Spin splitting of magnetoresistance oscillations and quantum Hall effect in $Ge/Ge_{1-x}Si_x$ superlattices in an oblique magnetic field

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A substantial change in the pattern of magnetoresistance oscillations and the quantum Hall effect upon a change in the orientation of the magnetic field with respect to the axis of the heterostructure has been observed in multilayer p-$Ge/Ge_{1-x}Si_x$ heterostructures ($x \simeq 0.3$). The effects observed result from the complex structure of the spectrum of the 2D hole gas. They are associated with a transformation of this spectrum in an oblique magnetic field.

In highly stressed multilayer $Ge/Ge_{1-x}Si_x$ heterostructures ($x=0.17$) with a p-type conductivity in the Ge layers, the magnetoresistance oscillation pattern $\rho_{xx}(B)$ is determined completely by the magnetic field component perpendicular to the planes of the layers,[1] $B_z = B\cos\theta$. The condition of a high stress means that the splitting (Δ) of the edge of the degenerate valence band Γ_8 is higher than the characteristic hole energy ϵ_F: $\Delta > \epsilon_F$. In this paper we are reporting a study of the $\rho_{xx}(B)$ oscillations and also the Hall resistance $\rho_{xy}(B)$ in an oblique magnetic field $B=\{0, B_y, B_z\}$ in a $Ge/Ge_{1-x}Si_x$ sample ($x=0.03$) with a relatively low stress in the Ge layers: $\Delta \simeq \epsilon_F$. The hole concentration found from the Hall coefficient in a weak magnetic field is $p=2.6\times10^{11}$ cm^{-2}, and the mobility is $\mu_p=1.4\times10^4$ cm^2/(V·s). Measurements were carried out at $T=1.7$ K in steady-state magnetic fields up to 12 T.

Figure 1 shows plots of $\rho_{xx}(B)$ and $\rho_{xy}(B)$ for the test sample for the case $B_y = B\sin\theta = 0$. In relatively weak magnetic fields (at large indices of the Landau levels), we worked from the period of the ρ_{xx} oscillations to determine the hole concentration: $p=3\times10^{11}$ cm^{-2}. This figure agrees well with the figure found from the Hall effect. From the temperature dependence and field dependence of the oscillation amplitude we found the effective cyclotron mass of the holes, $m=0.11m_0$, and the Dingle temperature, $T_D=3.6$ K. For the width of the Landau levels we have the estimate $\Gamma \simeq kT_D = 0.3$ meV, which corresponds in order of magnitude to the estimate found from the relation $\Gamma \simeq \hbar/\tau$ ($\Gamma \simeq 0.7$ meV). Here τ is the hole relaxation time found from the mobility: $\mu_p = e\tau/m$.

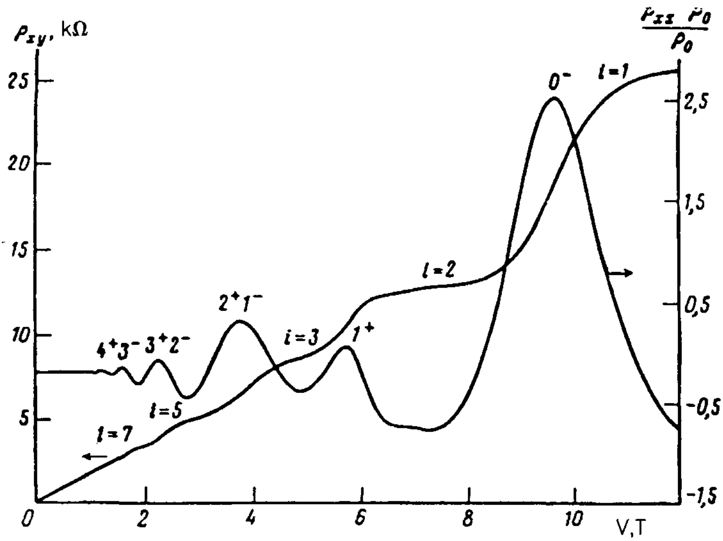

FIG. 1. The longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} versus the magnetic field B at $\theta=0$.

