Quantum Hall effect in multilayer p-Ge/$Ge_{1-x}Si_x$ heterostructures and energy spectrum of the 2D hole gas in a magnetic field

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The quantum Hall effect and the structure of magnetoresistance oscillations observed in multilayer p-Ge/$Ge_{1-x}Si_x$ heterostructure systems are analyzed on the basis of a picture of magnetic levels of the Ge valence band calculated from the model of an infinitely deep square quantum well. The odd numbers of the plateaus in the quantum Hall effect in weak magnetic fields and several features of the magnetoresistance oscillations are explained in terms of an involvement of a second subband of a spatial quantization of heavy holes in samples with relatively wide conducting layers and also in terms of a crossing of subband levels.

The quantum Hall effect has been observed and studied in detail in selectively doped, periodic, multilayer p-Ge/$Ge_{1-x}Si_x\langle B\rangle$ heterostructures with a high hole mobility.[1,2] Certain samples exhibit only odd-numbered plateaus of the quantum Hall effect in relatively weak magnetic fields, while other samples exhibit only even-numbered plateaus. For a simple parabolic band, this situation corresponds to different relations between the magnitudes of the spin and orbital splittings: Odd-numbered plateaus should be observed when the spin splitting is more than half of the orbital splitting, and even-numbered plateaus should be observed in the opposite case. However, the fact that both situations are observed in samples of the same material, along with the irregular changes observed in the width of the plateaus with increasing order number in a series of a given parity,[3] indicates that an analysis on the basis of a simple band is not sufficient.

Representative experimental results are shown in Figs. 1 and 2, at the top, for sample 1123ab (sample A), with $x=0.03$, and sample 1006-1 (sample B), with $x=0.10$. The internal uniaxial deformation in the 3D case for these values of x would correspond to a splitting of the valence band by respectively 10 and 20 meV. The Ge conducting layers and the $Ge_{1-x}Si_x$ barriers should, on the basis of the growth pro-

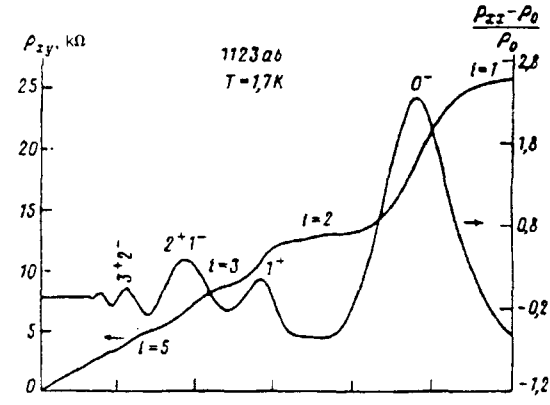
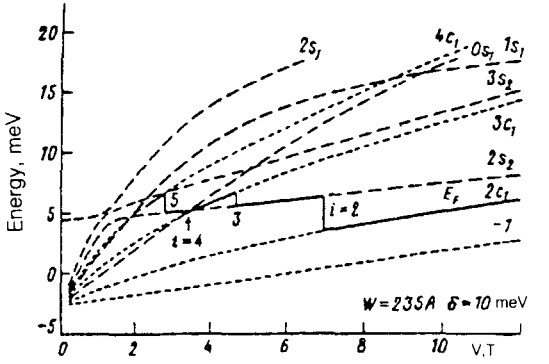


FIG. 1.



cedure, be of essentially the same thickness: 200 Å for each layer in the case of sample A, with 15 repetitions, and 125 Å for sample B, with 90 repetitions. The hole concentration p in the Ge layers is about 3×10^{11} cm^{-2} for both samples according to measurements of the Hall coefficient in a weak field and according to the period of the oscillations in the magnetoresistance. The mobility at 4.2 K is $(1.0–1.4)\times10^4$ cm^2/(V·s). Measurements were carried out in steady-state and pulsed magnetic fields. On the $\rho_{xy}(B)$ curves there are some plateaus of the quantum Hall effect, which correspond to values $\rho_{xy}^i = h/ie^2$ per conducting layer; here i is the number of completely filled Landau levels to the left of B_i. Since a plateau with $i=1$ is observed in both samples, we can draw the unambiguous conclusion that the Ge layers are isolated from each other and that all participate in the current transport in such a manner that the heterojunctions forming each of the potential wells do not divide the 2D hole gas in a well into two components.

