Dynamic breaking of color symmetry in Yang-Mills supersymmetry theories with matter

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In a supersymmetry quantum chromodynamics (with a small Lagrangian mass of the matter) and in an SU(5) theory with two anti-5-plets and two 10-plets (with a small Yukawa constant) there is a spontaneous breaking of color; vector bosons become very heavy; a weak-coupling theory arises; and there is no confinement.

It has recently been learned that an analysis of instantons in supersymmetry theories leads in certain cases to exact results for physical amplitudes.[1] Certain vacuum condensates, e.g., the gluino condensate, have been calculated. Similar instanton calculations were carried out in Refs. 2 and 3 under the additional assumption of a clustering of the correlation functions at large distances and with the help of relations for various condensates which follow from the Ward identities. Solving the resulting system of equations for the condensates leads to the important conclusion that a spontaneous breaking of supersymmetry is required in certain theories.[3]

Affleck *et al.*,[4] on the other hand, have used instanton calculations not to derive general relations but to analyze the effective potential for scalar fields in supersymmetry theories. They showed that a dynamic Higgs mechanism is generated in theories of the nature of a supersymmetry quantum chromodynamics with a small mass of the matter fields.

In the present letter we show that the clustering of correlation functions agrees with the explicit instanton calculations. The single-instanton approximation for calculating correlation functions can then be justified rigorously in a model with chiral matter fields,[3,5] in the limit of a small coupling constant for the Yukawa interaction (more on this below). It can be asserted that in these models there is a spontaneous breaking of both color symmetry and supersymmetry. This breaking occurs in a regime of *weak coupling*; i.e., the spectrum of particles and the amplitudes for the various processes can be calculated in terms of the fields which enter the Lagrangian.

To demonstrate the assertion of a decay of the correlation functions we consider the case of a simple supersymmetry gauge theory with the Lagrangian

$$\mathcal{L} = -\frac{1}{64g^2}\int d^2\theta W^\alpha W_\alpha + \frac{1}{4}\int d^4\theta[\bar{S}e^V S + \bar{T}e^{-V}T] + [\frac{m}{2}\int d^2\theta T\tilde{S} + \text{H.a.}], \quad (1)$$

where S and $\bar{T}$ are the matter superfields, which are color doublets; W_α is the superfield of the strength of the gluon field; V is a superfield which incorporates the vector potential of the gluon field (V and W_α are color triplets); g is the coupling constant; and m is the mass of the particle, assumed small: $m \ll \Lambda$, where Λ is a fundamental dimensional parameter of the theory, which appears in the asymptotic-freedom law for g^2.

To find physically interesting results we use the method of Ref. 1. In this theory it is useful to consider the correlation function[2]

$$<0 \mid T \{ W^2(x_1, \theta_1), S\bar{T}(x_2, \theta_2)\} \mid 0>, \tag{2}$$

where (x_i, θ_i) are coordinates in the superspace. In the limit $(x_1 - x_2) \to 0$ this correlation function is saturated by instantons of small dimensions, $\rho \sim |x_1 - x_2|$, and is a constant:

$$<0 \mid T \{ W^2(x_1, \theta_1), S\bar{T}(x_2, \theta_2)\} \mid 0> = \frac{Cg^2}{64\pi^4} \Lambda^5, \tag{3}$$

where C and Λ appear in the definition of the instanton density,

$$\mu(x_0, \rho, \alpha, \bar{\beta}, \eta_1, \eta_2) = C \Lambda^5 \, d\rho \, d^4x_0 \, d^2\alpha \, d^2\bar{\beta} \, d\eta_1 \, d\eta_2 . \tag{4}$$

Here x_0 is the position of the center of the instanton, ρ is its dimension, and $\alpha, \bar{\beta}$, and $\eta_{1,2}$ are Grassmann numbers, coefficients in an expansion in null fermion modes (normalized coefficients).

It follows from the supersymmetry that if correlation function (2) is a constant at short range then it will remain equal to this constant at large range. Also using the clustering of correlation function (2) at large range, we can assert

$$<\lambda\lambda> <st^+> = \frac{Cg^2}{64\pi^4} \Lambda^5 , \tag{5}$$

where λ is the gluino field, and s and t are scalar fields which enter the matter superfields S and T.

Supplementing (5) with a corollary of the Ward identity,[6]

$$\frac{1}{32\pi^2} <\lambda\lambda> = m <st^+> \tag{6}$$

we find two condensates. The condensate of the scalar field is parametrically large if $m \to 0$:

$$<st^+> = \left(\frac{Cg^2 \Lambda^5}{2^{11}\pi^6} \right)^{1/2} m^{-1/2}. \tag{7}$$

This result means that the scalar field is classical and that there is a spontaneous breaking of the color symmetry.

Because of the spontaneous breaking of color symmetry, the vector field acquires a large mass, and *the effective coupling constant is small at all distances.* We can thus test relation (5)—which is crucial to this entire analysis—through an explicit calculation of the condensate $\langle \lambda\lambda \rangle$ generated by instantons. An important point is that the integral over the dimensions of the instanton now converges and is determined by the dimensions of the instanton of order $\rho \sim 1/(g\sqrt{\langle st^+ \rangle})$. From the technical and fundamental standpoints a calculation of the condensate $\langle \lambda\lambda \rangle$ is entirely different from a calculation of correlation function (2) at short range; the latter calculation is determined by the contribution of instantons of arbitrarily small dimensions and is completely independent of the vacuum condensate of the scalar field.

