Nonlinear current-voltage characteristic of a metallic film

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A new phenomenon is predicted: the existence of a nonlinear current-voltage characteristic (IVC) of a thin metallic specimen at low temperatures. The nonlinearity is linked with the effect of the intrinsic magnetic field of the current I on the dynamics of conduction electrons. This magnetodynamic mechanism is now the only known mechanism of nonlinearity in metals that is not related to overheating. In the classical limit the voltage drop across the specimen is proportional to $I^{1/2}$. Quantization of electronic trajectories leads to oscillations of the IVC and a negative differential resistance. Numerical estimates demonstrate that the predicted effects can be observed experimentally.

1. In the linear regime, the static electrical conductivity of a metallic film $(d \ll 1)$ with diffuse boundaries is described by the expression[1]

$$\sigma = \frac{3}{4}\sigma_0 \frac{d}{l} \ln \frac{l}{d} , \tag{1}$$

where σ_0 is the conductivity of the bulk specimen, d is the thickness of the film, and l is the mean free path. The conductivity σ_1 is due to the group of effective electrons that do not collide with the boundaries of the specimen throughout the entire time of free flight. The relative number of such electrons is on the order of d/l.

The dc current I which flows along the film creates a nonuniform constant magnetic field within the specimen. This field is distributed antisymmetrically in the bulk of the plate, vanishing at the center of the plate and assuming values equal in magnitude but opposite in sign, H and $-H$, at the opposite faces,

$$H = \frac{2\pi}{c} \frac{I}{D} . \tag{2}$$

Here D is the horizontal length of the film in a direction orthogonal to the current, and c is the velocity of light. As the current I is increased, the intrinsic magnetic field increases and produces a progressively greater effect on the dynamics of the effective electrons. Because of the sign-alternating nature of the magnetic field in the specimen, the energy of interaction of an electron with this field $-e\mathbf{A}\mathbf{v}/c$ has a minimum at the center of the field ($\mathbf{A}$ is the vector potential and $-e$ and $\mathbf{v}$ are the charge and velocity of the electron). This means that there is a group of electrons which are captured in such a well which move in the plate along a trajectory winding near the surface in which the magnetic field changes sign. In a direction orthogonal to the boundary of the film they execute a periodic motion. With $d \ll R = cp_F/eH$ (p_F is the Fermi momentum) the relative number of capture electrons is on the order of $(d/R)^{1/2}$ and their effective conductivity is described by the expression

$$\sigma_{\text{cap}} \sim \sigma_0 (d/R)^{1/2} \sim I^{1/2}. \tag{3}$$

It is evident from a comparison of Eqs. (1) and (3) that when the conditions

$$d \ll (Rd)^{1/2} \ll l, \tag{4}$$

are satisfied, the electric field in the metal is determined completely by the group of captured electrons and the electrical conductivity of the film depends on the current I. Thus the action of the intrinsic magnetic field of the current leads to a nonlinear current-voltage characteristic (IVC): according to (3), a voltage drop in the specimen in the direction of the current flow is proportional to $I^{1/2}$.

The exact calculation of IVC involves a simultaneous solution of the equations of magnetostatics and Boltzmann's kinetic equation, which includes the Lorentz force resulting from the magnetic field of the current. Leaving the calculations for a more detailed publication, we will write the final result

$$U = r(I_0 I)^{1/2},$$

$$r = \frac{L}{dD\sigma_l}, \qquad I_0 = 0.51\,\frac{c^2 p_F D d \ln^2 d/l}{el^2} \tag{5}$$

Here r is the resistance of the film in the linear regime, L is the length of the specimen in the direction of the current flow, I_0 is the characteristic value of the current at which the transition to the nonlinear regime occurs, because of the inequalities (4), Eq. (5) can be used in the region where $I > I_0$.

2. Because of periodicity of the motion of captured electrons in a direction perpendicular to the plane of the plate, their state must be quantized. We shall examine the effect of quantization on the shape of the IVC, which turns out to be important when conditions (4) are satisfied.

As the current I is increased, the number of quantized electronic trajectories fitting into the specimen changes discretely; as a result are "jumps" in the conductivity which give rise to quantum oscillations of the IVC

$$U(I) = U_{cl}(I)\left\{1 - \frac{1{,}53T}{\epsilon_F N^{1/2}} \sum_{k=1}^{\infty} \frac{\sin(2\pi kN)}{k^{1/2}\,\mathrm{sh}(\pi^2 kNT/2\epsilon_F)}\right\}, \tag{6}$$

$$N = 0.30 \times p_F d (D/R)^{1/2}/\hbar.$$

Here $U_{cl}(I)$ is the classical IVC which is determined by Eqs. (5), T is the temperature of the specimen and ϵ_F is the Fermi energy. The integral part of the quantity N represents the number of quantum electronic states. Expression (6) is obtained in the semiclassical approximation when N is larger than unity.

Since T/ϵ_F is much less than unity, the oscillations of $U(I)$ must always be present, whose "period" would be $\Delta I = 2I/N$. At sufficiently low temperatures when

$$T \lesssim \epsilon_F / N^2, \tag{7}$$

the oscillations of the IVC become so sharp that sections with negative differential resistance appear on the curve of the dependence $U(I)$. In this case, in a regime with fixed voltage U, nonstationary domains of the electric and magnetic fields can appear in the specimen.

It should be emphasized that in contrast to other known quantum oscillations caused by the passage of a regular quantum level through the Fermi level, the oscillations of the IVC examined here occur due to the appearance of new, winding electronic trajectories that fit into the plate. Because of this difference, there is no need to impose a rigid condition on the temperature of the type $T \lesssim \Delta\epsilon_n$ in the Shubnikov–de Haas and de Haas–van Alfvén effects, which are attributable to the diamagnetic oscillations of the electronic density of states.

We shall make some numerical estimates. In a metallic film with $\epsilon_F \approx 3$ eV, $p_F \sim 10^{-19}$ gm/s $l \approx 0.1$ cm, $T \approx 4.2$ K, $d \approx 10^{-4}$ cm, the nonlinear effects examined above are manifested with a current density exceeding $j_0 \approx 10^5$ A/cm^2. The magnetic field on the surface of the specimen is $H \approx 10$ Oe, the power liberated per unit surface area is $P \approx 10^{-2}$ W/cm^2, and the number of quantized orbits of captured electrons is $N \approx 30$. Under these conditions, inequalities (4) and (7) are satisfied and $I > I_0$.

The mechanism proposed above for the appearance of the nonlinear IVC is very sensitive to the external magnetic field h_0. To observe the predicted effects it is necessary that

$$h_0 / H < (Rd)^{1/2} / l. \tag{8}$$

In conclusion we note that the capture of electrons by an inhomogeneous magnetic field produced by a current is the only known mechanism of electrodynamic nonlinearity in metals that does not involve overheating. In the high-frequency case it leads to nonlinear Landau damping,[2] excitation of current states[3,5] and dependence of the surface impedance on the amplitude of the external electromagentic wave.[6]

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