Unusual splitting of the spectrum of collective modes in superfluid ^{3}He-B

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(Submitted 7 February 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 7, 304–307 (10 April 1984)

The observed six-fold splitting of the spectrum of collective modes with $J = 2$ in a magnetic field, instead of the expected $(2J + 1)$-fold splitting, is explained by textural effects. The dependence found for the additional splitting of the central line with $J_z = 0$ on the texture of the order parameter can be used to identify vortex textures in rotating ^{3}He-B.

Collective modes in superfluid ^{3}He-B have been studied extensively in recent years (see the latest review in Ref. 1). These modes represent oscillations of the order parameter—the matrix $A_{\alpha i}$—relative to the equilibrium value $A^0_{\alpha i} = R_{\alpha i}$, where $R_{\alpha i}$ is an orthogonal 3×3 matrix. The degenerate equilibrium state of the liquid, described by the matrix $R_{\alpha i}$, corresponds to Cooper pairing with quantum number $J = 0$, where $J_i = L_i + R_{\alpha i}S_\alpha$, and L_i and S_α are the spatial and spin rotation operators, respectively. The operator J is the total angular momentum operator in a coordinate system chosen so that the unit vectors of the spin coordinate system $\hat{x}_\alpha$ are related to the unit vectors of the orbital system $\hat{x}_i$ by the relation $\hat{x}_\alpha = R_{\alpha i}\hat{x}_i$. The rotation matrix $R_{\alpha i}$ is defined by the axis of rotation $\mathbf{n}$ and the angle of rotation, which is fixed by the spin-orbit interaction and which is 104°.

The spectrum of collective modes of oscillations of the matrix $A_{\alpha i}$ against the background of a state with $J = 0$ is reminiscent of the spectrum of excited states of a spherically symmetric atom or molecule. It is characterized by the following quantum numbers: the angular momentum J, which has the values 0, 1, and 2 in the matrix $A_{\alpha i}$; the projection J_z of the angular momentum on the axis of quantization; and the parity T relative to complex conjugation. These 18 modes include four gapless Goldstone modes: sound with $J = 0$ and $T = -1$ and three spin waves with $J = 1$, $J_Z = -1, 0, 1$ and $T = 1$. In experiments with propagation of ultrasound, excitation of two five-fold degenerate modes with a gap is observed: the imaginary squashing mode (IS) with $J = 2$ and $T = -1$ and a real squashing mode (RS) with $J = 2$ and $T = 1$. The Zeeman effect is observed in a magnetic field H: five-fold splitting of the RS mode,[2] linear with respect to the field, $\omega(J_z) = \omega_0 + gJ_zH$.

A more precise experiment[3] showed, however, that there is an unusual additional splitting of the central RS mode with $J_z = 0$ into two lines. The magnitude of the splitting, which increases with increasing field, saturates at $H \sim 500$ G. It has been proposed that the splitting is related to a new degree of freedom in ^{3}He-B. We shall show that in reality the splitting is a result of the existence of a texture of the order parameter $R_{\alpha i}$, in which the axis of quantization of the projection of the momentum J_z in a magnetic field varies in space, even if the field is uniform. This phenomenon is

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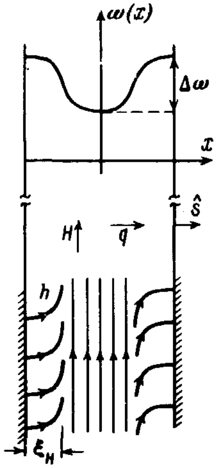

FIG. 1. Distribution of the field **h** of the axis of quantization of the angular momentum of collective modes in the texture between two plane parallel plates and $\omega(x)$—the dependence of the frequency of collective modes with $J=2$, $J_z=0$ on the coordinate x along the direction of propagation of the wave. The distance $\Delta\omega$ between the stationary points of the function $\omega(x)$ is equal to the observed splitting of this collective mode.

related to the fact that the spin moment S_α, which interacts with the magnetic field, enters into the total angular momentum J together with the matrix $R_{\alpha i}$. For this reason, the axis of quantization is not oriented along **H**, but is rotated by this matrix. Thus the axis of quantization $h_i = R_{\alpha i} H_\alpha / H$ depends on the texture of the order parameter $R_{\alpha i}$.

In the experiment of Ref. 3 the RS modes were excited by ultrasound propagating between two parallel plates, so that the wave vector **q** of the collective modes was oriented along the normal $\hat{\mathbf{s}}$ to the plates (x axis), and $\mathbf{H}\perp\hat{\mathbf{s}}$. The texture of the order parameter between the plates is determined by the competing action of the surface energy $F_s = -dH^2(\mathbf{h}\hat{\mathbf{s}})^2$ and of the magnetic anisotropy $F_H = -(4/5)aH(\mathbf{hH})$ (see Ref. 4). For simplicity, we shall examine immediately the case of a strong field, $H > 500$ G. Under the conditions of the experiment, in these fields the characteristic magnetic length ξ_H, at which the effect of the boundary on the order parameter vanishes, is less than the distance between the plates, $l = 4$ mm. For this reason, the vector **h** is oriented along **H** everywhere with the exception of regions near the plates, where at a distance $\sim\xi_H$ it rotates in order to minimize the surface energy F_s at the boundary. Thus, in this texture the axis of quantization **h** varies from $\mathbf{h}\|\mathbf{H}$ at the center between the plates ($x=0$) to $\mathbf{h}\|\hat{\mathbf{s}}$ at the boundary ($x = \pm l/2$), i.e.,

$$\mathbf{h} = \hat{\mathbf{s}} \sin\alpha(x) + \frac{\mathbf{H}}{H}\cos\alpha(x) \tag{1}$$

with $\alpha(0)=0$, $\alpha(\pm l/2) = \pm\pi/2$ (see Fig. 1).

