Phase transition in quantum chromodynamics

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Quantum-chromodynamics sum rules are constructed for describing the spectrum of hadronic matter at finite temperatures. Analysis of these sum rules in the ρ-meson channel indicates a first-order phase transition accompanied by the disappearance of confinement at a temperature $T_c \simeq 140$ MeV. The dynamic mass of the emitted quarks is found for temperatures near T_c. A phase transition accompanied by a restoration of chiral symmetry cannot occur before deconfinement.

The quark-hadron phase transition in quantum chromodynamics apparently represents the only feasible test of phase transitions in quantum field theory. The reasons are the relatively low values of the critical temperature, $T_c \sim 100$–300 MeV, and of the critical density, which is only slightly larger than the nuclear density,[1] and the possibility that hot, dense blobs of hadronic matter will form in high-energy hadron-hadron collisions.[2] There is considerable interest in determining the characteristics of a quark-hadron transition (its order, temperature, etc.).

In this letter we wish to propose a new method for studying hadronic matter at $T \neq 0$. The idea is to generalize the quantum-chromodynamics sum rules[3,4] to finite temperatures, in which case it is possible to find the excitation spectrum in various hadron channels with definite quantum numbers. We will show that at $T_c \simeq 140$ MeV a first-order phase transition occurs in a hadronic medium accompanied by the disappearance of the property of confinement. At temperatures near T_c the massless quarks acquire a nonzero dynamic mass $m_q^T \simeq 250$ MeV because of the existence of quark and gluon condensates. Analysis of the sum rules, furthermore, leads to the conclusion that a phase transition accompanied by the restoration of chiral symmetry[5] could not occur before deconfinement.

For definiteness, we consider the retarded Green's function (which has convenient analytic properties at $T \neq 0$) of the commutator of two vector currents $J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2$ with the quantum numbers of the ρ meson:

$$G^R_{\mu\nu}(\omega, \mathbf{p}, T) = i \int d^4x \, e^{ipx} \, \Theta(x^0) \ll [J_\mu(x), J_\nu(0)] \gg , \tag{1}$$

where $p = (\omega, \mathbf{p})$, and the double angle brackets specify an average over a Gibbs ensemble. The function $G^R_{\mu\nu}$, which is analytic in the upper half-plane of the complex variable[6] ω, is related to the spectral density $\rho(\omega, \mathbf{p}, T)$ by a standard dispersion relation. Making use of the property of asymptotic freedom in quantum chromodynamics, and applying a Borel transformation[4] in ω^2 to the dispersion relation for $G^R_{\mu\nu}$, we find the sum rule

$$\int_0^\infty \rho(\omega) e^{-\omega^2/M^2} d\omega^2/M^2 = \frac{1}{8\pi^2} \int_0^\infty \frac{d\omega^2}{M^2} \text{th}(\omega/4T) e^{-\omega^2/M^2}$$

$$+ \frac{1}{8\pi^2} \int_0^\infty \frac{d\omega^2}{M^2} 2n_F(\omega/2T) + \text{N.P.}, \tag{2}$$

where $n_F(\omega/2T) = [\exp(\omega/2T) + 1]^{-1}$, and *N. P.* means the nonperturbative parts of the Gibbs expectation values of the local operators which arise in the Wilson expansion for correlation function (1). (We have set $\mathbf{p} = 0$, and as ρ we have used $\lim_{\mathbf{p}\to 0} \rho_{00}/p^2 \equiv \rho$.) The temperature dependence of the nonperturbative effects (condensates) is generally not known, but it is important at $T \leqslant m_\pi$, since in this case the Gibbs expectation values are the same as the vacuum expectation values[1)] (the contributions of nonvacuum states are exponentially suppressed $\sim e^{m^\pi/T}$). At $T \lesssim m_\pi$, we can thus use as *N. P.* the condensate values known[4] for $T = 0$.

To determine the characteristics of the hadron spectrum at $T \neq 0$ we need to find a suitable approximation of the actual spectral density $\rho(\omega, T)$. The contribution of the resonance is given by the standard expression $\rho_{\text{res}}(\omega, T) = \text{fm}^2 \delta(\omega^2 - m^2)$, and a phenomenological model for the continuous spectrum can be found by analyzing the perturbation-theory spectral density for massive quarks at $T \neq 0$,

$$\rho = \Theta(\omega^2 - 4m_q^2) \text{th}(\omega/4T) \rho_0(\omega^2) + \delta(\omega^2) \int_{4m_q^2}^\infty du^2\, 2n_F(u/2T) \rho_0(u^2),$$

$$\rho_0(\omega^2) = \frac{1}{8\pi^2} (1 + 2m_q^2/\omega^2)(1 - 4m_q^2/\omega^2)^{1/2}, \tag{3}$$

through the replacements $\rho_0(\omega^2) \to 1/8\pi^2$, $4m_q^2 \to s_0$.

