Propagation of spin waves in a moving domain wall

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(Submitted 19 October 1983; resubmitted 6 February 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 8, 348–351 (25 April 1984)

The spectrum of spin waves in a moving domain wall is investigated. It exhibits characteristic "relativistic" peculiarities which are attributable to the difference between the longitudinal and transverse mass densities of the wall.

1. Spin waves propagating in a domain wall at rest have been studied in detail.[1–3] It is interesting to study, especially in connection with recent experiments,[4] the spin waves in a moving domain wall. We shall examine this problem for the example of a flat wall in a weak, orthorhombic ferromagnet, e.g., yttrium orthoferrite. This choice is determined by two factors. First of all, the simplest regime of motion of a domain wall, described by the self-similar, simple wave solutions of the nonlinear Landau-Lifshitz equations $f(x-ut)$, where u is the velocity of the domain wall which varies from zero to a limiting value $u_{\lim}=c$ equal to the velocity of spin waves in the linear section of the spectrum ($\omega=ck$), is realized in this material. Second, the mathematical description of this regime of motion in these materials is based on the Sine-Gordon equation, which has been studied in detail.

We shall represent the equation describing the motion of a domain wall in an orthorhombic antiferromagnetic crystal in the form

$$\ddot{\varphi} - c^2 \nabla^2 \varphi + \omega_g^2 \sin\varphi \cos\varphi = -\alpha \omega_E \dot{\varphi} - \omega_d \omega_H \sin\varphi, \tag{1}$$

where $\omega_g = \gamma(H_A H_E)^{1/2}$ is the gap in the spin-wave spectrum, $\omega_H = \gamma H, \omega_d = \gamma H_D, \omega_E = \gamma H_E$, H_D, H_A, H_E are the Dzyaloshinskii field, the effective anisotropy field, and the exchange field, respectively. The external field is assumed to be constant in time, and α is a dimensionless damping constant. This equation is derived in Ref. 5. The angle φ determines the orientation of the antiferromagnetism vector **l** in the turning plane) for example, in the ac plane in yttrium orthoferrite or in the ab plane in dysprosium orthoferrite at low temperatures).

The motion of a flat domain wall is described by the "one-dimensional" solution of Eq. (1)

$$\varphi_0 = 2 \arctan \exp \left[\pm \frac{y - ut}{\Delta(u)} \right], \tag{2}$$

where $\Delta(u) = \Delta_0(1 - u^2/c^2)^{1/2}, \Delta_0 = c/\omega_g, u = \mu H [1 + (\mu H/c)^2]^{-1/2}, \mu = \gamma \Delta_0 \alpha^{-1} \times (H_D/2H_E)$.

2. We shall examine small perturbations in a magnetic system described by Eq. (1) in the presence of a moving domain wall (2). For this purpose, we shall linearize Eq. (1)

$$\ddot{\Psi} - c^2 \nabla^2 \Psi + \omega_g^2 \cos 2\varphi_0 \Psi = -\alpha\omega_E \ddot{\Psi} - \omega_d \omega_H \cos\varphi_0 \Psi. \tag{3}$$

The solution of Eq. (3), which vanishes in the limit $y \to \pm\infty$, has the following form in a moving coordinate system $\tilde{q} = y - ut$

$$\Psi = \frac{\exp\{ i\omega [t - u\tilde{q}c^{-2}/(1 - u^2/c^2)] \} \exp(-i\mathbf{k}_\perp \mathbf{r}_\perp)}{\mathrm{ch}(\tilde{q}/\sqrt{1 - u^2/c^2})}, \tag{4}$$

where the frequency ω is related to the wave vector of transverse perturbations $\mathbf{k}_\perp$ by the equation

$$\omega^2 - i\omega\Gamma - k_\perp^2 c^2 (1 - u^2/c^2) = 0, \tag{5}$$

where $\Gamma = \alpha\omega_E$. We note that spin wave (4) has a nonzero component $k_y = k_\parallel = (\omega u c^{-2})/(1 - u^2/c^2)$, in contrast to the case of a domain wall at rest.

