Exchange interaction of positive ions with vortices in ^{3}He-B

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The exchange contribution to the interaction of vortices with positive ions in superfluid ^{3}He-B is found.

According to recent NMR experiments[1] the cores of vortices in rotating ^{3}He–B have a spontaneous magnetic moment. The possibility of direct observation of the localization of magnetization in the cores reveals the phenomenon of capture of charged ions by the cores. It is shown in this paper that the potential describing the interaction of positive ions with vortices depends on the mutual orientation of the magnetic moment of the vortices and the moments of ions oriented along the external field. The interaction potential determines both the density of the captured charge and the lifetime of ions in the vortices, which permits observing the indicated dependence experimentally.

In superfluid ^{4}He the attractive interaction between a vortex and an ion arises due to Bernoulli's force, which acts on an ion in the velocity field of the vortex.[2] In superfluid ^{3}He-B a different interaction mechanism plays the main role. In the case of p-pairing an impurity suppresses the superfluid state in its vicinity. As a result, the energy of the superfluid liquid in the presence of an impurity of atomic size increases by

$$\delta\Omega = \frac{k_F^2\,\sigma_{\rm tr}\Delta^2(T)}{16\pi T_c} \tag{1}$$

(see Ref. 3, where this formula was obtained for ^{3}He-B in the limit $T \rightarrow T_c$; see Ref. 4 for the generalization to the case of arbitrary temperatures). Here $\Delta\ (T)$ is the temperature-dependent energy gap, k_F is the Fermi momentum, and $\sigma_{\rm tr}$ is the transport cross section for scattering of quasiparticles of the normal Fermi liquid by the impurity. As the axis of the vortex is approached, the energy gap Δ decreases, vanishing on the axis in the case of a vortex with a normal core, a factor that creates the well for ions near the vortex axis. We can thus write the following expression for the interaction potential of an ion located at a distance r from the vortex axis:

$$\delta\Omega(r) = \frac{k_F^2\,\sigma_{\rm tr}\Delta^2\,(T)}{16\pi T_c} f(r)\,, \tag{2}$$

where $f(r) \rightarrow 1$ as $r \rightarrow \infty$, $0 \leqslant f(0) < 1$. Vortices with a superfluid core, where $f(0) \neq 0$, can also appear in ^{3}He-B. The explicit form of the function $f(r)$ for axially symmetric vortices with different core structures is given in Ref. 5 (see below).

Equation (2) was obtained under the assumption that the amplitude for scattering of quasiparticles of normal ^{3}He by an ion does not depend on their spins. On the other hand, positive ions in liquid helium are spheres, with a radius of several interatomic distances and consisting of helium which has solidified under the action of the excess electrostatic pressure. As pointed out by Édel'shteĭn,[6] the amplitude for scattering of quasiparticles of the liquid by a positive ion depends on the mutual orientation of the spins of the oncoming particle and the atom on the surface of the ion (exchange scattering), which distinguishes scattering by a positive ion from scattering by a negative ion—a spherical cavity that includes an electron. A possible observable effect of this difference is the logarithmic growth (see, for example, Ref. 7) of the mobility of positive ions with decreasing temperature at $T < 50$ mK. Exchange scattering that also contributes to the interaction potential of positive ions that interact with the vortices in superfluid ^{3}He-B. The interaction potential of an ion interacting with a vortex in an arbitrary superfluid Fermi liquid has the following form in the limit $T \to T_c$ (see Ref. 5):

$$\delta\Omega(\mathbf{r}) = \frac{1}{2}\int_0^{\Delta} \frac{d\Delta}{\Delta}\frac{T}{\hbar}\sum_n N_0 \int \frac{d\Omega_k}{4\pi}.$$

$$\times \mathrm{Sp}_4 \left\{ \frac{\pi\hbar N_0}{4\omega_n^2}\int \frac{d\Omega_{k'}}{4\pi}\, \hat{t}^N(\hat{\mathbf{k}},\hat{\mathbf{k}}')\,\hat{\Delta}(\hat{\mathbf{k}}',\mathbf{r})\,\hat{t}^N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + \frac{\pi\hbar}{i\omega_n^2}\, t_-^N(\hat{\mathbf{k}},\hat{\mathbf{k}})\,\hat{\Delta}(\hat{\mathbf{k}},\mathbf{r}) \right\}. \quad (3)$$

