Relativistic factorization of $eD \to e'pn$ cross sections in light-cone dynamics

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The cross sections for the electrodisintegrations $eD \to e'pn$ can be factorized on the basis of the h_∞ time-ordered perturbation theory. It is shown that in the relativistic region the concept of a momentum distribution of the nucleons is defined correctly only in the scale of the relative momentum of the intermediate nucleons in their center-of-mass frame. A simple method is proposed for experimentally observing the additional dependence of the relativistic wave function on the direction of the infinite momentum. The dependence of the relativistic wave function on the two arguments is shown to result from the virtuality of the Fock components.

The "purest" source of information about the momentum distributions of the nucleons in the deuteron is the exclusive reaction $eD \to e'pn$, which has recently been the subject of active experimental research in the relativistic region in terms of both Q^2 and the momenta of the nucleons in the deuteron,[1–3] $|\mathbf{n}|$. A need has accordingly developed for a systematic relativistic description of the electrodisintegration of the deuteron and for the introduction of the concept of relativistic momentum distributions of the nucleons. Relativistic bound systems can be described systematically in relativistic Hamiltonian dynamics,[4] various versions of which are examined in Refs. 5–8.

In this letter we use a time-ordered perturbation theory in an infinite-momentum frame (h_∞). The wave function determined on the light-front hypersurface $\rho x = 0$,

$\rho = (1, 0, 0, 1)$, is the same as the wave function in the infinite-momentum frame if we choose the infinite momentum $\vec{\mathscr{P}}$ to be directed along $\vec{\rho}$. Since the lowest-order phase shifts for NN scattering are real at energies up to ~ 1.5 GeV, we will restrict the discussion to the first row of the Fock column in the wave function of the deuteron: the two-nucleon component.

The momentum distributions of the nucleons are customarily studied under the conditions

$$M_t^2 - M^2 < 2M\nu; \quad \mathbf{q}^2 >> \nu^2; \quad (s - M^2)/M >> \epsilon_d; \quad q^2 = \text{const}, \tag{1}$$

where ϵ_d, M, and d are the binding energy, mass, and momentum of the deuteron; q is the momentum of γ^* $[q^2 = -\mathbf{q}_\perp^2 + 0(1/\mathscr{P}^2)]$; and $M_t^2 = (p_1 + n)^2$, $s = (q + d)^2$, $t = (d - n)^2$, and $\nu = dq/M, p_1$, where p and n are the proton and neutron momenta ($p_1^2 = p^2 = n^2 = m^2$). The t pole in the amplitude is the governing factor for exclusive reaction $D(ee'p)n$ in the quasielastic region under conditions (1).

According to the h_∞ time-ordered perturbation theory, the amplitude for the reaction $eD \to e'pn$ in the t channel is

$$T_{\xi r} = e_\mu^\xi U_\lambda^r \bar{u}(p) \left[F_\mu^{(1)} \frac{\hat{p}_1 + m}{m^2 - t} A_\lambda^{(1)} + F_\mu^{(2)} \frac{\hat{p}_1 - m}{4\eta^2 \mathscr{P}^2} A_\lambda^{(2)} \right] C\bar{u}^T(n), \tag{2}$$

where $F_\mu^{1(2)}$ and $A_\lambda^{1(2)}$ are the electromagnetic and strong vertices. The second term in (2) corresponds to the z diagram; U_λ^r and e_μ^ξ are the polarization vectors of the deuteron and of the virtual photon, respectively.

In principle, the suppression $\sim 1/\mathscr{P}^2$ of the z diagram which results from the small value of the energy denominator for certain components μ and λ can be offset by a numerator $\sim \mathscr{P}$ in (2). Our analysis of the matrix element with both vertices shows that the zeroth and longitudinal ($\mu = 0.3$) components of the electromagnetic current (for $\lambda = 0, \ldots, 3$) are "good"; for these components, the z diagram is suppressed $\sim 1/\mathscr{P}^2$.