On the $\rho_{xy}(B)$ curve we can clearly see some plateaus, which are characteristic of the quantum Hall effect:

$$\rho^{i}_{xy}=h/ie^2, \tag{1}$$

where $i=p/(eB_i/hc)$ is the number of completely filled Landau levels in fields greater than B_i, and eB/hc is the degree of degeneracy of the Landau level with a given spin projection. Comparison of the $\rho_{xx}(B)$ and $\rho_{xy}(B)$ curves leads to an unambiguous conclusion regarding the degree of degeneracy in terms of the spin of the Landau levels. Since plateaus with occupation numbers $i=1$, 2, 3, 5, 7, etc., are seen on the $\rho_{xy}(B)$ curve, the first two oscillation peaks (on the strong-field side), with 0^- and 1^+, are spin-split, while the other peaks are merged in pairs: $(1^-, 2^+)$, $(2^-, 3^+)$, etc. The numbers $N=0$, 1, 2,... specify the Landau levels, and the plus and minus signs specify the spin direction. The merging of peaks with different spin directions from neighboring Landau levels corresponds, for the case of a simple band, to the case in which the spin splitting is more than half the orbital splitting.

Figure 2 shows an increase in the splitting of the $(2^+, 1^-)$ peak, which is degenerate at $\theta=0$, with increasing value of the angle θ. This figure shows fragments of the curves of ρ_{xx} and ρ_{xy} as a function of B_z for various values of θ. In the interval 0–42°, the $(2^+, 1^-)$ peak gradually decreases in amplitude, and at $\theta>48°$ it splits in two: into a 1^- peak and a 2^+ peak [the splitting of the peaks can be seen well on the curve of $d^2\rho_{xx}(B_z)/dB_z^2$]. At $\theta=54°$, the spin-split 0^-, 1^+, 1^-, and 2^+ peaks are equidistant along the scale of the reciprocal field; at fields $B_z>2.5$ T the oscillation period doubles. The splitting of the $(2^+, 1^-)$ peak increases at $\theta>48°$, because the 2^+ component of the "doublet" shifts toward lower fields, while there is no change

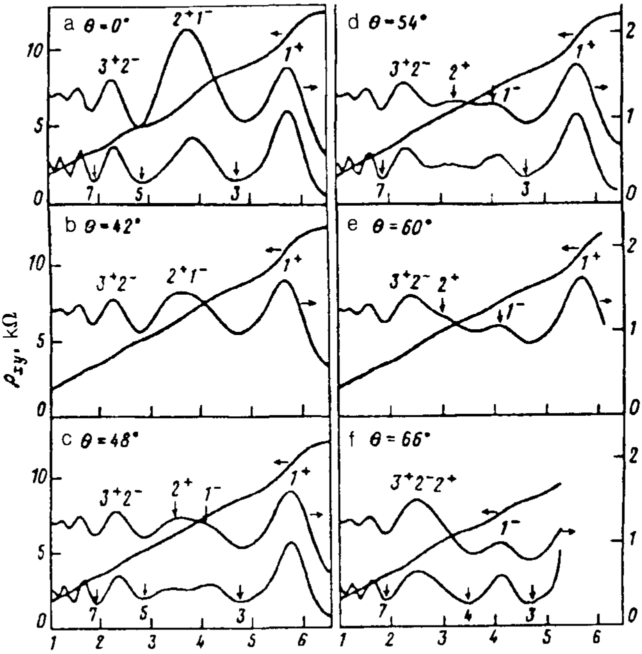

FIG. 2. ρ_{xx} and ρ_{xy}, along with the derivative $d\rho_{xy}/dB_x$ (the lower curves in frames a, c, d, and f), versus B_z for various values of θ. The minima of $d\rho_{xy}/dB_z$ are labeled in accordance with the numbers of the plateaus in the quantum Hall effect.

in the position of the other component, 1^-. At $\theta \simeq 66°$, the 2^+ component merges with the unsplit $(3^+, 2^-)$ peak, forming a sort of triplet (the 3^+, 2^-, and 2^+ peaks are not resolved, even on the plot of the second derivative of the magnetoresistance versus B_z). At sufficiently high indices of the Landau levels, $N>3$, i.e., at fields at which there is no spin splitting, the oscillation picture in the plot of ρ_{xx} versus B_z remains essentially the same as θ is increased.

