Strong-coupling corrections in a superfluid Fermi gas with repulsion

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Strong-coupling corrections to the coefficients $\beta_1 ... \beta_5$ are calculated for the quaternary terms in the Ginzburg–Landau free energy for a superfluid Fermi gas with repulsion. The stability conditions for the anisotropic A phase are analyzed. This theory is compared with the spin-fluctuation theory of Anderson, Brinkman, and Morel [Phys. Rev. **123**, 1911 (1961); A **8**, 2732 (1973)]. The possible realization of superfluid phases other than A and B is discussed.

1. The search for the microscopic nature of mechanisms which stabilize superfluid phases of ^{3}He other than the B phase (or, more generally, a triplet superfluid Fermi system) has continued to hold interest for many years now. A classic result in this field was established by Anderson, Brinkman, and Morel,[1] who proposed a spin-fluctuation mechanism for stabilizing the anisotropic A phase of ^{3}He back in the early 1970s. A more rigorous mathematical framework was constructed for that mechanism by Brinkman *et al.*[2] Later papers by Barton and Moore,[3] Jones,[4] Volovik and Mineev,[5] Mermin and Stare,[6] Bruder and Vollhardt,[7] and Marchenko[8] constituted a comprehensive group-theory analysis of all possible phases of a superfluid triplet Fermi system. Conditions on the coefficients $\beta_1 ... \beta_5$ of the quaternary terms in the Ginzburg–Landau free energy which would be required for the attainment of a local minimum on each of the 18 phases possible were also written in these papers.

Superfluid ^{3}He is of course the topic of primary interest in research on triplet superfluidity. Until now, numerous pieces of experimental evidence gave rise to the belief that only two superfluid phases—the isotropic B phase and the anisotropic A phase—could exist in this substance. Some recent experimental studies (Pecola *et al.*,[9] Frossati *et al.*,[10] Gould *et al.*,[11] etc.) and some theoretical papers by Capel,[12] however, hint at a possible realization of some superfluid phases other than A and B in ^{3}He. (We are talking about 3D ^{3}He; the existence of a planar phase in ^{3}He films is generally acknowledged.) Currently one of the most popular candidates for the role of a third superfluid phase is the so-called axiplanar noninert phase of Mermin and Stare, whose parameters are close to those of the A phase.

In the present letter we work from first principles to analyze the possible existence of various superfluid phases in a triplet Fermi system. For this purpose we select a very simple, exactly solvable model of a Fermi gas with a short-range repulsion. It can be shown in this model that both the temperature of the triplet superfluid transition T_{C1} itself (see the paper by Kagan and Chubukov[13]) and the values of the coefficients $\beta_1 ... \beta_5$ in the Ginzburg–Landau free energy depend on only a single microscopic parameter: the gas parameter ap_F (a is the scattering length, and p_F is the Fermi momentum). In the weak-coupling approximation we have

$$-\beta_5^{\text{W.C.}} = \beta_4^{\text{W.C.}} = \beta_3^{\text{W.C.}} = \beta_2^{\text{W.C.}} = -2\beta_1^{\text{W.C.}} = I,$$

$$I \sim N(0)/T_C^2, \quad T_{C1}/\epsilon_F \sim \exp[-1/(ap_F)^2]$$

(see the paper by Baranov and Kagan[14]), and the isotropic B phase is an absolute minimum. Strong-coupling corrections make the situation less trivial. We will show that incorporating strong-coupling corrections analogous to those discussed by Rainer and Serene[15] leads to the possibility that the anisotropic A phase will be stabilized. In this approximation, other likely candidates for the role of a third phase (axiplanar, planar, and polar phases) either lie above the B and A phases on the energy scale or are not even local minima.

