Effects of a Wess-Zumino term in the theory of a $d = 11$ supergravity

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(Submitted 13 March 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 10, 482–485 (25 May 1984)

The action of a $d = 11$ supergravity is renormalization-invariant in up to seventh-order perturbation theory because of a topological term in this action. A mechanism for the compactification $M^{n} \to (4\mathrm{D}\ \mathrm{space}) \times S^{7}$ is also discussed.

Cremmer *et al.*[1] proposed a theory for an $N = 1$, $d = 11$ supergravity in 1978. Their theory yields a high-symmetry $N = 8$ theory upon a dimensional reduction to $d = 4$. There is the hope that realistic supersymmetry models can be obtained upon the

 0021-3640/84/100586-05$01.00

spontaneous breaking of this theory. This theory is actually the only candidate for the role of a unified field theory based on a simple supergravity. An alternative theory is a conformal supergravity in $d = 4$ which does not contain dimensional coupling constants; it cannot be generalized to higher dimensionalities and cannot be treated by the Kaluza-Klein approach.[2]

Gravity theories, supergravity theories, and other field theories are not renormalizable in higher dimensionalities, so that at this point the only way we could allow the existence of such theories would be to accept them as finite, i.e., to assume that there are no quantum corrections to the original Lagrangian. The action of an $n = 1, d = 11$ supergravity is[1]

$$S = m_p^9 \int d'' \times e \{ -\frac{1}{2} R - \frac{1}{48} F_{MNPQ} F^{MNPQ}$$

$$+ \frac{4\sqrt{2}}{(4!)^3} \frac{1}{e} \epsilon^{M_1 \ldots M_{11}} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} A_{M_9 M_{10} M_{11}} - \frac{i}{2} \bar{\Psi}_M \Gamma^{MNP} \Psi_{p;N} + \frac{3\sqrt{2}}{(4!)^2}$$

$$\times (\bar{\Psi}_N \Gamma^{MNPQRS} \Psi_N + 12 \bar{\Psi}^P \Gamma^{QR} \Psi^S) F_{PQRS} + (\bar{\psi}\psi)^2 - \text{terms} \} , \quad (1)$$

where e_M^A is the elfbein field (or graviton field), Ψ_M is the gravitino field, and A^{MNP} is a third-rank antisymmetric tensor field. Together, these fields form a single supermultiplet of particles on the mass shell; $F_{MNPQ} = 24\partial [M^A NPQ]$. Action (1) is invariant under local supertransformations and Abelian gauge transformations of the field A_{MNP}:

$$\delta A_{MNP} = D_{[M} A_{NP]} = \partial_{[M} A_{NP]} . \quad (2)$$

A remarkable property of Lagrangian (1) is that it contains a Wess-Zumino term[1] $\epsilon^{M_1 \ldots M_{11}} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} A_{M_9 \ldots M_{11}}$, which follows directly from the supersymmetry properties. The coefficients of the various terms in (1) are fixed by the local supersymmetry. The only arbitrary parameter is the common factor which is determined by the "Planckian mass" m_p. We will now show that if there exists a quantization procedure which preserves both symmetries of the action, then (1) cannot be renormalized.

A correct method to calculate the loop corrections without introducing any extraneous z factors is the external-field formalism. We write the Lagrangian as $L = L_{\rm ex} + L_{\rm qu}$, and we write $e_M^A = E^A_{M_{\rm ex}} + e^a_{M_{\rm qu}}$, $A_{MNP} = A_{MNP_{\rm ex}} + a_{MNP_{\rm qu}}$, etc. The quantum Lagrangian L_{qu} contains no terms linear in the quantum field $a_{\rm qu}$ if the external fields satisfy the equations of motion. Furthermore, $l_{\rm qu}$ contains the external field A_{MNP} only in the form F_{MNPQ}. The only term of a different type, $\epsilon^{M_1 \ldots M_{11}} f_{M_1 \ldots M_4} f_{M_5 \ldots M_8} A_{M_9 \ldots M_{11}}$, can be rewritten in the form $\epsilon \cdots Ffa + \partial_M (\epsilon^{M \cdots} f \ldots A \ldots a \ldots)$; the term with the total derivative is zero in the quantum action, since the quantum fields fall of rapidly at infinity. After an integration over the quantum fields, we find from $L_{\rm qu}$ an expression which depends on only F_{MNPQ} (not A_{MNP}), so that it is not possible to derive a contraterm of the type $\epsilon^{M_1 \ldots M_{11}} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} A_{M_9 \ldots M_{11}}$ (finite or infinite). This result means that there are no corrections to the Wess-Zumino term in any order of perturbation theory, since we have nowhere used the quadratic approximation. Furthermore, there cannot be any corrections to other terms of the Lagrangian in this case, because of the local supersymmetry; we thus conclude that m_p is not renormalizable. This conclusion also essentially proves that there will be no new supersymmetry contraterms, but this point requires further discussion. We know that there are no such contraterms in up to seventh order in[3] $1/m_p^2$. It has also been verified that the single-loop corrections are finite in this theory[3] and that the anomalies cancel out.[4] We might note that our arguments are reminiscent of Steele's proof[5] that the Yang-Mills $N=4$ supersymmetry theory is finite. There the supersymmetry fixed the relative coefficients of the various terms of the Lagrangian, and the chiral vertices of the S^3 type could not have infinite renormalizations according to a superfield nonrenormalization theorem. Here we use another fact: the nonrenormalizability of the Wess-Zumino term (and not only an infinite nonrenormalizability), because the external-field method *explicitly* realizes all the symmetries of the Lagrangian and cannot give rise to contraterms with an implicit symmetry.[2]