In general, the spin structure of the DNN vertex in the h_∞ time-ordered perturbation theory can be written

$$A_\lambda^{(1)} = \Gamma_1 \gamma_\lambda + \frac{(p_1 - n)_\lambda}{2m} \Gamma_2 + \frac{\hat{\Delta}}{m} \left[\Gamma_3 \gamma_\lambda + \frac{(p_1 - n)_\lambda}{2m} \Gamma_4 \right] + \frac{\Delta_\lambda}{2m} \left[\Gamma_5 + \frac{\hat{\Delta}}{m} \Gamma_6 \right], \tag{3}$$

where $\Delta = p_1 + n - d = p_1 + q - p$, $\Delta = (\Delta_0, O)$, and $\Gamma_{1,\ldots,6}$ are generally real functions of the two independent arguments Δp_1 and Δd, $\Gamma_i(\Delta p_1, \Delta d) \equiv \Gamma_i(\eta \mathbf{k}^2, \mathbf{k}^2) \equiv \Gamma_i(x,z)$ and the kinematics is such that $\mathbf{k}^2 = m_\perp^2/4\eta(1-\eta) - m^2 = \mathbf{k}_\perp^2 + \mathbf{k}_3^2$; $m_\perp^2 = m^2 + \mathbf{k}_\perp^2$; $\Delta_0 = 2\mathbf{k}^2/\mathscr{P} = (2\mathbf{k}_\perp \mathbf{q}_\perp - \mathbf{q}_\perp^2)/2\eta\mathscr{P} + M\nu/\mathscr{P}$; $x = (m^2 - t)/M^2$; $z = (M_t^2 - M^2)/M^2$. The light-cone variables η and $k_\perp$ are related to the laboratory momenta by

$$\eta = 1 - (E_n \quad n_3)/M; \quad n_3 = (\sqrt{-q^2}\,|\mathbf{nq}| + \nu\mathbf{nq})/\mathbf{q}^2;$$
$$k_\perp = (\sqrt{-q^2}\,\mathbf{nq} - \nu|\mathbf{nq}|)/\mathbf{q}^2; \quad \mathbf{k}_\perp^2 = \mathbf{n}^2 - n_3^2; \quad E_n^2 = m^2 + \mathbf{n}^2. \tag{4}$$

The quantization axis is chosen along $\vec{\rho}$ $[\rho = (1,\vec{\rho}), \rho q = 0, \rho^2 = 0]$ in order to suppress the contribution of many-particle intermediate states to the extent possible.

For a qualitative analysis of the effects of the virtual nature of the intermediate states we adopt the simple model $\Gamma_{3,4} = -\Gamma_{1,2}/2$, $\Gamma_5 = \Gamma_6 = 0$, ignoring the x dependence of the functions Γ_i. Using the "good" components in (2), we find the following results for unpolarized particles in a coplanar reaction geometry:

$$\frac{d^3\sigma}{dE_2 d\Omega_2 d\Omega_p} = \mathcal{K}\left(\frac{d\sigma}{d\Omega_2}\right)_R G(x,z); \quad \mathcal{K} = \frac{E_n |\mathbf{p}| \delta(q^2,\nu,x)(k_1p_1)[k_2(k_1+p_1)]}{4E_1E_2\eta^2(1-\eta)\left[1-\frac{\mathbf{pq}}{\mathbf{p}^2}\frac{E_p}{(M+\nu)}\right]} \tag{5}$$

$$\frac{3(2\pi)^3}{m}G(x,z) = [3-2x(1+x)+xz(2+x)]\Gamma_1^2 - (2x+z)(1-x-x^2+xz/2)\Gamma_1\Gamma_2$$

$$+ x(2-x+z)(x^2-xz+z+z^2/4)\Gamma_2^2; \quad E_p^2 = m^2 + \mathbf{p}^2,$$

$k_{1,2} = (E_{1,2}, \mathbf{k}_{1,2})$ are the momenta of the initial and final electrons; and $(d\sigma/d\Omega_2)_R$ is the Rosenbluth cross section for scattering by the proton at the momentum p_1. The functions $\Gamma_{1,2}$ are related to the light-cone deuteron functions u and w (Ref. 9). The factor $\delta(q^2,\nu,x)$, which has a complicated structure, arises because the virtual nature at the electromagnetic vertex is taken into account here only for the good components of the electromagnetic current and also because we have used the requirement of gauge invariance. Under the kinematic conditions of Refs. 1 and 2 we have $\delta(q^2,\nu,x)\approx 1$ over the entire x range. At higher values of q^2 and x, however, the factor $\delta(q^2,\nu,x)$ may be quite different from unity (under the conditions of Ref. 3 we would have $\delta \sim 1.4$–1.6). We see that the function $G(x,z)$ is determined exclusively by the dynamics of the bound system and is independent of q^2 and ν, which are kinematic reaction conditions. On the basis of (5) we interpret $G(x,z)$ as the momentum distribution of the nucleons in the deuteron. The virtual nature of the intermediate states thus gives rise to qualitative changes in the dependence of the momentum distribution of the nucleons on $k_\perp$ and k_3 separately. A dependence of this sort on the two arguments $\mathbf{k}^2$ and $\vec{\rho}\mathbf{k}$ in terms of relativistic wave functions was first predicted by Karmanov.[5] We wish to define two limiting classes of processes. a) Processes with increasing $m_\perp^2$: $\mathbf{k}^2 = \mathbf{k}_\perp^2$, $k_3 = 0$, corresponding to motion along the surface of the light cone. A necessary condition here is