The strong-coupling corrections to the coefficients β_i which we found are

$$\Delta\beta_i = |\beta_i^{\text{W.C.}}| \frac{T_{C1}}{\epsilon_F} [\gamma_i (ap_F)^2 + \delta_i (ap_F)^3]$$

(γ_i and δ_i are numerical coefficients). They differ substantially both in absolute value and sign from the coefficients β_i of the Anderson–Brinkman–Morel spin-fluctuation theory.

2. As was shown in Ref. 14, in the zeroth approximation in the gas parameter ap_F (this is the weak-coupling approximation) the coefficients β_i of the quaternary terms in the Ginzburg–Landau free energy are

$$-\beta_5^{\text{W.C.}} = \beta_4^{\text{W.C.}} = \beta_3^{\text{W.C.}} = \beta_2^{\text{W.C.}} = -2\beta_1^{\text{W.C.}} = \frac{N(0)}{T_{C1}^2} \frac{7\zeta(3)}{120\pi^2}, \tag{1}$$

where $\zeta(z)$ is the Riemann zeta function [$\zeta(3) = 1.202$], $T_{C1} \approx \epsilon_F \exp[-32/(ap_F)^2]$ is the temperature of the triplet p-wave pairing in a Fermi gas with repulsion,[13] and $N(0)$ is the density of states at the Fermi surface. As is well known (Mermin and Stare[6] and Vollhardt and Wölfle[16]), the phase diagram becomes trivial for these relations among parameters: The isotropic B phase is the sole minimum (and the absolute minimum) in the absence of a magnetic field. Consequently, strong-coupling corrections must be taken into account in order to analyze the possibility that other phases would be realized. Figure 1 shows the diagrams of the next order for the free-energy functional which violate relations (1) among the coefficients β_i according to the approach of Ref. 14.

For clarity, we have used an unsymmetrized diagram technique, in which a wavy line corresponds to a completely unsymmetrized vertex Γ. In addition to the diagrams in Fig. 1, there is another series of diagrams (which contain only a single closed electron loop), which lead to changes in each of the coefficients β_i without changing the relations between them. We will save space by omitting these other diagrams.

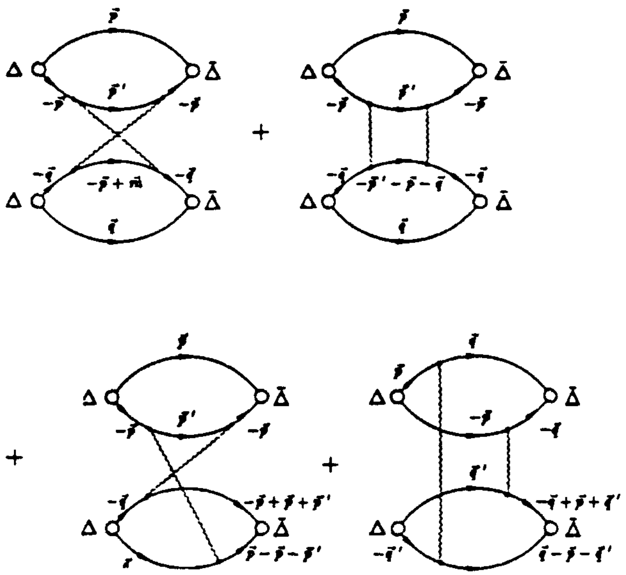

FIG. 1. Feynman diagrams making the contribution on the order of $(T_{C1}/\epsilon_F)|\beta_1^{\mathrm{W.C.}}|$ to the coefficients β_i, with a disruption of the weak-coupling relations, (1), among these coefficients.