The condensate $\langle\lambda\lambda\rangle$ turns out to be

$$<\lambda\lambda> = \int g^2 \frac{3}{\pi^2} \frac{\rho^4 d^4 x_0}{(x_0^2 + \rho^2)^4} v^2 \rho^3 d\rho \exp(-4\pi^2\rho^2 v^2) C\Lambda^5 = Cg^2\Lambda^5/64\pi^4 v^2, \tag{8}$$

$$<s> = <t> = v \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

so the relation (5) does in fact hold in the weak-coupling regime. Supersymmetry is not broken, in agreement with Ref. 7.

If we consider chiral theories, i.e., theories in which color symmetry forbids the introduction of a mass, then the corresponding relations of the type in (5) and (6) could not be allowed without assuming condensates, which must not exist in a supersymmetry vacuum.[3,5] This conclusion is regarded as a serious indication of a spontaneous breaking of supersymmetry.

An example of this type is the SU(5) gauge theory with two (anti-) 5-plets and two 10-plets $\Phi_i^\alpha, X_{\alpha\beta}^i$ ($i = 1,2$; $\alpha = 1,...,5$). In addition to a kinetic term, the Lagrangian has the Yukawa interaction

$$\mathcal{L}_{\text{Yukawa}} = \sum_i h_i \int d^2\theta\, \Phi_k^\alpha\, \Phi_l^\beta\, X_{\alpha\beta}^i\, \epsilon^{kl} + \text{H.a.}, \tag{9}$$

where $h_{1,2}$ are two independent coupling constants, and $\epsilon^{kl} = -\epsilon^{lk}$.

One of the basic assertions of this study is that in the limit $h_{1,2} \to 0$ we have a theory with a small effective coupling constant. All the relations for the condensates can be tested explicitly in this limit (as discussed above), and the breaking of color symmetry and supersymmetry can be considered proved.

In a situation in which supersymmetry is spontaneously broken, and there is a massless spinor particle—a goldstino—the system of equations for the condensates changes slightly because surface terms must be incorporated in the Ward identities.

In taking an expectation value over the supersymmetry vacuum, for example, we would evidently have[3]

$$<F_\beta^{*i}\, \eta_{\gamma\delta}^k\, \eta_{\rho\sigma}^l\, \epsilon^{\beta\gamma\delta\rho\sigma}> = <\{\bar{Q}\, \psi_\beta^{*i}\, \eta_{\gamma\delta}^k\, \eta_{\rho\sigma}^l\, \epsilon^{\beta\gamma\delta\rho\sigma}\}> = 0, \tag{10}$$

where F^β, ϕ^β, and ψ^β are respectively the F-term and the scalar and spinor fields of the supermultiplet Φ^β introduced above; and $\eta_{\gamma\delta}$ is the scalar field from the superfield $X_{\gamma\delta}$. Equation (10) now becomes the following equation in the general case:

$$< F^*\eta\eta> = \gamma_1\gamma_2, \tag{11}$$

where γ_1 is the residue for the transition of the supercurrent and the goldstino, and γ_2 is the corresponding quantity for the operator $\psi^*\eta\eta$. Furthermore, in the weak-coupling regime the scalar fields in (11) can be replaced by their vacuum classical values.

The predictions for the correlation functions which follow from the instanton calculations at short range [see (3), for example] remain unchanged when there is a spontaneous breaking of supersymmetry. We thus find a system of equations for the condensates which can, in principle, be used to determine all the condensates.

Here we will simply summarize the properties of the solution. The spectrum of particles consists of "heavy" particles, "light" particles, and a massless goldstino. The mass of the heavy particles, in particular, the vector bosons, is on the order of $m_H \sim g(\Lambda / h^{2/11})$. The mass of the light particles is on the order of $\Lambda h^{9/11}$.

In this example, h is a dimensionless number, $h \to 0$. If h is strictly equal to zero, then the theory has no lower state.[3,5] It may be, however, that in this case gravitational interactions lead to an effective Lagrangian which contains higher powers of the fields and reciprocal Planckian masses m_p. The existence of such small terms would ordinarily have no effect of any sort on the properties of the particles at low energies, but in the present case these small terms are the ones which would fix a lower state, i.e., a vacuum. A hierarchy of masses constructed from powers of Λ and m_p would arise.

The possible phenomenological applications will, of course, require a separate discussion. However, the very existence of theories in which both a breaking of color and a breaking of supersymmetry occur while the constant is small, and in which all physical quantities can, in principle, be calculated, seems to us to be a very interesting result.

Let us summarize our conclusions. In a supersymmetry quantum chromodynamics (with $m \sim 0$) or in an SU(5) theory with two Φ's and two X's ($h_{1,2} \sim 0$) there is a spontaneous breaking of color; a weak coupling arises; *there is no confinement*; and the supersymmetry is either conserved (in the first case) or spontaneously broken (in the second). In this regime the spontaneous breaking of supersymmetry is *necessarily* accompanied by a breaking of the color symmetry (since the goldstino reduces to the Lagrangian fields $\psi\lambda$, while the supercurrent is a color singlet). With $m \sim \Lambda$ or $h_{1,2} \sim 1$, or in an SU(5) theory with one Φ and one X, we would be dealing with a strong-coupling theory. It may be, however, that the color symmetry remains broken, in which case we would have an extremely unusual dynamic situation.

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