The measured oscillations are flexural oscillations of the domain wall. This is most easily demonstrated by using the "abbreviated" description of oscillations, introducing the coordinate of the center of the wall $q = q(x, z, t)$. The equation for $q(\mathbf{r}_\perp, t)(\mathbf{r}_\perp = (x, z))$ which is obtained from (1) with the help of the well-known procedure for eliminating secular terms (see Refs. 2, 5, and 6), has the form

$$\frac{\partial}{\partial t}(m\dot{q}) + m\dot{q}\Gamma - \nabla_\perp \sigma \nabla_\perp q = 2M_S H, \tag{6}$$

where $m = m_0(1-\dot{q}^2/c^2)^{-1/2}$, $\sigma = mc^2$, $m_0 = 2M_S\gamma^{-2}\Delta_0^{-1}H_D^{-1}$. If we set $q = ut + q$, where $\dot{\tilde{q}} \ll u$, then Eq. (6) can be written

$$m_\perp \ddot{\tilde{q}} + m_\perp \dot{\tilde{q}}\Gamma - c^2 m_\parallel \nabla_\perp^2 q = 0\,, \tag{7}$$

where $m_\parallel = m_0(1-u^2/c^2)^{-1/2}$, $m_\perp = m_0(1-u^2/c^2)^{-3/2}$. Setting $\tilde{q} = q_0 \times \exp[i(\omega t - \mathbf{k}_\perp \mathbf{r}_\perp)]$, from (7) we obtain (5). We see that (4) represents flexural oscillations of a moving domain wall.

3. The solution of Eq. (5) is

$$\omega = -i\Gamma/2 \pm [-(\Gamma/2)^2 + (c^2 - u^2)k_\perp^2]^{1/2}\,. \tag{8}$$

For $k_\perp^* = \Gamma/2c(1-u^2/c^2)^{1/2}$ the dispersion law changes from a reactive (wave) to a diffusion law. Thus, for small-scale perturbations with $k_\perp \gg k_\perp^*$ it follows from (8) that

$$\omega = v_\perp k + i\Gamma/2\,, \tag{9}$$

where $v_\perp = c(1-u^2/c^2)^{1/2}$ is the group velocity of flexural oscillations. Thus we have the important relation $v_\perp^2 + u^2 = c^2$, which shows that the velocity of Winter excitations depends on the velocity of the domain wall and cannot exceed the limiting value. This fact must be taken into account in analyzing, for example, the deceleration of a domain wall due to excitation of spin waves in it. Since $v_\perp \to 0$ as $u \to c$, the distance over which small-scale perturbations propagate $r_\perp \sim v_\perp/\Gamma$ also approaches zero. The characteristic dependence $v_\perp(u)$ is a result of the difference—familiar from relativistic mechanics—between the longitudinal $m_\parallel$ and transverse $m_\perp$ masses presented above.

Long-lived large-scale fluctuations ($k \ll k_\perp^*$) obey a diffusion-type dispersion law,

$$\omega = iDk_\perp^2\,, \qquad D = D_0(m_\parallel/m_\perp) = D_0(1-u^2/c^2),\quad D_0 = c^2/\Gamma\,. \tag{10}$$

The decay time of large-scale fluctuations $\tau = 1/Dk_\perp^2$ increases as $u \to c$. In this case, the second derivative with respect to time in Eq. (7) can be ignored, so that slow changes in front of the moving domain wall can be described by the diffusion equation $\dot{\tilde{q}} = D\nabla_\perp^2\tilde{q}$. For this reason, the evolution of the initial perturbation $\tilde{q}_{t=0} = \tilde{q}_0\delta(r_\perp)$ in the limit $u \to c$ obeys the diffusion law

$$\tilde{q} = \frac{\tilde{q}_0}{(4\pi Dt)^n} \exp\left(-\frac{r_\perp^2}{4Dt}\right),$$

where n is the dimensionality of the perturbation. Fluctuations of this type can be excited by local inhomogeneities as a result of the motion of the domain wall.

In conclusion we note that in order to assure the stability of a flat wall, an inhomogeneous external magnetic field is usually used in the experiment. In its presence, and taking into account the magnetostatic interactions, the spectrum of flexural oscillations for small value of k has a more complicated nature. Our results are valid for $k \gtrsim Q = M_S^2/m_0c^2$. For typical experimental conditions in $YFeO_3$ we would have $Q = 10^2$ cm^{-1}.

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Translated by M. E. Alferieff
Edited by S. J. Amoretty