μ_2, and then the expression $M_n(\mu_1, \mu_2)$ is minimized with respect to μ_1 and μ_2 with the given value of of $\epsilon_n(\tilde{\mu})$:

$$\left(-\frac{1}{2\tilde{\mu}}\frac{d^2}{dr^2}+\sigma|\mathbf{r}|-\frac{4}{3}\frac{\alpha_s}{|\mathbf{r}|}\right)\varphi_n(\mathbf{r})=\epsilon_n(\tilde{\mu})\varphi_n(\mathbf{r}). \tag{5}$$

In other words, μ_1^0 and μ_2^0 are found from the conditions $dM(\mu_1, \mu_2)/d\mu_i=0$ $(i=1, 2)$. Finally, the mass of the system is $M_n=M_n(\mu_1^0, \mu_2^0)$. Although a Schrödinger equation was used, the result found for the mass M_n is relativistic (since it works even in the case $m_1 \sim m_2 \sim 0$).

Current masses m_1 and m_2, a string tension σ, and a running coupling constant $\alpha_s(r)$ are introduced as initial parameters for the nonperturbative approach. As a result, a confinement dynamics is constructed for the spin-zero case. It leads to dynamic quark masses μ_1 and μ_2 and to a total mass $M_n=M_n(\mu_1^0, \mu_2^0)$ for the system. The mass M_n is classified on the basis of the angular momentum l and the radial main quantum number n_r.

2. The spin-dependent interaction is also analyzed by the method of vacuum correlation functions.[1]

Without resorting to a perturbative expansion in the parameter $1/\mu$, i.e., in the relativistic case, the following potential is derived (it is written here in standard Eichten–Feinberg form[4]):

$$\begin{aligned}V_{\mathrm{SD}}(r)=&\left(\frac{\sigma^{(1)}L^{(1)}}{4m_1^2}-\frac{\sigma^{(2)}L^{(2)}}{4m_2^2}\right)\left(\frac{1}{r}\frac{dV_0(r)}{dr}+\frac{2}{r}\frac{dV_1(r)}{dr}\right)\\&+\frac{\sigma^{(2)}L^{(1)}-\sigma^{(1)}L^{(2)}}{2m_1m_2}\cdot\frac{1}{r}\frac{dV_2(r)}{dr}+\frac{\sigma^{(1)}\sigma^{(2)}}{12m_1m_2}V_4(r)\\&+\frac{1}{12m_1m_2}[3(gs^{(1)}n)(\sigma^{(2)}n)-\sigma^{(1)}\sigma^{(2)}]V_3(r).\end{aligned} \tag{6}$$

This formula was derived in the potential case, in which the time scale of the motion of a quark, T_q, is much longer than the correlation length T_g which appears in the functions D and D_1: $T_q \gg T_g$ [D and D_1 determine the behavior of the bilocal correlation function $\langle\langle F(1)F(2)\rangle\rangle$; all the V_i are expressed in terms of these functions].

Since the perturbative contribution (P) and the nonperturbative contribution (NP) factorize in the lowest order of the cluster expansion, i.e., since

$\langle W(c)\rangle = \langle W\rangle_p \langle W\rangle_{\rm NP}$, the P and NP parts of the potential enter in an additive fashion. In other words, for any $i=0, 1, 2, 3, 4$ we have $V_i = V_i^{\rm P} + V_i^{\rm NP}$.

The perturbative part is

$$V_0^{\rm P} = -\frac{4}{3}\frac{\alpha_s}{r}, \quad V_1^{\rm P}=0, \quad \frac{dV_2^{\rm P}}{dr} = \frac{dV_0^{\rm P}}{dr} = \frac{4}{3}\frac{\alpha_s}{r^2}, \tag{7}$$

$$V_3^{\rm P} = \frac{4}{3}\frac{\alpha_s}{r^3}, \quad V_4^{\rm P} = -\frac{32\pi\alpha_s}{3}\delta^{(3)}(\mathbf{r}).$$