Here ω_n is the Matsurbara frequency; N_0 is the state density at the Fermi surface; $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$, and

$$\hat{\Delta}(\hat{\mathbf{k}},\mathbf{r}) = \begin{pmatrix} 0 & \Delta(\hat{\mathbf{k}},\mathbf{r}) \\ -\Delta^+(\hat{\mathbf{k}},\mathbf{r}) & 0 \end{pmatrix} \quad (4a)$$

is a 4×4 order-parameter matrix, which is a 2×2 matrix in the particle-hole space; each element of this matrix is a 2×2 matrix in spin space given in the case of p-pairing, by

$$\Delta(\hat{\mathbf{k}},\mathbf{r}) = i\Delta(T)(\vec{\sigma}\,\mathbf{d}(\hat{\mathbf{k}},\mathbf{r}))\sigma_y, \quad (4b)$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices. For an axially symmetric vortex with one quantum of circulation[8] we would have

$$d_\alpha(\hat{\mathbf{k}},\mathbf{r}) = \sum_{\mu,\nu} C_{\mu\nu}(r)\, e^{i\varphi(1-\mu-\nu)} R_{\alpha\beta}\, \lambda_\beta^\mu\, \lambda_i^\nu\, \hat{k}_i, \quad (5)$$

where r is the distance from the axis of a vortex layed out along the $\hat{z}$ axis, φ is the azimuthal angle, $R_{\alpha\beta}$ is the constant order-parameter matrix for the b phase, and $\lambda_\beta^\mu, \lambda_i^\nu$ are the spin eigenfunctions ($\lambda_{\beta,i}^{\pm} = (\hat{x}_{\beta,i} \pm i\hat{y}_{\beta,i})/\sqrt{2}$; $\lambda_{\beta,i}^0 = \hat{z}_{\beta,i}$) of a Copper pair ($\alpha$) and of the orbital angular momentum (i) with the projections $\mu = \pm 1, 0$ and $\nu = \pm 1, 0$. The coefficients $C_{\mu\nu}(r)$ for vortices with different discrete symmetries were found in Refs. 8 and 9. The 4×4 matrix of the scattering amplitude has the form

$$\hat{t}^N = \begin{pmatrix} \underset{\sim}{t}^N & 0 \\ 0 & (\underset{\sim}{t}^N)^+ \end{pmatrix} = t_+^N \hat{\tau}_0 + t_-^N \hat{\tau}_3 , \tag{6}$$

where τ_0 and τ_3 are the unit Pauli matrix and the z Pauli matrix in the particle-hole space.

In Ref. 5 it was assumed that the matrix $\underset{\sim}{t}^N$ does not have a spin structure. However, in the presence of exchange scattering the $\underset{\sim}{t}^N$ matrix has the form[1)]

$$\underset{\sim}{t}^N_{kk'} = u_{kk'} \sigma_0 + v_{kk'} \vec{\sigma} \mathbf{S}, \tag{7}$$

where $\mathbf{S}$ is a vector characterizing the total spin moment of atoms on the surface of the positive ion that participates in the exchange scattering.

