Thermodynamics of electrons in disordered conductors with Kondo impurities

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The thermodynamic properties of localized spins in disordered conductors are analyzed. The magnetic susceptibility has orbital components which do not contain small terms of order $(p_F l/\hbar)^{-1} < 1$. Nontrivial contributions to the heat capacity arise in only third order in the coupling constant, in contrast with the situation in the ordinary Kondo problem.

The exchange interaction of conduction electrons with magnetic impurities has several qualitative consequences for the dependence of the kinetic and thermodynamic properties on the temperature and the magnetic field. This circumstance was first pointed out by Kondo, who showed that a correction to the conductivity which depends logarithmically on the temperature arises in third-order perturbation theory. Just recently, the Kondo problem has been solved exactly.[1]

In a disordered system, the weak localization effects increase the probability for an electron to return and to interact with a magnetic impurity. The diffusive nature of the electron motion increases the interaction time. As a consequence of these effects, the Kondo logarithmic singularities are accompanied by some stronger singular corrections to the kinetic and thermodynamic quantities.

The Kondo problem in a disordered two-dimensional system was analyzed in Refs. 2 and 3. It was found that a doubly logarithmic temperature dependence arises in the spin susceptibility in second-order perturbation theory.

In the present letter we examine the orbital contributions to the magnetic susceptibility. These effects are remarkable in that they are not small quantities on the order of the parameter $(p_F l)^{-1} < 1$ ($l = v_F \tau$ is the mean free path, $p_F = m^* v_F$ is the Fermi momentum, and $\hbar \equiv 1$). The physical reason why there is no small parameter here is

that the probability for an electron to return to the position of the magnetic impurity is highly sensitive to the magnetic field. In this sense, these contributions are analogous to the corrections to the magnetic susceptibility which result from the interaction of electrons in a Cooper channel.[4,5]

All the results below refer to the region $\tau^{-1} > T > \max\{T_K, \tau_S^{-1}\}$, if the exchange interaction of the electron with the magnetic impurity has the antiferromagnetic sign, $J<0$; here $T_K = \epsilon_F \exp\{1/2J\}$ is the Kondo temperature, and τ_S is the characteristic time for the electron spin relaxation in scattering by magnetic impurities. When the exchange interaction has the ferromagnetic sign, $J>0$, we would have $T_K \gtrsim \epsilon_F$ and we would be dealing with the region $\tau^{-1} > T > \tau_S^{-1}$.

In the three-dimensional case the orbital contribution to the magnetic susceptibility (per impurity spin S) is

$$\delta\chi = -\chi_0 \left(\frac{m}{gm^*}\right)^2 \sqrt{\frac{2\pi}{3}}\, \zeta\left(\frac{3}{2}\right) \frac{\sqrt{T\tau}}{\ln^3 (T_K/T)}, \tag{1}$$

where $\chi_0 = (g\mu_B)^2 S(S+1)/3T$ is the magnetic susceptibility of the free spin, and $\zeta(x)$ is the Riemann zeta function.

In a film of thickness $a < \sqrt{D/T}$ ($D = v_F l/3$ is the electron diffusion coefficient), with the magnetic field directed normal to the film, we would have

$$\delta\chi = -\chi_0 \left(\frac{m}{gm^*}\right)^2 \frac{2\pi^2}{3} \frac{l}{a} \ln^{-3} (T_K/T). \tag{2}$$

In addition to (1) and (2) there are always the ordinary logarithmic corrections to the magnetic susceptibility, $\Delta\chi \sim \chi_0 \ln^{-1}(T_K/T)$.

The heat capacity presents an interesting situation: In a disordered conductor, a correction to the heat capacity arises even in third order in the coupling constant, while it does not appear until fourth order in the pure case.

For a three-dimensional sample the correction to the heat capacity per magnetic impurity is

$$\delta C = \frac{3S(S+1)\,\zeta(3/2)}{\sqrt{2\pi}\nu D^{3/2}} \frac{\sqrt{T}}{\ln^3 (T_K/T)}. \tag{3}$$

Here $\nu = (m^* p_F)/2\pi^2$ is the state density at the Fermi level.

In a film we would have

$$\delta c = \frac{4S(S+1)}{Da\nu} \ln^{-3} (T_K/T). \tag{4}$$

In a dirty conductor the effect described by expressions (3) and (4) can make a definite contribution to the heat capacity at all temperatures above T_K.

