New relaxation process in the phonon-impurity system in He^3-He^4 solutions

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Acoustic measurements and a theoretical analysis reveal two steps in the relaxation to equilibrium in the phonon system when an impurity is present. The relaxation time is found to be an order of magnitude shorter than in the theory derived previously, resolving the previous discrepancy between theory and experiment.

As was first shown by Khalatnikov and Zharkov,[1] the kinetic phenomena which occur in weak solutions of He^3 in He^4 at temperatures below 0.6 K are determined by an interaction between phonons and impurity excitations. These processes have been studied in measurements of the thermal conductivity,[2] second sound absorption,[3] etc. The results found experimentally for weak solutions, in which phonons are important, differ by more than an order of magnitude from the results calculated from the generally accepted theory of Baym *et al.*[4–7]

One of the most straightforward experimental methods for studying kinetic processes in a phonon-impurity system, and one whose results are among the least ambiguous to interpret, is to directly measure the velocity and absorption of acoustic phonons (first sound). In this letter we report the results of such experiments for solutions with a He^3 mole concentration $x \approx 10^{-3}$–10^{-4} over the temperature range 40–600 mK at wave frequencies of 10, 30, and 50 MHz. We have carried out very precise measurements of the quantity $\Delta c/c$, where c is the speed of sound and $\Delta c = c(T) - c(0)$, by a phase-comparison method. We have also measured the sound-absorption coefficient α. The change $\Delta c/c$ in the solution found experimentally can be

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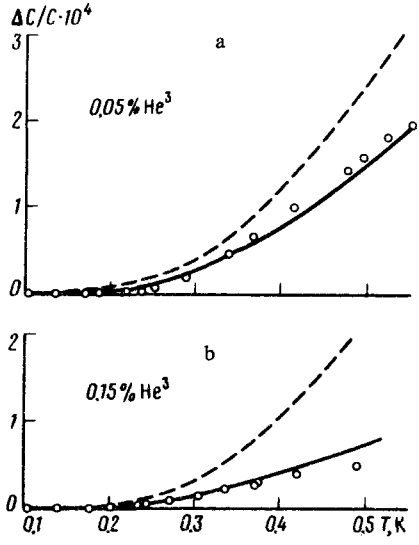

FIG. 1. Temperature dependence of the relative sound velocity at 10 MHz. a: $x = 5\times10^{-4}$. b: $x = 1.5\times10^{-3}$. Dashed curves—calculated from the theory of Refs. 4–7; solid curves—calculated from (6), with the impurity being taken into account.

written

$$\frac{\Delta c}{c} = \frac{\Delta c_i}{c} + \frac{\Delta c_{ph}}{c} + \frac{\Delta c_{phi}}{c}, \tag{1}$$

where the terms on the right side of (1) come from impurities, phonons, and the phonon-impurity interaction, respectively. An estimate of $\Delta c_{\rm phi}/c$ from the theory of Refs. 4–6 yields a negligibly small value. The change $\Delta c_i/c$ was calculated in Ref. 7, and the value of $\Delta c_{\rm ph}/c$ corresponds to the experimental data for He^4. We see from Fig. 1 that the results calculated on $\Delta c/c$ from the theory of Refs. 4–7 differ substantially from the experimental values.

An explanation of this situation regarding the kinetics of weak solutions requires a reexamination of the theory. We have taken the approach of solving the problem of the propagation of sound of arbitrary frequency. When we then take the hydrodynamic limit, we find the times which determine the thermal conductivity and the viscosity, while the kinetic limit gives us the renormalization of the velocity and the absorption of the sound observed in the experiments described above.

We begin with the complete linearized system of equations.[1,4–6] In the collision integrals we single out the eigenfunctions with zero eigenvalues, so that we can apply the τ approximation correctly in the impurity-impurity collision integral with a characteristic time $\tau_{33} \cong 10^{-11}x^{-1}T^{-1/2}$ and in the operator describing the three-particle phonon processes. These processes lead to an equilibrium in the given direction in a time $\tau_{\|} \cong 2.6\times10^{-10}T^{-5}$. Here and below, the temperatures are in kelvins and the times in seconds. The collision integral describing the phonon-impurity interaction can be written exactly by using the results of Ref. 5, according to which the scattering of a

phonon with a momentum ρ by an impurity is determined by the time

$$t_{phi} = \frac{11\rho_4}{m_4 c x p^4}, \tag{2}$$

where $\rho_4 = n_4 m_4$ is the He^4 density in the solution. From this system of equations we find the dispersion relation $\omega = \omega(k)$, whose real part gives us the renormalization of the sound velocity, and whose imaginary part gives us the absorption α. It turns out that the impurity part, $\Delta c_i/c$, agrees with the result derived by Baym[7] by perturbation theory with allowance for thermal expansion, while the value of α_i agrees with the result found in Ref. 6. In the calculations, it is convenient to combine the phonon-impurity part with the phonon part. As a result, the time determining the relaxation of the phonon gas in the presence of an impurity is found to be

$$\tau_{phi} = \int_0^\infty t_{phi} (t_{phi} + \tau_{\|})^{-1} p^4 n' dp \Big/ \int_0^\infty (t_{phi} + \tau_{\|})^{-1} p^4 n' dp, \tag{3}$$

where n' is the derivative of the phonon distribution function with respect to the energy.

