Forced motion of a domain wall in the field of a spin wave

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It is shown that the interaction of a domain wall with a packet of spin waves causes a displacement of the wall, and in the presence of a plane wave the domain wall moves with a constant velocity toward the wave. An exact solution corresponding to the interaction of the domain wall with a plane spin wave is presented in an integrable case.

In a ferromagnet with uniaxial anisotropy, ignoring the dipole-dipole interaction and relaxation processes, Walker's rule, which is related to the conservation of the projection of the magnetic moment on the anisotropy axis, prohibits motion of the domain wall. We shall show that the interaction of a domain wall with a packet of spin waves leads to a displacement of the wall, and in the presence of a plane spin wave the wall must move toward the wave with a constant velocity.

The dynamics of the magnetization vector $\mathbf{S} = (S_1, S_2, S_3)$, $\mathbf{S}^2 = 1$ is described by the phenomenological Landau and Lifshitz equations,[1]

$$\dot{S}_3 = \frac{i}{2} \nabla (S_+ \nabla S_- - S_- \nabla S_+), \tag{1}$$

$$\dot{S}_+ = i \nabla (S_3 \nabla S_+ - S_+ \nabla S_3) + \frac{i}{2} h'_a (S_3) S_+, \tag{2}$$

$$S_+ = S_1 + i S_2, \quad S_- = S_1 - i S_2$$

and $h_a(S_3)$ is the anisotropy energy density. We shall limit ourselves to normal incidence of a spin wave (wave packet) onto a flat domain wall, choosing the direction x along the direction of propagation of the wave.

Equation (1) has the form of a conservation law for the projection of the magnetic

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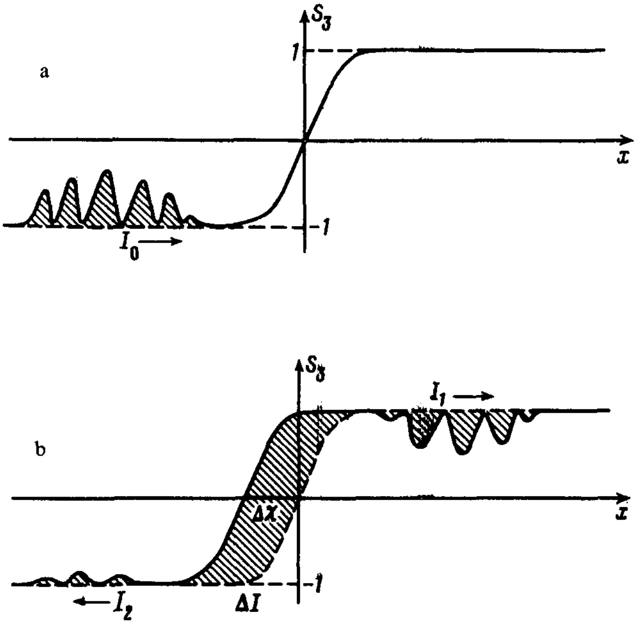

FIG. 1. I_0, I_1, I_2, and $\Delta I = 2\Delta x$ denote the shaded areas in the figures.

moment on the anisotropy axis. It is this relation that leads to Walker's rule, since a displacement of the domain wall changes the magnitude of the indicated projection of the magnetic moment of the specimen.

1. We shall examine the passage of a localized packet through a domain wall. Let the packet be located on the left side of the wall before the collision (Fig. 1a) and assume that it makes a positive contribution I_0 to the conserved integral

$$I = \int_{-L}^{L} S_3 dx \tag{3}$$

(the interval of integration includes the wall and the packet). After the interaction (Fig. 1b), as it evolves the packet passes through the wall and is partially reflected. The contribution of the wave passing through the wall to the integral (3) is negative and the contribution of the reflected wave is positive. But, since the amplitude of the reflected wave cannot be too large (this contradicts the energy-conservation law), an unbalance ΔI arises in the law of conservation of the projection of the moment (3), which can be compensated for only by a displacement of the domain wall toward the left. The magnitude of the displacement evidently is

$$\Delta x = \frac{1}{2}\,(I_0 + I_1 - I_2), \tag{4}$$

where $I_{0,1,2}$ are the incident, transmitted, and reflected packets, respectively, numerically equal to the modulus of the projection of the magnetic moment onto the anisotropy axis.