To demonstrate the derivation of these expressions, we note that the Hamiltonian of the exchange interaction is

$$\mathcal{H} = -\frac{J}{\nu} \mathbf{S} \boldsymbol{\sigma}_{\alpha\beta} \psi_\alpha^+ \psi_\beta, \tag{5}$$

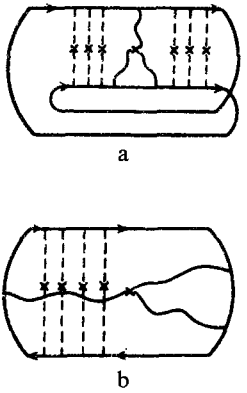

FIG. 1.

where the operators S represent the impurity spin, $\vec{\sigma}_{\alpha\beta}$ are the Pauli matrices, and the operators ψ_α^+ and ψ_β create and annihilate an electron with a spin projection α and β.

Figure 1a shows the Cooper correction $\delta\Phi_C(T,H)$, while Fig. 1b shows the diffusion correction $\delta\Phi_D(T)$ to the thermodynamic potential. The wavy lines correspond to the spin propagator

$$<T_\tau S^i(\tau_1) S^k(\tau_2) S^f(\tau_3)>_{\omega_1\omega_2\omega_3}.$$

We note that the diffusion which appears in the expression for the thermodynamic potential describes fluctuations of the electron spin density which decay because of the scattering of electrons by the same magnetic impurities.[6] The spin-spin scattering also suppresses the Cooper contribution to the thermodynamic potential. Nontrivial corrections arise under the condition $T\tau_S > 1$, under which we can ignore the spin damping of the diffusion and of the cooperon.

A direct calculation yields

$$\begin{aligned} \delta\Phi &= \delta\Phi_D(T) + \delta\Phi_C(T, H), \\ \delta\Phi_D(T) &= \delta\Phi_C(T, H=0). \end{aligned} \tag{6}$$

It is sufficient to calculate the Cooper contribution $\delta\Phi_C(T,H)$. A cooperon in a magnetic field is described by[7]

$$C = [Dq_z^2 + \omega_H(n + \frac{1}{2}) + |\omega_m|]^{-1}, \tag{7}$$

where $\omega_m = 2\pi mT$, $\omega_H = (4DeH)/c$, and q_z is the momentum component along the magnetic field.

Using (7), we find (d is the dimensionality of the sample)

$$\delta\Phi_C(T,H) = -\frac{4J^3S(S+1)}{\nu a^{3-d}}\sum_{\omega_m>0}^{\tau^{-1}} \frac{\omega_H}{D}\sum_{n\geqslant 0}\int\left(\frac{dq_z}{2\pi}\right)^{d-2}\frac{1}{Dq_z^2+\omega_H(n+1/2)+\omega_m}. \tag{8}$$

Carrying out the summation in (8), and focusing on the region of low frequencies and small momenta, we find

$$\delta\Phi_C(T,H) = -\frac{J^3S(S+1)\,\omega_H}{\pi\nu a^{3-d}D}\left(\frac{T}{2D}\right)^{\frac{d-2}{2}} g_{\frac{2-d}{2}}\left(\frac{\omega_H}{4\pi T}\right), \tag{9}$$

where

$$g_\mu(x) = \int_0^\infty \frac{dt\,t^\mu}{(l^t-1)\,\mathrm{sh}\,xt}. \tag{10}$$

The integral in (10) converges if $\mu > 1$, and the quantity $g_\mu(x)$ at $\mu = -1/2$ and 0 is understood in the sense of an analytic continuation along μ.

Expression (9) has been derived in third order in J; in general, we would have to use the substitution $J \to (2\ln T_K/T)^{-1}$.

Results (1)–(4) can be found from (6) and (9) by differentiating with respect to the magnetic field and the temperature.

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[1]P. B. Vigman, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 392 (1980) [JETP Lett. **31**, 364 (1980)].
[2]F. J. Ohkawa, in: Anderson Localization (Y. Nagaoka and H. Fukyama eds.), Springer-Verlag, New York, 1982.
[3]F. J. Ohkawa, H. Fukyama, and K. Yosida, J. Phys. Soc. Jpn. **52**, 1701 (1983).
[4]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **67**, 647 (1974) [Sov. Phys. JETP **40**, 321 (1975).
[5]B. L. Al'tshuler, A. G. Aronov, and A. Yu. Zyuzin, Zh. Eksp. Teor. Fiz. **84**, 1525 (1983) [Sov. Phys. JETP **57**, 889 (1983)].
[6]B. L. Al'tshuler, A. G. Aronov, and A. Yu. Zyuzin, Solid State Commun. **44**, 137 (1982).
[7]B. L. Al'tshuler, D. E. Khmelnitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. **B22**, 5142 (1980).

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