Sample A has plateaus with order numbers $i=1$, 2, 3, 5, and 7. For sample B we can see plateaus with $i=1$ and 2 clearly, and we can also discern a plateau with $i=4$. This sequence of numbers corresponds to a periodic arrangement of the magnetoresistance minima along the scale of the reciprocal magnetic field. In the second derivative of the magnetoresistance we can also identify a poorly defined minimum with $i=6$. Those plateaus whose order numbers differ by 1 are associated with nondegen-

erate magnetic levels; those whose order numbers differ by 2 are associated with doubly degenerate levels. The magnetoresistance peaks in these figures have labels of 0, 1, 2, ..., which specify the Landau levels of a simple band; the plus or minus sign specifies the spin projection. As was mentioned previously, the coalescence of the peaks corresponding to different spin projections of neighboring Landau levels for sample A and the fact that in this field range we see only those plateaus of the quantum Hall effect which have odd numbers might mean that the spin splitting is greater than half the orbital splitting. However, the even numbers of the features in the case of sample B indicate the opposite relation.

We calculated the structure of the Ge valence band, quantized by a magnetic field, in the model of an infinitely deep square quantum well, as was done in Ref. 4 (see the results in the lower parts of Figs. 1 and 2). As the Luttinger parameters we used[5] $\gamma_1=13.38$, $\gamma\equiv(\gamma_2+\gamma_3)/2=4.965$, and $\kappa=3.41$. The magnetic levels in the spatially quantized valence band are grouped in two independent sets, labeled *c* and *s*. Here we might draw an analogy with the 3D case, in which, for a zero projection of the wave vector onto the magnetic field, k_H, the levels are again grouped in two series, *a* and *b*. However, since we are initially assuming $k_H\neq 0$ in the 2D case (this assumption follows from the imposition of the boundary conditions along the magnetic field), we cannot establish an unambiguous correspondence between these series. Only the lowest level (-1) is of the same nature, $b(-3/2)$, in two and three dimensions. In the designations of the levels, the first number $(-1, 0, 1, \ldots)$ specifies the level number, while the subscript $(1, 2, \ldots)$ on the letter specifies the order number of the subband of the spatial quantization of heavy holes. Within a single series, the magnetic levels with identical numbers belonging to different spatial-quantization subbands repel each other ($2s_1$ and $2s_2$ in the figures).

In the case of an extremely strong stress, and also in very thin layers, in which case the splittings of the band due to the deformation and the spatial confinement are both significantly greater than the Fermi level, all the Landau levels of the valence band are equidistant, and they shift in a linear fashion with the magnetic field.[6] At a small deformation (or in the absence of deformation), on the other hand, and also in relatively wide layers, there is a complex valence-band spectrum. The Landau levels are not equidistant, and they are nonlinear functions of the magnetic field. In this case, as we see in the figures, it is difficult to draw any analogies with the simple band. The structure of the band becomes more complicated because of the mixing of states of heavy and light holes which occurs with increasing magnetic field, as well as the mixing of states of different spatial-quantization subbands. The nature of the mixing depends on the magnetic number of the levels, and it differs in sets *c* and *s*. Accordingly, the crossings of the magnetic levels of different subbands are accompanied by self-crossings of levels belonging to a common subband.

