Flute solitons in a plasma with shear

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A flute instability can give rise to vortices localized near magnetic lines of force, according to the two-fluid approximation. The vortices in turn cause a plasma convection across magnetic surfaces without perturbing the magnetic field.

The flute instability and the variety of forms it takes occupy a prominent place in the theory of plasmas, because these instabilities set an upper limit on the plasma pressure which can be maintained in a magnetic confinement system. As the amplitude of the perturbations grows, saturation may set in. A nonlinearity found in Ref. 1 causes unstable perturbations to convert into an array of solitons: localized vortices of plasma flow across the magnetic field. The vortices move at an arbitrary velocity. Vortices with a small characteristic dimension a, which do not perturb the magnetic field, were studied. It was believed that when a was large the shear would perturb the magnetic field, thereby increasing the energy of the vortices. Large vortices therefore appeared unfavorable from the energy standpoint. In the present letter we use the two-fluid approximation to show that this is not the case for vortices which are frozen in the electrons.

Following Ref. 2, we simplify the equations describing the flute waves. We make use of the circumstance that the characteristic frequency is small in comparison with ω_{Bi} (the ion cyclotron frequency) and that the ion Larmor radius r_{Bi} is small in comparison with a, which is in turn much smaller than the characteristic longitudinal dimension. Under these conditions we can restrict the analysis to a small neighborhood of the magnetic surface of interest (ordinarily a resonant surface), on which the magnetic field is $\mathbf{B}_0$. Assuming a/R and $a\kappa_e$ to be small ($1/R = -\partial \ln B_0/\partial x + 1/R_1$, where R_1 is the radius of curvature of $\mathbf{B}_0$, and κ_e is the characteristic dimension for the change in the electron pressure), we find, in the local approximation, two-dimensional equations in the x, y plane, which is perpendicular to $\mathbf{B}_0$ and whose y axis lies on the magnetic surface. The current continuity equation $\mathrm{div}\mathbf{j} = 0$ becomes

$$\rho_0 d\Delta\chi/dt + \omega_{Bi}^{-1}\,\mathrm{div}\,\{p_i, \nabla\chi\} + R^{-1}\,\partial p/\partial y = \{\psi, \Delta\psi\}/4\pi,$$

$$\chi = \frac{c\phi}{B_0};\ \{p, \chi\} \equiv \frac{\partial p}{\partial x}\frac{\partial \chi}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial \chi}{\partial x};\ \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \{\chi, \ldots\}. \qquad (1)$$

Here ϕ is the electric potential; $p = p_i + p_e$; $p_{i,e}$ are the ion and electron pressures; ρ_0 is the constant part of the plasma density; and ψ is the transverse magnetic flux, which is equal to the z component of the vector potential. According to Ref. 2, the other components can be ignored. The flux ψ describes both the perturbed magnetic field and the unperturbed field, which produces the shear. The right side of (1) is proportional to the divergence of the longitudinal current $\mathbf{j}_\parallel$. Assuming that the ion component of $\mathbf{j}_\parallel$ is much smaller than the electron component, we write equations for the pressure as follows:

$$dp_i/dt = 0;\ \ dp_e/dt = \{\psi, \Delta\psi\}\,p_{0e}/\rho_0\omega_{Bi}4\pi, \qquad (2)$$

where p_{0e} is the constant part of the electron pressure. From the equation of motion for electrons along the magnetic field we find

$$d\psi/dt = \{p_e, \psi\}/\rho_0\,\omega_{Bi}. \qquad (3)$$

We have thus derived the two-dimensional system of equations (1)–(3) in the two-fluid approximation. This system of equations agrees with the three-dimensional equations of Ref. 3, aside from the second term in (1), which we believe is written more correctly in the form derived in Ref. 4, so that energy is conserved. We seek a localized steady-state solution of system (2), (3). As in Ref. 1, we assume that this solution is traveling along the y axis at a velocity u. Under the locality conditions, u is an eigenvalue of the solution

$$p_i = p_{0i}[1 + \kappa_i(x - \chi/u)];\ \ \chi = \chi(x, y - ut); \qquad (4)$$

$$p_e = p_{e0}[1 + \kappa_e(x - \chi/u)];\ \ u = u_e; \qquad (5)$$

$$\psi = \psi(x);\ \ u_{i,e} = \pm c\kappa_{i,e}T_{0i,e}/eB_0, \qquad (6)$$

and ψ is an arbitrary function of x. The magnetic field which produces the shear is directed along the y axis and is $B_y = \partial\psi/\partial x$, $B_y(0) = 0$. This field is not perturbed, because solution (4)–(6) is frozen in the electrons (is traveling at the electron drift velocity u_e). Substituting (4)–(6) into (1), we find from the latter

$$\Delta\chi - \omega_{Bi}x(u_e - u_i)/u_eR = F[\chi - (u_e - u_i)x], \qquad (7)$$

where F is an arbitrary function.

Equation (7) is solved by the method of Refs. 1, 5, and 6. We introduce a circle of radius a. Choosing F to be different linear functions inside and outside this circle, we find

$$\chi = b_0 J_0 + b_1 J_1 x/r + b_2 x; \quad r \leqslant a;$$
$$\chi = b_3 K_0 + b_4 K_1 x/r; \quad r \geqslant a; \; r^2 = x^2 + (y - ut)^2. \tag{8}$$

Here $J_{0,1}$ are Bessel functions of argument $k'r$, and $K_{0,1}$ are modified Hankel functions of argument $k''r$, which fall off as $\exp(-k''r)$; $k'' = (\omega_{Bi}/u_e R)^{1/2}$. The constants b_1–b_4 and k' are determined by requiring that F be single-valued and that χ and $\nabla\chi$ be continuous at $r = a$. The constants a and b_0 are left arbitrary. If $b_0 \neq 0$, the vorticity $\Delta\chi$ is discontinuous at $r = a$; this discontinuity is permissible. We find that the electric field is independent of the time in a coordinate system moving at the velocity u_e. There is thus no Landau damping (a resonant interaction of the vortex with the electrons).

When a flute-dissipative instability acts to replenish these vortices, they will evidently fill the entire unstable region. A clear picture of a vortex turbulence of this type was found in Ref. 7 in experiments with a rotating shallow pool of water. The flute vortices are similar to the vortices of electrostatic drift waves with[8] $\nabla T_e = 0$, except that for the latter the phase velocity along z is much lower than the electron thermal velocity or the Alfvén velocity, and the condition $u/u_e > 1$ also holds. Under the condition $\nabla T_e \neq 0$, however, this similarity is disrupted.[9] Another distinction is that the flute vortices can occur because of the condition $R < \infty$, while the drift vortices can occur because of the condition $u > u_e$.

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