The nonperturbative part is

$$V_0^{\rm NP} = \sigma r, \quad \frac{dV_1^{\rm NP}}{dr} = -\frac{dV_0^{\rm NP}}{dr} = -\sigma,$$

$$\frac{dV_2^{\rm NP}}{dr} = O\left(\frac{1}{r}\right), \quad V_3^{\rm NP} \approx V_4^{\rm NP} = O\left(\exp\left(-\frac{r^2}{T_g^2}\right)\right). \tag{8}$$

For the nonperturbative part, (8), we use the asymptotic behavior at $r \gg T_g$. This approximation is valid since lattice calculations yield[5] $T_g \approx 0.2$–0.3 fm, and the size scale of the system is $r \approx 1$ fm.

Finally, in the p-wave case we find the following formulas for the masses:

$$m(^1P_1) = M_0, \quad m(^3P_2) = M_0 + a - 0.1c, \quad m(^3P_1) = M_0 - a + 0.5c,$$

$$m(^3P_0) = M_0 - 2a - c. \tag{9}$$

Here the center of the multiplet is

$$M_0 = \frac{\Sigma_{J=0}^2 (2J+1) m(^3P_J)}{\Sigma_{J=0}^2 (2J+1)} = m(^1P_1),$$

since the spin–spin interaction V_4 vanishes for the p wave. We have $M_0 = M_n$ from (1), and a is a radial spin–orbit matrix element for a resultant spin $\mathbf{S}_+ = \mathbf{S}_1 + \mathbf{S}_2$. In other words, $a \equiv \langle V_{LS+}\rangle c$ is a matrix element of tensor forces [$c \equiv \langle V_T\rangle$]. The calculations are carried out in the c.m. frame.

An off-diagonal spin–orbit matrix element leads to a mixing of states with identical parity, $^3P_1$ and $^1P_1$. As a result, $P_1^{\rm high}$ and $P_1^{\rm low}$ are physical states. The corresponding masses are

$$m(|P_1^{\rm high}\rangle, \quad |P_1^{\rm low}\rangle) = (1/2)\,[M_0 - a + 0.5c \pm \sqrt{(a-0.5c)^2 + 8d^2}], \tag{10}$$

where d is a spin–orbit matrix element for the spin difference $\mathbf{S}_- = \mathbf{S}_1 - \mathbf{S}_2$; i.e., we have $d \equiv \langle V_{LS-}\rangle$ "+" $P^{\rm high}$ and "−" $P^{\rm low}$.

A mixing does not occur in a system in which the current masses of the quarks are equal ($m_1 = m_2$), since in this case the dynamic masses of the quarks are equal ($\mu_1 = \mu_2$), so we have $d = 0$.

The mixing angle θ which appears in the matrix for the transformation from the old basis ($|^3P_1\rangle$, $|^1P_1\rangle$) to the new one ($|P_1^{\rm high}\rangle$, $|P_1^{\rm low}\rangle$), in the form

$$R=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

is given by

$$\sin\theta=\left(\frac{m(^1P_1)-m(P^{\text{low}})}{m(P_1^{\text{high}})-m(P_1^{\text{low}})}\right)^{1/2}=\left(\frac{m(^3P_1)-m(P_1^{\text{high}})}{m(P_1^{\text{low}})-m(P_1^{\text{high}})}\right)^{1/2}. \tag{11}$$

3. Working from Eqs. (9)–(11), we can easily take the limit which leads to the Dirac equation, i.e., $m_2/m_1 \ll 1$. In this limit we have $a=d$ and $c=0$. Accordingly, after the mixing we have only the two levels $M=M_0+a$ and $M=M_0-2a$, each of which is doubly degenerate. Depending on the sign of a, we have two regimes:

(1) $a>0$, $\sin\theta=\sqrt{2/3}$, $m(^3P_2)>m(^3P_0)$. After a mixing in the lowest state P_1^{low}, the state $^3P_1$ is predominant. This state is degenerate with $^3P_0$. Correspondingly, P_1^{high} is dominated by the fraction $^1P_1$, which is degenerate with $^3P_2$. In other words, in this case we have a Coulomb order of levels: $m(^3P_2)>m(^1P_1)>m(^3P_1)>m(^3P_0)$.