Figure 1 shows the results of an analysis of sum rule (2). At $T_c \simeq 140$ MeV, there are abrupt changes in the spectral characteristics f, m^2, and s_0, which imply a first-order phase transition in the hadronic matter. The most interesting feature in parts *a* and *b* of Fig. 1 is the sharp and substantial change in the beginning of the continuous spectrum: The parameter s_0 decreases by a factor of six and becomes smaller than the mass of the resonance. It follows that at $T > T_c$ a new channel arises with a scale value s_0 (140 MeV) $\simeq$ (500 MeV)^2. This behavior of s_0 implies a transition of the system into a deconfinement phase, where colored particles (quarks and gluons) exist along with composite entities. The mass of the quarks in the medium is estimated from the relation $4(m_q^T)^2 = s_0$ to be $m_q^T \simeq 250$ MeV. Interestingly, this value is approximately equal to the masses customarily assigned constituent quarks.

The mass of the resonance changes considerably less, $m(T = 0)/m(T \simeq 140$ MeV$) \simeq 1.3$, but the ρ-meson contribution to the spectral density decreases substantially: $\text{fm}^2(T = 140 \text{ MeV})/\text{fm}^2(T = 0) \simeq 0.3$. If the ρ meson does exist, as a resonance (and can now decay into its constituent quarks, since $s_0/m^2 \simeq 0.7 < 1$), it would be a rather

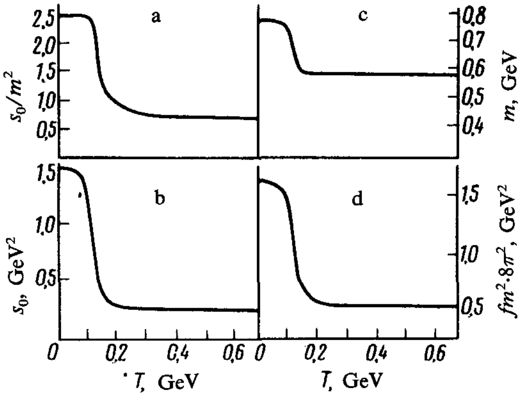

FIG. 1. Temperature dependence of the spectral characteristics in the ρ-meson channel. a—The ratio of the beginning of the continuum to the square mass of the ρ meson; b—the beginning of the continuum, s_0; c—mass of the ρ meson, m; d—contribution of the ρ meson to the spectral density, fm^2.

porous formation [$fm^2 \sim |\Psi(0)|^2$, where $\Psi(0)$ is the value of the wave function at the origin of coordinates].

How is the temperature of the deconfinement phase transition related to the restoration of chiral symmetry? The condensate of quark fields plays a governing role in the analysis of the sum rules (3) near the resonance. If this condensate vanishes at some temperature T_F, then the sum rules will have the unique solution $s_0 = 0$, $fm^2 = 0$. In a medium in which the continuous spectrum begins at the origin, and there are no resonances, there should also be no confinement. We can thus conclude that at least the condition $T_c \leqslant T_F$ must hold. We might note that the inequality $T_c \leqslant T_F$ was derived by Kogut *et al.*[7] through a lattice approximation and a numerical integration; it was also derived by Krasnikov[8] on the basis of the properties of the triangle anomaly at $T \neq 0$. Krasnikov also concluded that the quarks have a nonvanishing mass at $T > T_c$.

In summary, by working from sum rules for finite temperatures we have shown that a first-order phase transition related to a disappearance of confinement occurs in hadronic matter at $T_c \simeq 140$ MeV. A phase transition accompanied by the restoration of chiral symmetry occurs at temperatures higher than that at which confinement occurs. The dynamic mass of the quarks in the interval $T_c < T < T_F$ is determined by the nonzero condensates and has the value $m_q^T \simeq 250$ MeV.

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[1)]The condensates actually begin to change at $T > m_\pi$. Estimates from the vector dominance model show that the expectation values $\langle\langle(\bar{q}\gamma_\mu\lambda_q^a)^2\rangle\rangle$ and $\langle\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a)^2\rangle\rangle$ (the ones which basically determine the sum rules for the ρ meson) are essentially independent of T up to $T \sim 200$ MeV.

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