It is not difficult to show that at the order of T_{C1}/ϵ_F the contributions of all diagrams (with one or two closed electron loops) to the coefficients β_i are given by the expression

$$-\frac{T^3}{2}\sum_{\omega_1\omega_2\omega_3}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}\,|\Gamma(\mathbf{p}_i,\omega_i)|^2\,K(\mathbf{p}_1,\omega_1)K(\mathbf{p}_3,\omega_3)$$

$$\times G(\mathbf{p}_2,\omega_2)G(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3,\ \omega_1+\omega_2-\omega_3)$$

$$-\frac{T^3}{4}\sum_{\omega_1\omega_2\omega_3}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}\,|\Gamma(\mathbf{p}_i,\omega_i)|^2\,K(\mathbf{p}_1,\omega_1)K(\mathbf{p}_3,\omega_3)$$

$$\times G(\mathbf{p}_2,\omega_2)G(\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3,\ \omega_1-\omega_2-\omega_3)$$

$$+T^3\sum_{\omega_1\omega_2\omega_3}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}\,|\Gamma(\mathbf{p}_i,\omega_i)|^2\,F^+(\mathbf{p}_1,\omega_1)K(\mathbf{p}_3,\omega_3)$$

$$\times F(\mathbf{p}_2,\omega_2)G(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3,\ \omega_1+\omega_2-\omega_3)$$

$$-\frac{T^3}{8}\sum_{\omega_1\omega_2\omega_3}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_3}{(2\pi)^9}\,|\Gamma(\mathbf{p}_i,\omega_i)|^2\,F(\mathbf{p}_1,\omega_1)F^+(\mathbf{p}_3,\omega_3) \qquad (2)$$

$$\times F(\mathbf{p}_2,\omega_2)F^+(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3,\ \omega_1+\omega_2-\omega_3), \qquad (3)$$

where $G=(i\omega-\xi)^{-1}$ is the Matsubara Green's function; $\omega=\pi T(2n+1)$ is the frequency; $\xi=(p^2-p_F^2)/(2m)$ is the spectrum; $K(\mathbf{p},\omega)=|\hat{\Delta}|^2G^2(-\mathbf{p},-\omega)G(\mathbf{p},\omega)$; $F(\mathbf{p},\omega)=G(\mathbf{p},\omega)\hat{\Delta}G(-\mathbf{p},-\omega)$; $\hat{\Delta}_{\alpha\beta}=\Delta(T)i(\sigma_2\sigma_i)_{\alpha\beta}\mathbf{A}_{ik}\cdot\mathbf{n}_k$ is the triplet superfluid gap; σ_i are the Pauli matrices; and $\Gamma(\mathbf{p}_i\omega_i)\equiv\Gamma(\mathbf{p}_1\omega_1,\mathbf{p}_2\omega_2,\mathbf{p}_3\omega_3,\mathbf{p}_4\omega_4)$ is the two-particle vertex (the complete antisymmetrized vertex).

In the first approximation in T_{C1}/ϵ_F, we can evaluate the total two-particle vertex Γ at vanishing frequencies and on the Fermi surface: $\Gamma(\mathbf{p}_i,\omega_i)\rightarrow\Gamma(p_F\mathbf{n}_i,0)$, where $\mathbf{n}$ is a unit vector. In the case of a Fermi gas the total two-particle vertex is

$$\begin{aligned}\Gamma_{\alpha\beta,\gamma\delta}&(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3)\\ &\equiv\Gamma_s\delta_{\alpha\gamma}\delta_{\beta\delta}+\Gamma_a\sigma_{\alpha\gamma}\sigma_{\beta\delta}\\ &=[\tfrac{1}{2}g+2g^2\Pi(\mathbf{p}_2-\mathbf{p}_3)+g^2C(\mathbf{p}_1+\mathbf{p}_2)-g^2\Pi(\mathbf{p}_1-\mathbf{p}_3)]\delta_{\alpha\gamma}\delta_{\beta\delta}\\ &\quad-[\tfrac{1}{2}g+g^2C(\mathbf{p}_1+\mathbf{p}_2)+g^2\Pi(\mathbf{p}_1-\mathbf{p}_3)]\sigma_{\alpha\gamma}\sigma_{\beta\delta},\end{aligned} \tag{4}$$

where

$$\Pi(q)=\frac{mp_F}{4\pi^2}\left[1+\frac{4p_F^2-q^2}{4p_Fq}\ln\frac{2p_F+q}{2p_F-q}\right]$$

is a polarization operator,

$$C(p)=\frac{mp_F}{2\pi^2}\left[1+\frac{\sqrt{1-p^2/4p_F^2}}{2}\ln\frac{\sqrt{1-p^2/4p_F^2}-1}{\sqrt{1-p^2/4p_F^2}+1}\right]$$

is a Cooper loop, regularized at large momentum, and $g=4\pi a/m$ is a coupling constant.