The formation of a plateau on the plot of $\rho_{xy}(B)$ corresponds to passage of the Fermi level through a region of localized states between Landau levels. For the analysis we used an idealized model in which the middle of a $\rho_{xy}(B)$ plateau and a minimum of the magnetoresistance correspond to a jump in the Fermi level to the neighboring level. Although at a constant carrier concentration the *positions* of the quantum-Hall-effect plateaus along the magnetic field scale and the positions of the

corresponding magnetoresistance minima, which are determined entirely by the formula $B_i=(h/e)(p/i)$, are insensitive to the pattern of levels in the ideal case, the *sizes* of the plateaus and the *amplitudes* of the minima do depend on the relative positions of the levels, and this dependence is strong. If the lowest unfilled level in a field B_i (with order number $i+1$) comes so close to the uppermost filled level (with order number i) that the gap of localized states between the two levels collapses, then the corresponding plateaus of the quantum Hall effect and minima of the magnetoresistance disappear. For sample A this is the situation, in particular, at $i=4$ and with a potential well of width 235 Å. In this case, in a field $B_4=3.5$ T, the Fermi level passes through a simultaneous intersection of three levels ($2s_2$, $3c_1$, and $0s_1$). Of these three levels, the two which lie to the left of B_4 are filled (Fig. 1).

The disappearance of the plateaus from the plot of ρ_{xy} and the disappearance of the ρ_{xx} minimum with $i=4$ in the case of sample A result from the superposition of magnetic level $2s_2$ of the second heavy-hole subband on a level of the first subband. As a result, the order numbers of the structural features with $i>4$ become one higher than in the absence of the second subband, and the odd series turns out to be predominant. Working from the analogy with a simple band, we conclude that this situation is possible if the spin splitting is greater than half the orbital splitting, although the picture of levels of the first subband, considered separately, corresponds to a small spin splitting. Aronzon *et al.*[3] have suggested that a second subband of a spatial quantization of holes has an effect in corresponding heterostructure systems. Estimates of the effect of a second subband, which are derived without calculations of the scheme of magnetic levels, must be approached with caution, since the Fermi level is not fixed in a magnetic field.

Since the distance between spatial-quantization levels is inversely proportional to the square width of a well, for sample B, with the considerably narrower Ge layers, the second subband is higher than in sample A, and it does not participate in current transport (Fig. 2). Correspondingly, for sample B at large quantum numbers we see only those features which have even numbers, since the Fermi level moves through levels which are grouped in pairs $[Nc_1, (N+3)s_1]$, in which the even-numbered levels outweigh the odd-numbered ones, and the overall picture of the magnetoresistance is more regular. On the other hand, we can see a fine structure near the second minimum of the second derivative of the magnetoresistance of sample B. This fine structure may be due to a poorly defined step with $i=3$ or, possibly, a crossing of the Fermi level from the level $3c_1$ to the level $0s_1$ near their intersection between fields B_2 and B_3 as the magnetic field is reduced.

When the Fermi level moves through the second spatial-quantization subband, the structure of the quantum Hall effect and the magnetoresistance should be very sensitive to parameters of the potential well and the material. For sample A, for example, even a small decrease in the width of a well leads to radical changes in the ratios of the heights of the steps with $i=3$–5 (Fig. 1). As a result, a plateau with $i=4$ should appear, while the plateaus with $i=3$ and 5 should shrink sharply. The minima of the magnetoresistance should undergo corresponding changes. Consequently, the well width of 235 Å, at which we observe the best agreement between the calculated picture of levels and the experimental data, allows only small deviations within the

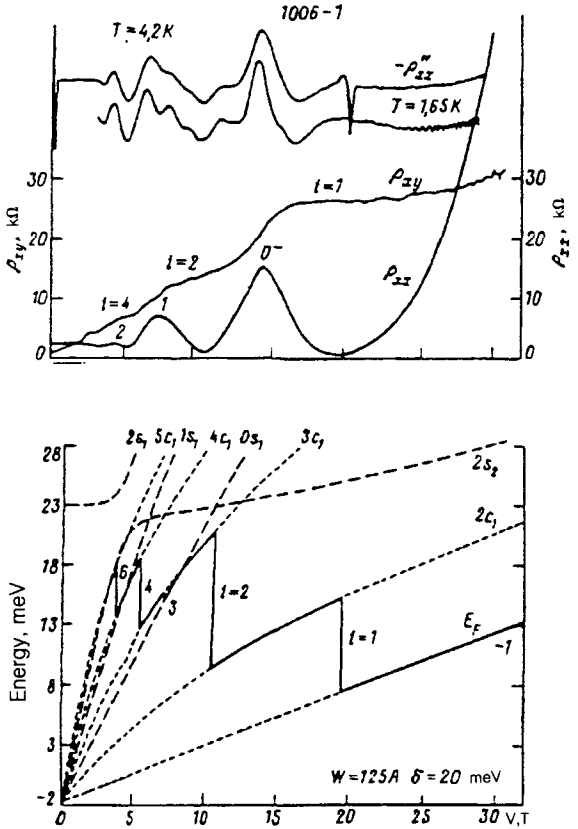


FIG. 2.