The identification of oscillation peaks offered above corresponds to the structure of the quantum Hall effect. Since the Hall plateaus are not always clearly defined, Fig. 2 shows plots of $d\rho_{xy}/dB_z$, in addition to $\rho_{xx}(B_z)$ for certain values of θ (0°, 48°, 54°, and 66°). The minima of this derivative correspond to fields B_z^i in which the Hall resistance reaches the values ρ_{xy}^i given by relation (1). We see in this figure that at $\theta = 48°$, as at $\theta = 0$, there are some well-defined plateaus with $i = 3$, 5, and 7 in fields $2 < B_z < 6$ T, although on the derivative $d\rho_{xy}/dB_z$ we can also see a poorly defined minimum corresponding to $i = 4$. For $\theta = 54°$, plateaus with $i = 3$ and 7 are well defined; there are also some plateaus with $i = 4$ and 5, not as well defined. Finally, at $\theta = 66°$, the only plateaus are those with occupation numbers $i = 3$, 4, and 7. The plateaus with $i = 5$ and 6 are thus missing. Such a situation would be possible only as a result of a merging of oscillation peaks from three Landau levels.

A clear idea of the characteristic values of i for each value of θ can be obtained

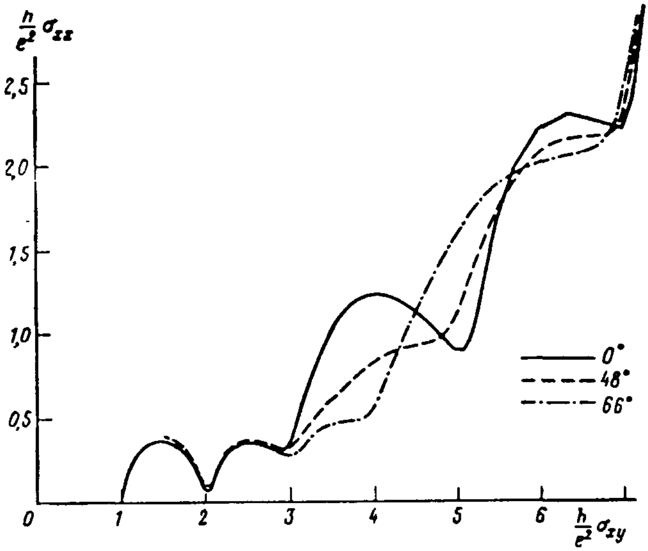

FIG. 3. Relationship between the longitudinal component σ_{xx} and the Hall component σ_{xy} of the conductivity for various values of the angle θ.

from a plot of σ_{xx} versus σ_{xy} in units of e^2/h (Fig. 3). The minima of σ_{xx} correspond to those values of $i=\sigma_{xy}/(e^2/h)$ at which there are plateaus on the curve of the Hall resistance. We see in Fig. 3 that, as θ is varied, the $\sigma_{xx}(\sigma_{xy})$ curve is transformed only in weak magnetic fields, corresponding to the interval $3<i<7$; i.e., there is a regrouping of Landau levels with increasing θ in this region of occupation numbers.

For a 2D gas of electrons with a simple dispersion relation, the cyclotron frequency is determined by exclusively that component of the magnetic field which is normal to the plane of the 2D layer (B_z), while the spin splitting is determined by the magnitude of the total magnetic field,[2] $|B|$. For a 2D hole gas of the Γ_8 valence band, on the other hand, in the case of a strong, uniform compression of the 2D layer, both the orbital splitting and the spin splitting depend on B_z alone.[3] In both cases, the energies of the Landau levels of the 2D system are linear in the magnetic field and are equidistant.

Calculations of the spectrum of carriers of the Γ_8 band under spatial-quantization conditions predict that in the absence of a stress,[4] or for a stress which is not too high,[5,6] the dispersion relation for holes in the lower spatial subbands is very nonparabolic. This situation may lead to a nonlinear dependence of the energies of the Landau levels on the magnetic field. Indeed, perturbation-theory calculations for a sample in an oblique magnetic field show[7] that the energies of the Landau levels are nonlinear functions of both B_z and B_y, and that the extent of the nonlinearity depends strongly on the index of the Landau level, N. As a result, there may be a crossing of levels with different values of N; i.e., points of a random degeneracy of Landau levels may arise at certain magnetic fields. The values of these fields are extremely sensitive to changes in external parameters (e.g., the orientation of the magnetic field). This circumstance is undoubtedly pertinent to both the splitting of the (2^+1^-) peak as θ is varied and the observation of the $(3^+2^-2^+)$ "triplet" at $\theta > 66°$.

In summary, the transformation of the pattern of magnetoresistance oscillations and of the quantum Hall effect observed in p-Ge/Ge$_{1-x}$Si$_x$ heterostructures with $x \simeq 0.3$ as the orientation of the magnetic field is varied reflects the particular nature of the spectrum of Landau levels of the 2D gas of holes of the complex valence band of Ge (the Γ_8 band).

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