Some rather involved transformations of the integrals in (2) show that, as in Ref. 15, the strong-coupling corrections for the coefficients β_i are determined by bilinear combinations of spherical harmonics of the total vertex, evaluated for $\mathbf{p}_1=\mathbf{p}_3$. In other words, we have

$$\Delta\beta_i=|\beta_1^{\mathrm{W.C.}}|\frac{T_{C1}}{2\epsilon_F}[r_{1i}(\Gamma_{0s})^2+r_{2i}(\Gamma_{0a})^2+r_{3i}(\Gamma_{0s}\Gamma_{0a})+\cdots],$$

where r_{1i}, r_{2i}, r_{3i},... are numerical factors; and

$$\Gamma_{0s}\equiv\int\Gamma_s(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_1,\mathbf{p}_2)\frac{d\cos\theta}{2}=\frac{g}{2}+g\frac{ap_F}{\pi}\frac{(2\ln 2+1)}{3},\ \theta=\widehat{\mathbf{p}_1\mathbf{p}_2},$$

$$\Gamma_{0a}=-\frac{g}{2}-g\frac{ap_F}{\pi}\frac{(5-2\ln 2)}{3},$$

$$\Gamma_{1s}=g\frac{ap_F}{\pi}\frac{2(7\ln 2-1)}{5},$$

$$\Gamma_{1a}=-g\frac{ap_F}{\pi}\frac{2(2+\ln 2)}{5}.$$

All the other harmonics (Γ_2, Γ_3, etc.) are much smaller than $|\Gamma_1|$. A direct calculation of the numerical factors r_i leads to the following final result for the coefficients β_i:

$$\begin{aligned}
\beta_1&=|\beta_1^{\text{W.C.}}|\left\{-1+\frac{T_{C1}}{2\epsilon_F}\left[-75.4\left(\frac{2ap_F}{\pi}\right)^2+0.2\left(\frac{2ap_F}{\pi}\right)^3\right]\right\},\\
\beta_2&=|\beta_1^{\text{W.C.}}|\left\{2+\frac{T_{C1}}{2\epsilon_F}\left[-7.0\left(\frac{2ap_F}{\pi}\right)^2+29.4\left(\frac{2ap_F}{\pi}\right)^3\right]\right\},\\
\beta_3&=|\beta_1^{\text{W.C.}}|\left\{2+\frac{T_{C1}}{2\epsilon_F}\left[-6.4\left(\frac{2ap_F}{\pi}\right)^2-13.3\left(\frac{2ap_F}{\pi}\right)^3\right]\right\},\\
\beta_4&=|\beta_1^{\text{W.C.}}|\left\{2+\frac{T_{C1}}{2\epsilon_F}\left[-48.3\left(\frac{2ap_F}{\pi}\right)^2-108.8\left(\frac{2ap_F}{\pi}\right)^3\right]\right\},\\
\beta_5&=|\beta_1^{\text{W.C.}}|\left\{-2+\frac{T_{C1}}{2\epsilon_F}\left[-108.9\left(\frac{2ap_F}{\pi}\right)^2-183.2\left(\frac{2ap_F}{\pi}\right)^3\right]\right\}.
\end{aligned}\tag{5}$$