framework of the model used here. Corrections to the well width found here may arise when we incorporate possible deviations of the well from a square shape, when we allow the well to have a finite depth, and when we take into account the actual anisotropy $\gamma_2 \neq \gamma_3$ (Ref. 7). These changes in the structure of the experimental curves can also be expected as a result of changing the hole concentration. All these points constitute topics for further research.

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Exponent of 10/3 for the relaxation time of polymer chains

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A new model predicts a behavior $\tau \sim N^{10/3} N_e^{-4/3} [1-(N_e/N)^{1/2}]^{2/3}$ for the maximum relaxation time for a melt of polymer chains of N monomers (with an average of N_e monomers per entanglement). This prediction agrees with the experimental result $\tau \sim (N/N_e)^{3.4} N_e^2$.

Models based on the concept of reptation[1–5] for a melt of polymer chains consisting of N monomers, with an average of N_e monomers between successive entanglements along one chain, correctly predict the behavior $D \sim N_e/N^2$ for the translational diffusion coefficient and the relation $\tau \sim \eta$ for the viscosity η and the maximum relaxation time. However, the prediction $\tau \sim N^3$ {along with the correction $\tau \sim N^3 N_e^{-1} (1-\mu) N_e/N)^{1/2}]^3$, where $\mu \geqslant 1.47$ for a continuous Routh chain[4,5] (μ is actually used as an adjustable parameter)}, apparently does not explain the experimentally established behavior[6–9] $\eta \sim \tau \sim N^{3.4}$. There is a corresponding discrepancy between theory and experiment for the maximum relaxation time of chains in a network. There is a brief but comprehensive discussion of the present state of the problem, including the most recent studies, in Ref. 10.

I share the opinion[10,11] that the failure of the prediction is a consequence of the single-chain approximation. It is not at all obvious that the time over which a chain creeps within a tube (in reptation theory), i.e., the tube renewal time (which could be calculated in a different way, without leaning on the concept of reptation), is the same as the maximum relaxation time. The chain remains entangled with the same (spatially closest) chains (with the same region of the network in the case of chains in a network), but in a different way.

Weiss *et al.*[11] have offered the estimate $\tau \sim N^{10/3}/(\ln N)^{2/3}$ as the time of the first creep of $\sim N^{1/2}$ chains outside a sphere of radius $\sim N^{1/2}$ for a melt. A chain inside a sphere of radius $\sim N^{1/2}$ has $\sim R^3/N \sim N^{1/2}$ "neighboring" chains. Despite the successful prediction of an exponent of 10/3 for a melt, however, the specific physical suggestion of Ref. 11 looks a bit eccentric after a renormalization in accordance with $\tau \sim N^{10/3} N_e^{-4/3}/[\ln(N/N_e)]^{2/3}$. It is difficult to agree with the hypothesis that the behavior of an ensemble of $\sim (N/N_e)^{1/2}$ chains, i.e., the autocorrelation function of degree $\sim (N/N_e)^{1/2}$, would make the governing contribution to the dependence of τ on N. Furthermore, the discussion of Ref. 11 cannot be extended to the case of chains inside a network. The agreement of the exponents of 3.4 found experimentally in the two cases is a consequence of a common physical mechanism. The relation $\tau_{\text{relax}}/\tau_{\text{diff}} \sim N^{1/3}$, where τ_{relax} is the relaxation time of the tube, i.e., the time for a total loss of memory about the chain which was in the tube at zero time, and τ_{diff} is the time required for the traversal by diffusion of a distance on the order of the inertial radius

of the chain, was derived in Ref. 10. If we assume $\tau \sim \tau_{\text{relax}}$, we find $\tau \sim N^{10/3}$ (or $\sim N^{10/3} N_e^{-4/3}$). A development of the technique of Ref. 10 can apparently make it possible to also derive a correction factor which gives the rate of convergence on an exponent of 10/3, for comparison with that found in the present study.