Result (3) reflects different mechanisms which are operating to bring the phonon system to equilibrium in the presence of impurities. For weak solutions, for example, the phonon-impurity relaxation time is much longer than the time $\tau_{\|}$, which corresponds to the limit $\tau_{\|} \to 0$. From (3) we then find

$$(\tau_{phi}^0)^{-1} = -(6\pi^2 \rho_{ph})^{-1} \int_0^\infty t_{phi}^{-1} p^4 n' dp, \tag{4}$$

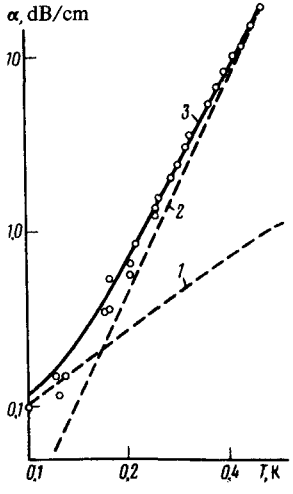

FIG. 2. Temperature dependence of the sound absorption coefficient at 30 MHz with $x = 1.5 \times 10^{-3}$. 1—Impurity contribution according to Ref. 7; 2—phonon contribution calculated from (6); 3—sum of the phonon and impurity contributions.

where ρ_{ph} is the density of the normal component due to phonons. According to (4), the time τ^0_{phi} can be found from expression (2), with $p = 2\pi T/c$; we find $\tau^0_{\text{phi}} \cong 7\times 10^{-12} x^{-1} T^{-4}$. It follows that two steps are involved in the relaxation to an equilibrium in the phonon system: First, an equilibrium is established between energetic phonons with $p \cong 2\pi T/c$ and impurities, and then all the other phonons instantaneously ($\tau_{\|} \to 0$) become aligned with the energetic phonons. Here $\tau^0_{\text{phi}} > \tau_{33}$.

In weak solutions we are thus seeing a fundamentally new relaxation mechanism, which is quite different from the relaxation process in Baym's theory,[4-6] in which all the phonon times are omitted and in which it is assumed that the equilibrium in the phonon system is determined exclusively by phonon-impurity scattering. This situation corresponds to the limit $\tau_{\|} \to \infty$, for which expression (3) gives us

$$\tau^{\infty}_{phi} = -(6\pi^2 \rho_{ph})^{-1} \int_0^{\infty} t_{phi}\, p^4 n' dp. \tag{5}$$

The divergence at the lower limit in (5) was eliminated in Refs. 4–6 by taking into account the absorption of long-wave phonons as a measure of the viscosity of the impurity gas. As a result, $\tau^{\infty}_{\text{phi}}$ is determined by the time (2), where p is on the order of the momentum of a thermal, rather than energetic, phonon. For weak solutions $\tau^0_{\text{phi}} \gg \tau_{\|}$ we should choose the time τ^0_{phi}, which is more than an order of magnitude shorter than τ_{phi} at the given concentration. If we now change the time $\tau^{\infty}_{\text{phi}}$ to τ^0_{phi} in all the equations of the kinetic theory of weak solutions, we eliminate the large discrepancy between the theory and the experiments of Refs. 2, 3, etc.

For the renormalization of the velocity and the absorption of sound due to phonons and the phonon-impurity interaction we find the following expressions:

$$\left(\frac{\Delta c}{c}\right)_{ph} = \frac{3}{2}\frac{\rho_{ph}}{\rho_4} \operatorname{Re}\left(\frac{u_2}{4} - \frac{1}{3} + \Phi\right); \quad \alpha_{ph} = \frac{3}{2}\frac{\rho_{ph}}{\rho_4}\frac{\omega}{c} \operatorname{Im}\left(\frac{1}{3}\lambda^2_q - \Phi\right), \tag{6}$$

where

$$\Phi = (u + 1 + \lambda q - \lambda)^2 \frac{qQ - 1}{1 + Q - qQ}; \quad u = \frac{\rho_4}{c}\frac{\partial c}{\partial \rho_4}; \quad u_2 = \frac{\rho_4^2}{c}\frac{\partial^2 c}{\partial \rho_4^2}; \quad \lambda = \left(1 - \frac{m_3}{m} + \frac{\rho_4}{mc^2}\frac{\partial \epsilon_0}{\partial \rho_4}\right);$$

m_3 and m are respectively the mass and effective mass of the impurity; ϵ_0 is the binding energy of the impurity in the solution; $q = 1 - \delta + i/\omega\tau_{phi}$; and $c\delta = \langle v_{ph} - c \rangle$ is the deviation of the phonon group velocity from c, averaged with a weight factor $p^2 n'$. The solid curves in Figs. 1 and 2 are calculated from (6), with the impurity contribution; these curves agree with the experimental data. We might also note that Eqs. (6) describe the frequency dependence observed experimentally.

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