Substituting (4)–(7) into (3) and performing the required integration and summation, we obtain

$$\delta\Omega(\mathbf{r}) = \frac{k_F^2 \Delta^2(T)}{16\pi T_c} \left\{ (\sigma_{\rm tr} + \sigma_1 \overline{\mathbf{S}^2}) f(r) + \sigma_2 \overline{S}_\alpha R_{\alpha i} \hat{z}_i M(r) - 2\sigma_1 \overline{g}(\mathbf{r}) \right\}. \tag{8}$$

Here

$$f(r) = \frac{1}{3} \sum_{\mu\nu} | C_{\mu\nu}(r) |^2 ,$$

$$M(r) = \frac{1}{3} \frac{m(r)}{m_0} = \frac{1}{3} \sum_{\mu\nu} \mu | C_{\mu\nu}(r) |^2, \tag{9}$$

$$g(\mathbf{r}) = \frac{1}{3} \sum_{\nu} | \sum_{\mu} C_{\mu\nu}(r) S_\alpha R_{\alpha\beta} \lambda^\mu_\beta \, e^{-i\mu\varphi} |^2 ,$$

$$\sigma_1 = -\left(\frac{m_3}{2\pi}\right)^2 \int \cos\theta \, | v_{\mathbf{q}} |^2 \, d\Omega , \tag{10}$$

$$\sigma_2 = -\frac{2m_3}{k_F} (\operatorname{Im} v^N)_{\mathbf{kk}} - \left(\frac{m_3}{2\pi}\right)^2 \int \cos\theta \, (v_{\mathbf{q}} u^*_{\mathbf{q}} + v^*_{\mathbf{q}} u_{\mathbf{q}}) d\Omega ,$$

$$| \mathbf{q} | = | \mathbf{k} - \mathbf{k}' | = 2k_F \sin\frac{\theta}{2} .$$

Graphs of the functions $f(r)$ for vortices with different core structure are given in Ref. 5, and graphs of the functions $m(r)/m_0$ are given in Ref. 8. The functions $f(r)$, $m(r)/m_0$, and $g(r)$ change in a jump-like manner with phase transitions in the vortex cores (see Refs. 1 and 8).

The average value of $\overline{\mathbf{S}}$ for an ion consisting of solid paramagnetic ^{3}He is zero. There are, however, two possibilities here. First of all, a nonzero average spin $S = N\mu H/T$ can be created in an external field. Here μ is the nuclear magneton, and N is the number of atoms on the surface of the ion ($N \sim 10$). Second, there are indications[10] that at temperatures lower than approximately 1 mK (the exact value depends on the pressure) liquid helium near a solid surface transforms into the ferromagnetic state. In this case the surface of the ion will have a spontaneous spin moment $S(T) \lesssim N$.

The orientation of $\overline{\mathbf{S}}$ depends on the orientation of the external field, so that the potential $\delta\Omega(r)$ describing the interaction of positive ions and vortices must change under the transformation $\mathbf{H} \rightarrow -\mathbf{H}$ to twice the second term in the braces in (8). The amplitudes of the ordinary u and exchange scattering v are unknown. However, if they are of the same order of magnitude and $\overline{\mathbf{S}} \gtrsim 1$ in fields $H \sim 10^3$ G and $T \sim 1$ mK [for a v vortex $f(0) \sim 0.4$ (Ref.5) we have $m(0)/m_0 \sim 0.2$ (Ref. 8)], then the change $\delta\Omega$ accompanying a change in the sign of the field may be comparable to the depth of the potential well in the absence of exchange scattering and, therefore, can be observed in experiments on the capture of positive ions by vortices in rotating ^{3}He-B.

We note, in conclusion, that in a superconductor, i.e., in the case of s-pairing, exchange scattering by an isolated impurity with spin $\mathbf{S}$ increases the energy of the superconductivity by an amount

$$\delta\Omega = \frac{1}{16\pi} \frac{\Delta^2(T)}{T_c} k_F^2 \sigma^v \overline{S^2}, \qquad (6)$$

and ordinary scattering does not contribute to $\delta\Omega$, $\sigma^v = (m_3/2\pi)^2 \int |v_q|^2 d\Omega$.

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[1)] Since we are interested here only in the fundamental possibility of observing exchange interaction of ions and vortices in a narrow temperature interval near T_c, it is not necessary to include the weak (logarithmic) dependence of $v_{kk'}$ on the energy of the quasiparticles.

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