In the present study we estimate the maximum relaxation time of the melt in a different way: as the time over which one chain "spreads out," as the result of translational diffusion, over a spatial region of size L such that the chain is, on the average, topologically decoupled from its original "partners," i.e., from the chains which were spatial neighbors. We can estimate the decoupling time for two Gaussian chains of N monomers. We assume the length of a monomer to be l, and we assume that both chains are inside a sphere of radius L. We estimate the degree of entanglement: We paste a film of minimum area on the first chain. The area of the film is then $\sim Nl^2$. The degree of entanglement is proportional to the number of intersections of the second chain with the film, i.e., $\sim N^2/(L/l)^3$, because of the ability of the chains to form knots. If the second chain intersects the film n times, then the probability that the chains are not entangled falls off no more slowly than exponentially with n ($n \ll N$), although the mathematical expectation of the absolute value of the Gauss invariant of the entanglement is $\sim \sqrt{n}$.

If $L/l \sim N^{1/2}$, then the probability for nonentanglement of two annular chains of N monomers is correspondingly estimated to be $\exp(-\beta N^{1/2})$. In connection with this result, we refer the reader to Eq. (18) of Ref. 12 and the derivation of that equation. Let us compare the results with the results of numerical simulations,[13,14] in which a dependence $P_0 = 1 - A_0 \exp(-\alpha_0 R^3)$ is proposed for the probability for the nonentanglement of two annular chains whose centers of mass are separated by a distance R. The quantity A_0 tends toward one with increasing N, and we have $\alpha_0 \sim N^{-1.7}$. If we take an average of P_0 over the interior of a sphere of radius $\sim N^{1/2}$, we find the quantity $(1 - A_0)$, which takes on the values[13,14] 0.34, 0.25, 0.18, and 0.13 for chains of lengths 20, 40, 60, and 80, respectively. The correspondence is satisfactory.

A more accurate estimate follows from the known analogy[15] with the motion of a charged particle in a magnetic field. We denote by **H** the magnetic field of a unit current flowing along the first chain. The energy of the field outside the chain is $\sim (N - N^{1/2})l$. An estimate follows from considerations of the renormalization-group type. Switching from N to gN, we find the equation

$$g(N + xN^y)l + (g + xg^y)N^{1/2}l = [gN + (xgN)^y]l \tag{1}$$

and thus $x = -1$ and $y = 1/2$.

Correspondingly, the degree of entanglement is $\sim N^2(1 - N^{-1/2})/(L/l)^3$. If this degree of entanglement is to be ~ 1, i.e., if the chains are to be decoupled on the average, we must have $L \sim N^{2/3}(1 - N^{-2/3})^{1/3} l$. This situation would require a time $\tau_2 \sim L^2/D \sim N^{10/3}(1 - N^{-1/2})^{2/3}$.

We find an identical estimate for the time τ_k ($k \geqslant 3$) for decoupling, in the sense used above, of a chain with $(k-1)$ partners. It is reasonable to suggest that the

maximum relaxation time satisfies $\tau \sim \tau_2 \sim \tau_k$, where k is bounded. Taking N_e into account, we then find

$$\tau \sim N^{10/3} N_e^{-4/3} [1-(N_e/N)^{1/2}]^{2/3}. \quad (2)$$

For a chain in a network with a sparse cross-linking (e.g., the number of monomers between linkages is $\sim N$), this discussion is modified in the following way: The "intersection" of a chain with its own trajectory at zero time is estimated to be $\sim N^2/(L/l)^3 \sim 1$; using N_e, we find $\tau_0 \sim N^{10/3} N_e^{-4/3}$.

The arguments above support the hypothesis that as $N \to \infty$, the relaxation time has an asymptotic behavior $\tau \sim N^{10/3}$.

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