We turn now to the reasons for the compactification of the 11-dimensional space-time according to theory (1). A discussion of compactification usually involves finding certain solutions of the equations of motion of the boson sector of the theory and testing their stability with respect to small *local* perturbations. The results are $ADS_4 \times S^7$ solutions, with loop corrections[6] or fermion condensates; and $M^4 \times S^7$ solution is also possible. In contrast with previous investigators, we wish to show here not simply that there exists a self-consistent vacuum solution but also the inevitability or at least the possibility of an evolution of the theory toward the formation of vacuum condensates and a contraction of the dimensionality of the internal seven-dimensional manifold. Although other attempts have been made along this line in model-based theories (see Ref. 7, for example), we are analyzing a "true" $d=11$ supergravity here. For the compactification in this case it is not necessary to deliberately introduce matter fields, since the theory unavoidably contains the antisymmetric field A_{MNP}. We will describe two possible compactification mechanisms, of which the second appears to be the more plausible.

Under the assumption that there are no fermion condensates, the classical equations for the boson sector of the theory (1) are

$$R_{MN} - \frac{1}{2} g_{MN} R = - \frac{1}{48} \left(8 F_{MPQR} F_N^{PQR} - g_{MN} F_{PQRS} F^{PQRS}\right), \tag{3}$$

$$\nabla_M F^{MNPQ} = - \frac{\sqrt{2}}{(2(4!)^2 \sqrt{G^{11}})} \epsilon^{NPQM_1 \ldots M_8} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} . \tag{4}$$

The possibility of a compactification stems from the existence of solutions of Eq. (4) (Ref. 8):

$$F_{\mu\nu\rho\sigma} = \frac{f}{\sqrt{g^4}} \epsilon_{\mu\nu\rho\sigma} , \tag{5}$$

$$F_{mnpq} = \lambda \epsilon_{mnpqrst} S^{rst} \qquad (6)$$

Here λ is an arbitrary numerical constant, S^{rst} is the twisting on S^7 corresponding to the connectedness of an absolute parallelism, $R^{\ m}_{npr}(r+S)=0$, and $f= \pm 2\sqrt{2}/r_7$. The last equation is a condition for the existence of the Englert topologically nontrivial solution (6) on an S^7 sphere with the radius r_7. In the static case, the determinant g^7 does not depend on the time, so that we could write $f=$ const in (5); this would not be true in a dynamic analysis. We assume that the mixed components of the vacuum condensate F_{MNPQ} are null components in terms of Lorentz invariance. We can determine the curvature tensors $R_{\mu\nu}$ and R_{mn} in terms of the vacuum values of the fields $F_{\mu\nu\rho\sigma}$ and F_{mnpq} by working from (3). Substituting the results into action (1), we find $S=\int d^{\,n}x\sqrt{G}(1/3)f^2$. In principle, it would be possible to obtain a "dynamic" term $\dot{f}^2$ in the action. The seven-dimensional curvature contains time derivatives of the radius r_7, not simply a static part $1/(r_7)^2$. We now use the topological condition $f\sim 1/r_7$. The Lagrangian, which becomes $L=\dot{f}^2+f^2$ (with a plus sign!), describes an instability: a motion of a one-dimensional particle in a potential $V(f)=-f^2$. In other words, we conclude that f unavoidably increases, and r_7 decreases, over time.

A second compactification mechanism arises in a more systematic analysis of Eqs. (3). We now assume that there are no Englert fields; i.e., $F_{mnpq}\equiv 0$. In this case the condition $f\sim 1/r_7$ drops out, and the metric is factorized into a four-dimensional metric and a seven-dimensional metric, with all components possibly dependent on the time. Equations (3) then have solutions of the following type, at least at large values of r_7, in the beginning of the compactification:

$$ds^2 = dt^2 - a^2(t) g_{\alpha\beta} dx^\alpha dx^\beta - b^2(t) g_{mn} dx^m dx^n; \quad \alpha, \beta = 1, 2, 3,$$

$$m, n = 1, \dots 7,$$

where $a(t)$ and $b(t)$ are determined by different "cosmological terms," $+(8/3)f^2$ and $-(7/3)f^2$. Correspondingly, the four-dimensional part of the metric describes ADS_4, i.e., a solution with an oscillating scale factor, while the seven-dimensional metric corresponds to a swelling or shrinking sphere. In the latter case, we obtain a compactification of the internal dimensions. Significantly, this compactification occurs under completely plausible initial conditions; we are actually assuming that the antisymmetric fields A_{MNP} received some "initial kick" $f\sim\dot{A}_{\mu\nu\rho}$. In contrast with the four-dimensional $A_{\mu\nu\rho}$, the seven-dimensional A_{mnp} do not affect the Einstein equations at the beginning of the compactification, so that A_{mnp} enters (3) divided by the radius of a seven-dimensional sphere, which is large at the outset: $F_{mnpq}\sim A_{mnp}/r_7$. The mixed fields $A_{\mu\nu m}$, ..., the evolution of the gauge fields included in the metric components $g_{\mu} n$, and other questions will be discussed in a detailed paper.

We wish to thank A. I. Vaĭnshteĭn, N. A. Voronov, V. I. Zakharov, V. A. Novikov, L. B. Okun', K. A. Ter-Martirosyan, and M. A. Shifman for interest in this study and for useful discussions.

[1]By "Wess-Zumino term" we mean an expression which changes upon gauge transformations to a total derivative (an expression which is implicitly invariant).

[2)]We recall that the external-field method usually (as in the present case) performs an infrared regularization of the theory.

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Translated by Dave Parsons
Edited by S. J. Amoretty