(2) $a<0$, $\sin\theta=-\sqrt{1/3}$, $m(^3P_0)>m(^3P_2)$. After a mixing in the lowest state, P_1^{low}, the state $^1P_1$ is predominant, while in P_1^{high} the state $^3P_1$ is correspondingly predominant. In other words, we have $m(^3P_0)>m(^3P_1)>m(^1P_1)>m(^3P_2)$.

For arbitrary a there is accordingly a degeneracy between states with the following quantum numbers: The state which is degenerate with $^3P_2$ is mainly a $^1P_1$ state, while the state which is degenerate with $^3P_0$ is mainly a $^3P_1$ state. This conclusion agrees with the solution of the Dirac equation with degenerate levels: $^3L_{L+1}$ with ^1L_L and $^3L_{L-1}$ with ^3L_L, where L is the orbital angular momentum of the system.

4. Numerical calculations of the masses for D, D_S, B, B_S, and K mesons have been carried out on the basis of Eqs. (9)–(11), derived above. The following behavior was found. With mixing being taken into account, all the mesons have a Coulomb order of levels; i.e., $m(^3P_2)>m(P_1^{\text{high}})>m(P_1^{\text{low}})>m(^3P_0)$. For all systems, the spin–orbit component of the spin-dependent potential outweighs other components. As a result, the spin of the orbital, $a\equiv\langle V_{LS+}\rangle$, determines the relative positions of levels, as in the case of the Dirac equation ($a>0$ gives the Coulomb arrangement of levels). For the B and B_S mesons, we find, within the error of the calculations, a $^3P_2 - P_1^{\text{high}}$ level degeneracy and a $^3P_0-P_1^{\text{low}}$ level degeneracy, in agreement with the degeneracy in the case of the Dirac equation.

The masses for the K meson are slightly high in comparison with the experimental data (~ 100 MeV). In part, this discrepancy can be attributed to our use of only the asymptotic value at $r\gg T_g$ for the nonperturbative part. This approach leads to purely perturbative tensor forces (i.e., $c\equiv\langle V_T\rangle$ is determined exclusively by the Coulomb interaction). However, the primary reason why the mass is too high is apparently our neglect of a new dynamic regime: a regime of quark zero modes, which leads to a breaking of chiral symmetry.[8] These effects may be important specifically in the

case of K mesons, since the spin degrees of freedom, which are important in the case of light quarks, contribute a nonzero density of quasizero quark modes.[8]

The masses and splittings of the other mesons coincide, within the errors, with the corresponding values in the literature[7] and with experimental data. (At the moment there are no comprehensive experimental data for B and D mesons.)

Possible sources of errors are the approximations made in the derivation of the potentials (the asymptotic value for the Wilson loop and the approximation of locality for the spin-dependent potentials) and our neglect of various effects: 1) the breaking of chiral symmetry; 2) the interaction of mesons with their own decay channels (both open and closed). The second of these factors, however, does not introduce any substantial correction,[6] while the first factor and the incorporation of a nonlocality lead to an average error of ± 50 MeV for mesons containing light quarks and ± 25 MeV for systems of heavy quarks.

Within the error ($\sim 12\%$), the mixing angle agrees with values in the literature.[6,7] For the D_S and B_S mesons we have $\theta \approx 45°$. This figure means that the $^3P_1$ and $^1P_1$ states are represented equally in P_1^{high} and P_1^{low} ($\theta = 41°$; Refs. 6 and 7). For the D and B mesons we have $\theta \approx 55°$, which corresponds to $\sin\theta = \sqrt{2/3}$. Consequently, the situation with regard to these systems is close to the limit $m_2/m_1 \ll 1$ of this Dirac equation ($\theta \approx 48°$; Refs. 6 and 7). For the K meson we have $\theta = 28°$, which agrees, within the calculation errors, with values in the literature[6,7] ($\theta = 34°$).

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