This general discussion does not depend on the function $h_a(S_3)$ and does not require that the amplitudes of the spin wave packets be small. We assumed only that the domain wall does not have internal degrees of freedom. Excitation of a domain wall with a magnetic moment $\tilde{I}$ would have decreased displacement (4) by $\tilde{I}/2$. The absence of such excitations in our formulation of the problem can be proved rigorously for models (1) and (2) in the one-dimensional integrable case[1)]

$$h_a(S_3) = -\beta^2 S_3^2 . \tag{5}$$

2. A more detailed quantitative description, based on the scattering matrix for spin waves scattered by the domain wall, can be given for waves with small amplitude. The scattering matrix can be calculated on the basis of a linear theory, assuming that the domain wall is fixed in the first approximation. Let a plane spin wave with small amplitude a_k be incident from the left onto the domain wall. We then find

$$\begin{aligned} S_+ &\cong a_k \exp(i\omega_k t - ikx) + b_k \exp(i\omega_k t + ikx) \quad \text{for } x \ll -L, \\ S_+ &\cong c_k \exp(-i\omega_k t + ikx) \quad \text{for } x \gg L, \end{aligned} \tag{6}$$

where L is the anisotropy scale, b_k is the amplitude of the reflected wave, and c_k is the amplitude of the transmitted wave. From the energy conservation law it follows that

$$|a_k|^2 = |b_k|^2 + |c_k|^2. \tag{7}$$

In the integrable case (5) the domain wall ($S_3 = \mathrm{th}(\beta x)$, $S_+ = i/\mathrm{ch}(\beta x)$) is nonreflective: $b_k \equiv 0$, $c_k = a_k[(k+i\beta)/(k-i\beta)]$.

Let us integrate relation (1) from $-L_1$ to $L_1 (L_1 \gg L)$. In calculating the integral on the right side we shall use asymptotic expressions (6),

$$\frac{\partial}{\partial t}\int_{-L_1}^{L_1} S_3\,dx = k|c_k|^2 + k|a_k|^2 - k|b_k|^2 . \tag{8}$$

Incorporating (7), we find

$$\int_{-L_1}^{L_2} S_3\,dx = 2k|c_k|^2 t + \text{const}. \tag{9}$$

Thus the domain wall must deform in such a manner as to satisfy relation (9). In our formulation of the problem, as already noted, the domain wall does not have internal degrees of freedom and, for this reason, the indicated deformation is a displacement in the negative direction with constant velocity

$$V = -k|c_k|^2 . \tag{10}$$

In the integrable case (5) the exact solution, which corresponds to the interaction of a spin wave with a domain wall, can be explicitly calculated with the help of the inverse-problem method and it has the form

$$S_3 = \frac{\sqrt{(k^2+\beta^2)(1-\delta^2)}\,\mathrm{sh}A + \delta k \sin B}{\sqrt{k^2+\beta^2}\,\mathrm{ch}A + \delta\beta\cos B},$$

$$S_+ = e^{i\phi_1}\frac{\delta\sqrt{k^2+\beta^2}\,(\mathrm{ch}\,A\cos B + i\,\mathrm{sh}A\sin B) + \beta - ik\sqrt{1-\delta^2}}{\sqrt{k^2+\beta^2}\,\mathrm{ch}\,A + \delta\beta\cos B} \tag{11}$$

$$A = \beta\sqrt{1-\delta^2}\,(x - vt - x_0),\ B = \sqrt{1-\delta^2}\,(k^2+\beta^2)t - kx + \phi_2,\ v = -\frac{k\delta^2}{\sqrt{1-\delta^2}},$$

where δ, k, ϕ_1, ϕ_2, x_0 are arbitrary constants. We note that solution (11) is valid for any amplitude $0\leqslant\delta<1$ of the spin wave.

For a packet of waves with small amplitude, based on relation (7), the formula for the displacement of the domain wall assumes the form

$$\Delta x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |c_k|^2\, dk = I_1. \tag{12}$$

3. The domain structure of magnets stabilizes at large distances due to mechanisms that we ignore. These mechanisms are, for example, the dipole-dipole interaction, the defects of the crystal lattice, the surface and geometric effects, etc. Dissipation processes and the more complicated anisotropy of real crystals should also be taken into account. Of course, when the mentioned corrections to the model are included, the integral I in (3) is no longer conserved. However, if the transit time of the spin wave packet through the domain wall is much shorter than the characteristic time of the interaction responsible for the breakdown of the integral, then the arguments presented above can be justified.

When a packet passes through a sequence of walls, each wall is displaced in a direction opposite to the motion of the packet and the displacement of each subsequent wall is smaller than that of the preceding wall due to reflection and dissipation of the spin waves; i.e., the density of domain walls must decrease slightly. Inclusion of inhomogeneities, impurities, and different surface and volume defects can lead to the inverse process—accumulation of domain walls.

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[1)]The integrability of this model was established for the first time by Borovik.[2]

[1]L. D. Landau and E. M. Lifshitz, L. D. Landau sobranie trudov (Collected Works of L. D. Landau), Nauka, Moscow, 1969, Vol. 1, p. 128.
[2]A. E. Borovik, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 629 (1978) [JETP Lett. **28** 581 (1978)].

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