In principle, expression (4) can be used to calculate and compare the free energies of all 18 possible phases of ^{3}He. We will content ourselves in the present letter with a look at the phases which are cited most commonly in the literature: the B phase, the A phase, the polar phase, the planar phase, and the axiplanar phase. In our approximation it is a simple matter to show that the polar, planar, and axiplanar phases either lie above the B phase on the energy scale or completely fail to qualify as even local minima of the energy. The situation regarding the A phase is less trivial. According to Ref. 16, the conditions for stabilization of the A phase are

$$\beta_4+\beta_5-2\beta_1-\beta_3<-\tfrac{1}{2}(\beta_1+\beta_3)<0.\tag{6}$$

At the order $(ap_F)^2$ we find $\beta_4+\beta_5-2\beta_1-\beta_3\equiv 0$ from (4), and the A phase is not yet stabilized.

In the next order, $(ap_F)^3$, we find

$$\beta_4+\beta_5-2\beta_1-\beta_3=-\frac{T_{C1}}{2\epsilon_F}\times 558.3\left(2\frac{ap_F}{\pi}\right)^3<0,$$

and inequalities (5) can be written

$$\begin{aligned}
&\frac{T_{C1}}{2\epsilon_F}\left[81.8\left(\frac{2ap_F}{\pi}\right)^2+13.1\left(\frac{2ap_F}{\pi}\right)^3\right]<1,\\
&\frac{T_{C1}}{2\epsilon_F}\left[81.8\left(\frac{2ap_F}{\pi}\right)^2+571.3\left(\frac{2ap_F}{\pi}\right)^3\right]>1.
\end{aligned}\tag{7}$$

In real ^{3}He we have $T_{C1}/\epsilon_F\sim 10^{-3}$. Nevertheless, because of the very large numerical coefficient (~ 600) of $(T_{C1}/2\epsilon_F)(2ap_F/\pi)^3$ in the second inequality in (6), the conditions for the realization of the A phase can in principle be satisfied for $ap_F > 1$. At zero pressure we would have $ap_F = 2$ ($2ap_F/\pi = 4/3$). With increasing pressure, ap_F increases, making it easier to satisfy inequalities (6).

3. Let us compare our results with the results of some previous studies. The complete vertex of the Anderson–Brinkman–Morel spin-fluctuation theory is given by the expression $\Gamma(q) = (g/2)/[1-(g/2)\Pi(q)]$. The polarization operator $\Pi(g) \approx (mp_F/2\pi^2)/(1-q^2/12p_F^2)$ is expanded in a series near the ferromagnetic instability corresponding to small values $q \to 0$. Expression (3), which we have derived for the total vertex, contains, in addition to the polarization operator, a Cooper loop. It is not based on the assumption of proximity to the ferromagnetic instability $[(g/2)\Pi(0) \not\to 1]$. The strong-coupling corrections to the coefficients β_i, which we have found, differs considerably in both sign and absolute value from the Anderson–Brinkman–Morel corrections.

It can be shown that our strong-coupling corrections are analogous to the strong-coupling corrections discussed by Rainer and Serene for a dense Fermi system. The Fermi-gas approach has the advantages that (first) we know the microscopic mechanism for stabilization of triplet pairing in a Fermi gas with repulsion, and (second) we can derive an explicit expression for the complete two-particle vertex in the gas approximation and thereby pursue the expressions for the coefficients β_i to the point of numbers.

We note in conclusion that in strong magnetic fields $H > T_{C1}/\mu_B = 4T$ (μ_B is the Bohr nuclear magneton) the $S_z = 0$ projection of a triplet Cooper pair is suppressed paramagnetically. As a result, the phase diagram contains only the A phase even in the weak-coupling approximation. The strong-coupling corrections do not change the situation qualitatively. Accordingly, our only hope for finding new phases (other than A and B) in this approximation must be pinned on weak magnetic fields ($H < 4$ T). The possible existence of a third phase in weak magnetic fields was pointed out in a theoretical paper by Capel *et al.*[12] A detailed analysis of the phase diagram in this region will be the subject of a separate study.

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