Mechanism for an absolute parametric instability of an inhomogeneous plasma

V. I. Arkhipenko, V. N. Budnikov, E. Z. Gusakov, I. A. Romanchuk, and L. V. Simonchik

Institute of Physics, Academy of Sciences of the Belorussian SSR; A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

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The structure of plasma oscillations in a region of parametric spatial amplification has been studied experimentally for the first time. A new mechanism for an absolute parametric instability has been observed. This mechanism operates when a pump wave with a spatial structure more complicated than a plane wave propagates through a plasma which is inhomogeneous along more than one dimension.

A spatial inhomogeneity of a plasma has a strong stabilizing effect on parametric instabilities, raising their excitation thresholds and causing them to reach saturation at a comparatively low level because of a convective transport of energy away from the interaction volume.[1] The parametric processes which occur in this case are not instabilities. Various mechanisms for the excitation of a "genuine" absolute parametric instability in this situation have been analyzed in the literature. Most of this theoretical work, however, has used the model of a plane-layer plasma and the approximation of a plane pump wave, whereas experimentally the plasma will often have a complex configuration, and the spatial structure of the pump will be quite different from a plane wave. Together, these two circumstances can lead to new mechanisms for the excitation of an absolute instability. In the present letter we report a study of one such mechanism, involving the decay $l \to l' + s$ in a magnetized plasma which is inhomogeneous in two dimensions.

The experiments were carried out in the linear Granit device.[2] An oblique plasma wave (the fundamental Trivelpiece-Gould radial mode of an axially inhomogeneous plasma-filled waveguide) is launched by a waveguide device (Fig. 1) in a plasma which

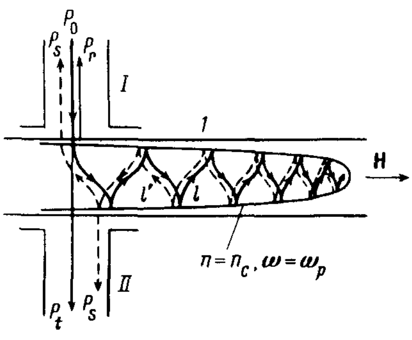

FIG. 1. Excitation and propagation of the oblique plasma wave.

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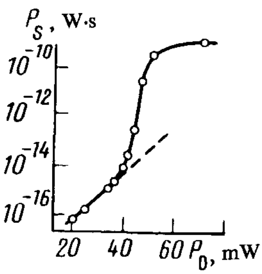

FIG. 2. Spectral density of the scattered signal versus the power of the pump wave. Solid curve—experimental; dashed—theoretical.

is inhomogenous both radially and axially: $\omega_{Pe} = \omega_{Pe}(r, z)$. The frequency of the wave is $f_0 = 2.525\times10^9$ s^{-1}. Near the point with $\omega_{pe}(0,0) = 2\pi f_0$, where $(\omega_{Pe}/2\pi f_0)^2 = 1 - (z/a) - (r^2/b^2)$, the wave is converted into a "warm" plasma wave. The field of this wave and its longitudinal retardation $k_0(z, f_0)$, determined from the equation

$$3r_d^2\, k_n^2 - \frac{z}{a} - \frac{2(2n+1)}{k_n b} = 0 \qquad (1)$$

$$(a = 6\ \text{cm} \quad b = 0.4\ \text{cm})$$

with $n = 0$ ($a = 6$ cm, $b = 0.4$ cm), increase rapidly.[3] In this region we would primarily expect a parametric growth of the reflected fundamental ($n = 0$) Trivelpiece-Gould mode and an ion acoustic wave. Since the decay conditions for this process, $2k_0(z, f_0) = 2\pi f_s/c_s$, are satisfied at only a single point z_0, we would expect[1] that the spectral density of the scattered wave would be given by an expression $P_s = AT_{sl}$, where the coefficient A would be determined by the level of the acoustic noise incident on the decay region, and the convective gain T_{sl} would satisfy $\ln T_{sl} \sim P_0$.

We observed the development of parametric processes at a power $P_0 > 20$ mW, at which the spectrum of the signal reflected from the plasma acquired a narrow satellite ($\delta f = 9\times10^5$ s^{-1}) shifted in the red direction by $f_0 - f = 3\times10^6$ s^{-1}. A plot of the amplitude of this satellite versus the pump power (Fig. 2) reveals three regions. In the first region, $P_0 < 37$ mW, the behavior $P_s(P_0)$ agrees well with the theoretical prediction. At a gain $T_{sl} > 2\times10^3$ the dependence becomes far stronger than exponential, and the spectrum of the satellite contracts. At $P_0 = 54$ mW, the level of the scattered signal corresponds to a nearly total parametric reflection of the wave. With a further increase in the power, the shift f_0–f' decreases, apparently because of a total reflection of the wave in the dense plasma upon scattering by sound of a lower frequency $f_s = f_0 - f'$.

It is logical to suggest that the reason for the steepening of the $P_s(P_0)$ curve is that we are approaching the threshold for the absolute instability $l_0 \rightarrow l_0' + s$. To identify the mechanism for this instability, we carried out an experiment to visualize the spatial structure of the acoustic noise by an enhanced-scattering method.[3] A low-power Trivelpiece-Gould mode at the frequency $f_{pr} = f_0 + \Delta f$ was excited in the plasma; this

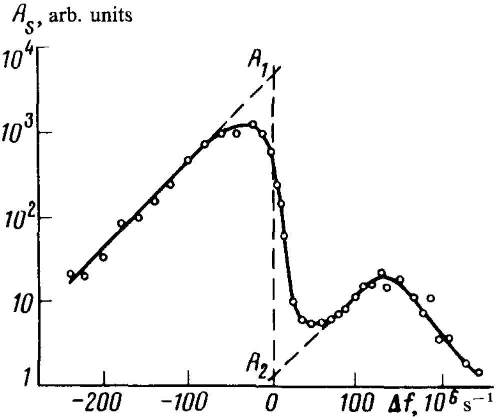

FIG. 3. Spectral density of the satellite of the probe wave versus its frequency $f_{pr} = f_0 + \Delta f$.