$$E_n - m = n_3, \tag{6}$$

b) Processes with fixed $m_\perp^2$: $\mathbf{k}^2 = k_3$, $\mathbf{k}_\perp = 0$, corresponding to motion in the direction perpendicular to the surface of the light cone. This case occurs under the condition

$$\mathbf{n}^2 = \mathbf{n}_3^2 . \tag{7}$$

All the variables in (6) and (7) are in the laboratory frame. Using expressions (6) and (7), we can easily distinguish between the longitudinal (b) and transverse (a) processes experimentally.

We should point out that to ignore the virtual nature of the situation here (the energy nonconservation $\sim 1/\mathscr{P}$ in the h_∞ time-ordered perturbation theory), as has been the widespread practice,[7,8] leads to a qualitatively different expression for the momentum distribution of the nucleons $[\delta(q^2,\nu,x)\equiv 1]$:

$$G(\mathbf{k}^2) = \frac{\epsilon}{m}[u^2(\mathbf{k}^2) + w^2(\mathbf{k}^2)]/(2\pi)^3,\ \epsilon^2 = m^2 + \mathbf{k}^2,\quad \int G(\mathbf{k}^2)\frac{m\,d^3\mathbf{k}}{\epsilon(k)} = 1. \qquad (8)$$

In deriving (8) we used the assumption[8] that the deuteron state in the infinite-momentum frame is formed long before the interaction and that the expansion is in real NN states; we are using the angular condition of Ref. 6 and thus the crude density matrix

$$\rho_{\lambda\sigma} = g_{\lambda\sigma} - d_\lambda d_\sigma/M^2 \underset{\Delta\to 0}{\approx} g_{\lambda\sigma} - (p_1+n)_\lambda(p_1+n)_\sigma/M_t^2.$$

Numerical calculations of the G function on the basis of Eqs. (5) and (8) show that they are the same only for processes (a) (and only if the x dependence of the functions Γ_i is ignored!). The reason is that longitudinal polarizations of the deuteron, which increase with $\mathscr{P}$, play an importnat role in processes (b); these polarizations cancel the terms $\sim 1/\mathscr{P}$ in (3). Consequently, the assumption of Ref. 8 and thus expression (8) for the momentum distributions of the nucleons are apparently justified only for processes (a), which "evolve" along the surface of the light cone (and even in this case, they are valid only approximately).

Figure 1 shows functions $G(x,z)$ calculated from (5): $G_\perp$ for processes (a) and $G_\parallel$ for processes (b). The relativistic momentum distributions of the nucleons are independent of q^2 and ν only in the scale, for both processes (a) and processes (b). The $|\mathbf{n}|$ scale,[1)] which is widely used in research on the momentum distributions of the nucleons in nuclear physics, cannot be used in the relativistic region, since different values of $\mathbf{n}^2$ will correspond to a given value of $\mathbf{k}^2$ when q^2 and ν are different. This circumstance is the reason for the false discrepancy (by a factor of 5.6 at $|\mathbf{n}| = 0.335$ GeV/c) between the data reported by the SLAC and Saclay groups.[3] The Saclay data are presented in Ref. 3 in the $|\mathbf{n}|$ scale, while the extraction procedure and the kinematic conditions used by the SLAC group lead to $y^2\approx\mathbf{k}^2$ ($\mathbf{k}_3^2\approx(\nu^2/\mathbf{q}^2)y^2$; $\mathbf{k}_\perp^2\approx(\mathbf{q}_\perp^2/\mathbf{q}^2)y^2$; $\nu^2\ll\mathbf{q}^2,\mathbf{q}_\perp^2\sim\mathbf{q}^2$). Figure 1 shows the SLAC and Saclay experimental data after our processing with the help of (4) and (5) in a common k scale. The discrepancy does not exceed 40%. The theoretical curve calculated from Eqs. (5)

