Transverse shift of a focal spot due to switching of the sign of circular polarization

N. B. Baranova, A. Yu. Savchenko, and B. Ya. Zel'dovich
Nonlinear Optics Laboratory, Technical University, 454080 Chelyabinsk, Russia

(Submitted 5 January 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **59**, No. 4, 216–218 (25 February 1994)

An axially symmetric optical lens made of isotropic material produces a transverse shift $\Delta x \propto \sigma\lambda/2\pi$ of the focal spot "center of gravity" under the switch of circular polarization sign $\sigma = \pm 1$ of a plane incident wave $(\hat{e}_z + i\sigma\hat{e}_y)\exp(ikz)$. This can be accomplished if most of the upper half ($y>0$) of the input plane is illuminated. This simple effect is discussed in the general context of the spin–orbit interaction of the photon.

Light propagation even through a locally isotropic medium certainly influences the polarization state. The simplest example is the trivial change of polarization direction $\mathbf{b}_{\text{old}} \to \mathbf{b}_{\text{new}}$ connected with the refraction $\mathbf{s}_{\text{old}} \to \mathbf{s}_{\text{new}}$ of the propagation direction unit vector $\mathbf{s}$. Even if the Fresnel reflection is ignored (it can be made small if the refractive index varies smoothly), the transverse character of the electromagnetic wave should be retained, so that the minimum change would be $\mathbf{b}_{\text{new}} \propto \mathbf{b}_{\text{old}} - \mathbf{s}_{\text{new}}(\mathbf{b}_{\text{old}} \cdot \mathbf{s}_{\text{new}})$ (see Refs. 1 and 2). An inverse effect—the influence of (circular) polarization on the ray's trajectory—was predicted and observed experimentally as the "optical magnus effect"[3–5] (see also Refs. 6 and 7).

At first glance, in a vacuum (or in isotropic homogeneous medium) the propagation and polarization are disconnected: a plane wave $\mathbf{b}\exp(ik\mathbf{s}\cdot\mathbf{r})$ holds constant its polarization vector $\mathbf{b}$ in the entire 3D space. However, if one works with a superposition of many plane wave components, as in the case of a converging wave with a spherical wavefront, one cannot satisfy the transversality condition $(\mathbf{b}\cdot\mathbf{s})=0$ by a single vector $\mathbf{b}$ for all the constituents. In the present letter we examine the peculiar effects of interference of the truly 3D vector electromagnetic fields.

We start with the approximate solution of the Helmholtz equation: Hermit–Gaussian beams (see, e.g., Ref. 8)

$$M_{00}(x,y,z) = \frac{1}{(1+iz/z_0)} \exp\left[ikz - \frac{x^2+y^2}{2a_0^2(1+iz/z_0)}\right], \tag{1}$$

$$M_{01}(x,y,z) = \frac{y}{a(z)} M_{00}(x,y,z)\exp\left(-i\arctan\frac{z}{z_0} - i\frac{\pi}{2}\right). \tag{2}$$

Here $a_0 = \Delta r(\text{HWe}^{-1}\text{M})$ is the half-width of the M_{00}-beam focal waist by the criterion e^{-1} of the intensity at a maximum, $z_0 = ka_0^2 = \Delta z(\text{HWHM})$ is the waist length, and $a(z) = a_0(1+z^2/z_0^2)^{1/2}$ is the z-dependent radius of the M_{00}-beam.

A superposition

$$M(x,y,z) = M_{00}(x,y,z) + \alpha M_{01}(x,y,z), \tag{3}$$

with a real value of $\alpha > 0$ describes the beam with the illumination mostly in the upper half ($y > 0$) of the space at the input: at $z \to -\infty$

$$I(x,y,z) \propto \left[1+\alpha^2 \frac{y^2}{a^2(z)} + 2\alpha \frac{y}{a(z)}\right] \exp\left[-\frac{x^2+y^2}{a^2(z)}\right]. \tag{4}$$

After the passage through the focal waist ($z \to +\infty$) the beam is localized mostly in the lower half of the space, $y < 0$.