wave was scattered by parametrically excited acoustic noise at the point $z_{pr} \cong z_0 + (2\Delta f)/(f_0)a$. We recorded the dependence of the amplitude (A_s) of the scattered signal on Δf (Fig. 3). The A_s peaks correspond to regions of sound generation, while the exponential decrease in A_s at $\Delta f < -7\times 10^7$ s^{-1} and $\Delta f < 1.6\times 10^8$ s^{-1} is due to damping. From this decrease we can determine the propagation constant, $k_s'' \cong 2.3$ cm^{-1}, and we can estimate the temperature ratio:$T_i/T_e \cong 1/15$ ($T_e = 2$ eV). The large A_s peak corresponds to the region of the decay $l_0 \to l_0' + s$. The rate of the spatial amplification which is occurring here can be determined from the decrease in the amplitudes of the acoustic wave incident on the resonance region and of the acoustic wave amplified there: $T_{sl} = 10^{A_1 - A_2} = 3\times 10^3$. This value is approximately equal to the gain at the beginning of the steep part of the $P_s(P_0)$ curve. The high level of the incident acoustic wave, 10^6 times the equilibrium level, apparently means that this wave is excited in an interaction of an intense satellite of the pump wave with a small admixture of the first radial Trivelpiece-Gould mode in the pump wave. The resonance point for this process, z_1, is determined from the condition $k_0(z_1, f_0) + k_1(z_1, f_0) = 2\pi f_s/c_s$. Using (1), we can easily show that in this region a probe wave with $\Delta f = 1.6\times 10^8$ s^{-1} should be scattered, in accordance with the experimental data. An estimate shows that both of the soundwaves—both the incident wave and that amplified at $P_0 = 70$ mW— are excited as the result of a complete conversion of the energy of the first overtone of the Trivelpiece-Gould mode and the fundamental mode itself, respectively, into the energy of the satellite. We can then estimate the power which is transferred to the first overtone, P_1, from the ratio of the peaks in Fig. 3 at the level of 10^{-2} of the power in the fundamental Trivelpiece-Gould mode. Despite the low level, the first overtone has an important effect: It excites an acoustic wave of a complex transverse structure through the decay $l_1 \to l_0' + s$, sending back into the decay region part of the energy taken from this wave. In other words, this mode gives rise to a "feedback loop." Two circumstances are important for this mechanism for an absolute instability: (a) the fact that the natural modes in a plasma, which is inhomogeneous in two dimensions, are different from plane waves; (b) the fact that the pump wave is not purely a single-mode wave.

Under these circumstances the threshold for the absolute instability is determined from the balance among the gain T_{sl}, the loss as the waves propagate between the points z_0 and $z_1 - b_s$ and b_l and the slight feedback: $T_{ls} - T_{sl} T_{ls} b_l b_s = 1$ or

$$\frac{4P_1 \qquad (k_0 k_1)^3 (k_0^2 + k_1^2)\ T_{sl}}{\pi f_0 n_c T_l (k_0 + k_1)^4 \left[3r_d^2 b\, k_0 k_1 (k_0 + k_1) + \dfrac{3k_0^2 + k_1^2}{k_0 k_1} \right]} \tag{2}$$

$$= \exp \left\{ \frac{\nu a}{2\pi f_0} (3k_1 - k_0) + 2k_s''(z_0 - z_1) \right\},$$

where $k_1 = k_1(z_1, f_0)$, $k_0 = k_0(z_1, f_0)$, and $\nu = \nu_{en} = 4\times 10^7\ s^{-1}$. Substituting definite parameter values into (2), we find the value $T_{sl} \cong 10^3$ to be the threshold for the onset of the instability. This result agrees with the results reported above.

To verify the major role played by the region between the points z_0 and z_1 for the onset of the absolute instability, we carried out an experiment in which this instability was stimulated by a probe wave of frequency $f_0 + \Delta f$, which was (according to the calculations) capable of causing a further amplification of the ion-acoustic noise ($f_s = 3\times 10^6\ s^{-1}$) in the feedback loop only if $|\Delta f'| < 10^8\ s^{-1}$. The results showed that at $\Delta f < 9\times 10^7\ s^{-1}$ a pronounced increase in the satellite of the pump wave ($P_0 = 41$ mW) is caused by a probe wave of extremely low power $P_{pr} = 2$–4 mW, which causes a slight amplification of acoustic noise, $T_{sl} \lesssim 3$. As the frequency difference is increased, $\Delta f > 10^8\ s^{-1}$, the necessary power increases sharply: $P_{pr} > 30$ mW ($T_{sl} > 10^2$).

We do not believe that this mechanism for an absolute instability—involving the inhomogeneity of the plasma in more than one dimension and the multimode structure of the pump wave—is peculiar to the vicinity of a conversion point of the focus type in a plasma which is inhomogeneous in two dimensions. We believe instead that this mechanism can also have important consequences in other situations.

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[1]A. D. Piliya, in: Proceedings of the Tenth Conference on Phenomena in Ionized Gases, Oxford Univ. Press, London, 1971, p. 320.
[2]V. I. Arkhipenko *et al.*, Fiz. Plazmy 7, 396 (1981) [Sov. J. Plasma Phys. 7, 216 (1981)].
[3]V. N. Budnikov *et al.*, Fiz. Plazmy 6, 1050 (1980) [Sov. J. Plasma Phys. 6, 576 (1980)].

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