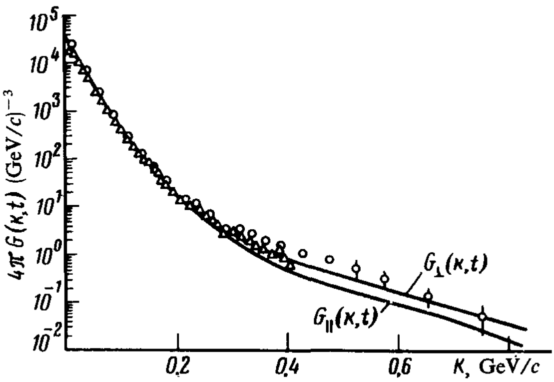

FIG. 1.

and (8) with the Paris-potential wave function[10] agrees well with the experimental data. We should also point out that the SLAC data at $k \geqslant 0.2$ GeV/c seem to be substantially distorted because of the approximate processing procedure[3] [cf. (5) and (8) with (3)–(6) from Ref. 3].

Experimental study of the momentum distributions of the nucleons in the deuteron at $k > 0.2$ GeV/c for processes (a) and (b) is of major interest. Comparison of these processes will make it possible to observe an effect due to the dependence of the relativistic wave function on the additional argument $k_3 = \vec{p} \cdot \mathbf{k}$, which is not used in the model of Ref. 6 and which is introduced in Ref. 5. For this purpose we need direct measurements of the processes $eD \to e'pn$, with ep coincidences being detected under conditions (6) and (7). The relationship between q^2 and ν is fixed by the position of the maximum of the quasielastic peak.

Let us briefly summarize the results of this study. 1) It has been shown possible to single out the momentum distributions of the nucleons in the relativistic region. 2) It has been shown for the first time that the relativistic momentum distributions of the nucleons must be studied, and (a point deserving even more emphasis) the results obtained at various values of q^2 and ν must be compared, in the k scale. 3) For the first time, a method has been proposed for experimentally detecting the additional dependence of the relativistic wave function on the direction of the vector $\vec{p}$. 4) It has been shown that the pronounced discrepancy between the experimental reported by the SLAC and Saclay groups is a consequence of the use of a nonrelativistic $|\mathbf{n}|$ scale.

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[1)]Here $|\mathbf{n}|$ and $|\mathbf{k}|$ are the relative momenta of the intermediate nucleons in the laboratory frame and in their center-of-mass frame, respectively.

[1]Yu. P. Antuf'ev, V. L. Agranovich, V. S. Kuz'menko, and P. V. Sorokin, Yad. Fiz. **25**, 1123 (1977) [Sov. J. Nucl. Phys. **25**, 736 (1977)].
[2]M. Bernheim, A. Bussiere, J. Mougey, *et al.*, Nucl. Phys. A **365**, 349 (1981).
[3]P. Bosted, R. G. Arnold, S. Rock, and Z. Szalata, Phys. Rev. Lett. **49**, 1380 (1982).
[4]P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[5]V. A. Karmanov, Zh. Eksp. Teor. Fiz. **71**, 399 (1976) [Sov. Phys. JETP **44**, 210 (1976)]; Zh. Eksp. Teor. Fiz. **83**, 3 (1982) [Sov. Phys. JETP **56**, 1 (1982)].
[6]M. V. Terent'ev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)].
[7]L. A. Kondratyuk and M. V. Terent'ev, Yad. Fiz. **31**, 1087 (1980) [Sov. J. Nucl. Phys. **31**, 561 (1980)]; L. A. Kondratyuk and M. I. Strikman, Preprint ITEP-25, Moscow, 1983, p. 50.
[8]L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 217 (1981).
[9]E. V. Inopin, S. I. Nagornyĭ, and A. E. Inopin, Preprint KhFTI-57, Khar'kov Physicotechnical Institute, Khar'kov, 1982, p. 15.
[10]M. Lacombe *et al.*, Phys. Lett. B **101**, 139 (1981).

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