This means that the spectrum of RS modes in a magnetic field depends on the coordinate x. Since the wavelength of the RS modes is small, so that $q^{-1} \ll \xi_H$, l, we can use the approximation of local oscillators, under the assumption that at each point x there is a unique spectrum of collective modes which are excited by ultrasound independent of the other sections of the texture. We shall examine the mode with $J_z = 0$. Its frequency, which depends on the angle between $\mathbf{q}$ and the direction of the axis of quantization $\mathbf{h}$, can be represented in the following general form:

$$\omega^2(x) = \omega_0^2 + c_1^2 q^2 + c_2^2 (\mathbf{q}\,\mathbf{h})^2 . \tag{2}$$

The intensity of excitation of this RS mode by ultrasound with frequency ω and wave vector $\mathbf{q}$ is proportional to the spectral density,

$$P(\omega) = \frac{1}{l} \int_{-l/2}^{l/2} dx\, \delta(\omega - \omega(x)) = \left(l \left| \frac{\partial \omega}{\partial x} \right|_{x = x(\omega)} \right)^{-1} . \tag{3}$$

The function $P(\omega)$ becomes infinite at two frequencies, satisfying the condition for the steady state, $\partial\omega/\partial x = 0$: $\omega = \omega(0)$ and $\omega = \omega(\pm l/2)$. These frequencies correspond to the two lines in the spectrum of absorption of ultrasound due to excitation of the RS mode with $J_z = 0$. The distance between the lines, i.e., the observed additional textural splitting of this RS mode, is

$$\Delta\omega(J_z = 0, H > 500\text{ G}) = \omega(\pm l/2) - \omega(0) \approx \frac{q^2 c_2^2}{2\omega_0} . \tag{4}$$

This splitting does not depend on the magnitude of the magnetic field at $H > 500$ G. At lower fields, this expression must be multiplied by $\cos^2\alpha(0)$, where $\alpha(0)$ is no longer zero, but depends on the type of texture and has different values for different textures. This corresponds to the experimental situation: In fields $H < 500$ G the splitting depends on the history. At $H = 0$ or at $\mathbf{H} \| \hat{\mathbf{s}}$ the texture is uniform and there is no splitting, also consistent with experiment. There is also quantitative agreement. It is easy to show that the splitting of RS modes with different J_z in zero magnetic field, which arises due to removal of degeneracy with respect to J_z with finite $\mathbf{q}$, is expressed in terms of the same phenomenological constant c_2: for example,

$$\omega(|J_z| = 1, H = 0) - \omega(|J_z| = 2, H = 0) \approx \frac{q^2 c_2^2}{2\omega_0} . \tag{5}$$

Thus the quantities (4) and (5) must coincide and indeed experiment[3] shows that these quantities are close to each other.

This discussion may be summarized as follows. 1) By analogy with (4) there must also be a textural splitting of the remaining four RS modes with $J_z \neq 0$. However, it has not yet been observed, because the splitting of these modes by ultrasound is much smaller. 2) In a strong field, such that $\xi_H < l$, which is inclined at an angle β to the normal $\hat{\mathbf{s}}$ to the plates, the magnitude of the textural splitting is $\Delta\omega = (c_2^2 q^2/2\omega_0)\sin^2\beta$. 3) Since $\xi_H \sim 1/H$, the field at which the splitting $\Delta\omega$ saturates is inversely proportional to the distance between the plates. 4) The splitting of collective modes under the action of textures can be used to identify vortex textures arising in ^{3}He-B in a rotating vessel (see Ref. 5) and Maki solitons. In particular, on the basis of the splitting of the RS mode with $J_z = 0$ it is possible to determine independently of NMR experiments the vortex parameter λ, which characterizes the orientational effect of vortices on the order parameter $F_V = (2/5)a\lambda H^2(\hat{\Omega}\,\mathbf{h})^2$, where $\hat{\Omega}$ is the direction of the axis of rotation. If the ultrasound is directed along $\hat{\Omega}$ and **H** is oriented at an angle β to $\hat{\Omega}$, then the splitting $\Delta\omega$ and the parameter λ are related by the relation

$$\lambda = \frac{\cos\beta}{\cos\mu} - \frac{\sin\beta}{\sin\mu}, \quad \Delta\omega = \frac{c_2^2 q^2}{2\omega_0}\sin^2\mu . \qquad (6)$$

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Translated by M. E. Alferieff
Edited by S. J. Amoretty