Approximate expression for the 3D vector field corresponding to the scalar beams (1)–(3) can be written in the form

$$\mathbf{E}(x,y,z) = \mathbf{b}M(x,y,z) + \frac{i}{k}\hat{\mathbf{e}}_z \cdot \left(b_x \frac{\partial M}{\partial x} + b_y \frac{\partial M}{\partial y}\right), \tag{5}$$

where $\mathbf{b} = b_x\hat{\mathbf{e}}_x + b_y\hat{\mathbf{e}}_y$ is a constant polarization vector in the (x,y) plane. Expression (5) guarantees that $\mathrm{div}\mathbf{E} = 0$ with the required accuracy. Let us now consider circular polarizations, $b_x = 1$, $b_y = i\sigma$, with $\sigma = \pm 1$. Each individual mode—M_{00} and M_{01}—possesses x-symmetric distribution of total intensity $I + |E_x|^2 + |E_y|^2 + |E_z|^2$ at the focal plane $z = 0$:

$$I_{00}(x,y,0) = I_{00}(-x,y,0), \quad I_{01}(x,y,0) = I_{01}(-x,y,0). \tag{6}$$

Their superposition contains $|E_x|^2$ and $|E_y|^2$ components of the intensity, which are still symmetric with respect to $x \to -x$. Finally, just the intensity of the "forbidden" z-component of the field $|E_z|^2$ contains the interference term $\propto \sigma\alpha$, which is antisymmetric relative to $x \to -x$:

$$|E_j(x,y,0)|^2 = \exp\left(-\frac{x^2+y^2}{a_0^2}\right) f_j(x,y,0); \tag{7}$$

$$f_x = f_y = 1 + \alpha^2 \frac{y^2}{a_0^2}, \tag{8}$$

$$f_z = \left[\frac{a_0^2 y^2 + a_0^2 x^2 + \alpha^2 (y^2 - a_0^2)^2 + \alpha^2 x^2 y^2}{a_0^4} + \frac{2\sigma\alpha x}{a_0}\right] (ka_0)^{-2}. \tag{9}$$

For example, at the line $y = z = 0$ and for $\alpha = 1$ the profiles are $f_x = 1$, $f_y = 1$, $f_z = (1 + \sigma x/a_0)^2/(ka_0)^2$. We see that the shift of the "forbidden" component distribution $|f_z|^2$is large, $\Delta x(\ |E_x|^2) \sim \sigma a_0$, i.e., about the size of the radius of the focal waist. However, the total fraction of the "intensity" connected with $|E_z|^2$ is small. It is determined by the angular width $(ka_0)^{-1} = \theta_0 = \Delta\theta$ (HWe^{-1}M) of the M_{00} Gaussian mode: this fraction is about $\theta_0^2 = (ka_0)^{-2}$. For $\theta_0 \sim 1$ rad the above-mentioned fraction becomes about 1, and the shift is $\Delta x \sim \sigma\lambda/2\pi \sim a_0$.

It should be emphasized that the observation of such a shift by looking at the focal waist through a microscope is impossible. The reason is that at the stage of the divergent wave propagation from the focal plane toward the microscope objective lens the effect of the opposite sign identically nullifies that shift. This means that some depolarizing or phase-randomizing luminescent molecules or atoms must be placed in the focal plane. The use of dichroic luminophores with a considerably enhanced absorbtion of the "forbidden" E_z-polarization would be especially convenient.

In summary, we have demonstrated that there are at least two types of (circularity sign) σ-dependent effects of spin–orbit interaction of a photon. One of them is the optical magnus effect, which appears in a medium with inhomogeneous refractive index and hence with the "curved" rays; it can be accumulated in a suitable geometry of the path and is the effect which is the inverse of the Berry's phase. The other effect is the one suggested in this letter; it exists even in a vacuum and occurs as a result of the interference of various vector constituents of a spherical wave consisting of "straight" individual rays. We are now checking the hypothesis that all σ-dependent phenomena in a locally isotropic, smoothly inhomogeneous medium can be generally reduced to the two effects mentioned above.

[1] M. Born and E. Wolf, *Principles of Optics,* Pergamon Press, Oxford, 1987.
[2] Y. A. Kravtsov and Y. I. Orlov, *Geometrical Optics of Inhomogeneous Media,* Springer-Verlag, Berlin, 1990.
[3] B. Ya. Zel'dovich and V. S. Liberman, Sov. J. Quantum Electron. **20**, 427 (1990).
[4] A. V. Dooghin, N. D. Kundikova, V. S. Liberman, and B. Ya. Zel'dovich, Phys. Rev. A **45**, 8204 (1992).
[5] V. S. Liberman and B. Ya. Zel'dovich, Phys. Rev. A **46**, 5199 (1992).
[6] C. Imbert, Phys. Rev. D **5**, 787 (1972).
[7] O. Costa de Beauregard and C. Imbert, Phys. Rev. Lett. **28**, 1211 (1972).
[8] A. Gerard and J. M. Birch, *Introduction to Matrix Methods in Optics,* A. Wiley-Interscience Publication, 1975.

Published in English in the original Russian journal. Reproduced here with the stylistic changes by the Translations Editor.