

Simulation of the spiral structure of galaxies in a rotating liquid

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The hydrodynamic theory of the formation of the spiral structure of galaxies with a rapidly rotating core is confirmed by an experiment in rotating shallow water. There is an analogous (“supersonic”) discontinuity in the velocity profile of the water. The water exhibits a theoretically predicted new centrifugal instability, which generates a spiral pattern.

According to the generally accepted wave theory for the spiral structure of flat galaxies,^{1,2} the spiral pattern is a density wave in a galactic disk which is rotating at a certain angular velocity around its center. The question of the mechanism for the excitation of density waves, on the other hand, is a physical problem. We report here an attempt to study this problem at the level of a hydrodynamic simulation.

In 1972, one of the present authors (A.M.F.) hypothesized³ that the excitation of a spiral structure occurs in gaseous subsystems of galaxies and is a consequence of a hydrodynamic instability associated with a differential rotation or density gradient. Subsequent development of this theory^{4–6} showed that hydrodynamic instabilities of this sort can occur in the observed galaxies which contain a region with a sharp decrease (“discontinuity”) in the rotation velocity at the edge of the rapidly rotating core.⁷ In the gaseous subsystem of the galactic disk, this discontinuity sets the stage for the onset of a centrifugal instability which generates a spiral pattern. It turns out that

the perturbed gravitational force in the gaseous galactic disk can be ignored in comparison with the perturbed pressure force (gravitation is limited to a role of maintaining equilibrium). The young stars which are constantly forming from the gas in the spiral arms of galaxies and which make these arms so noticeable, because of their large luminosity-to-mass ratio, indicate that these spiral arms form in the gaseous disks of galaxies. (The elevated density of the older stellar population in a spiral arm is a secondary effect caused by the appearance of a "spiral" gravitational potential due to the increase in the gas density.) Let us compare the theoretical predictions with the data from astronomical observations.

1) The centrifugal instability is expressed in the excitation of density waves in the form of lagging spirals (the ends of the spirals are displaced in the direction opposite the rotation direction)⁶; observations confirm this conclusion.⁸ 2) The number (m) of arms of the spiral pattern is determined by the "Mach number" $M = R\Omega_1/c_s$, falling off with increasing M (R is the radius of the discontinuity, Ω_1 is the angular rotation velocity of the core, and c_s is the velocity of sound). In the Local Galaxy⁹ and in the nearby galaxies M31 (Ref. 7) and M81 (Ref. 10) (with a rapidly rotating core and a negative angular-velocity gradient) the Mach number is definitely $M > 3$, so that the number of spiral arms could not exceed two according to the Morozov's theory.⁶ Observations (see Refs. 7, 9, and 10, for example) show that these galaxies have two arms. 3) The frequency (Ω_p) at which the spiral pattern rotates is proportional to Ω_1 ; specifically, $\Omega_p = \alpha\Omega_1/2$, where $\alpha \simeq 1$. The observed ratio Ω_p/Ω_1 is several times lower than the theoretical prediction, apparently because a linear theory is being compared with data on the definitely nonlinear process which generates spirals in real galaxies. In summary, the observations confirm all the qualitative conclusions of the theory.

This theory lends itself to a simple laboratory simulation. This simulation is possible because the equations which describe the dynamics of the gaseous subsystem of the galactic disk are completely equivalent to the equations of rotating shallow water, with c_s being replaced by the scale velocity of gravity waves in shallow water $(g^*H_0)^{1/2}$, where H_0 is the unperturbed depth of the liquid, and g^* is the acceleration due to the resultant of the gravitational and centrifugal forces. Correspondingly, the hydrodynamic instabilities of a gaseous galactic disk are equivalent to the instabilities of shallow water, which has an analogous rotational-velocity profile.⁴⁻⁶

The apparatus used for this model test of the theory is shown in Fig. 1. A velocity "discontinuity," with a width roughly equal to the depth of the liquid, is formed in a

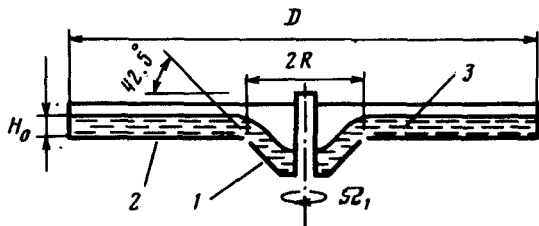


FIG. 1. The experimental apparatus. 1—Rotating cone; 2—fixed disk; 3—layer of "shallow water."

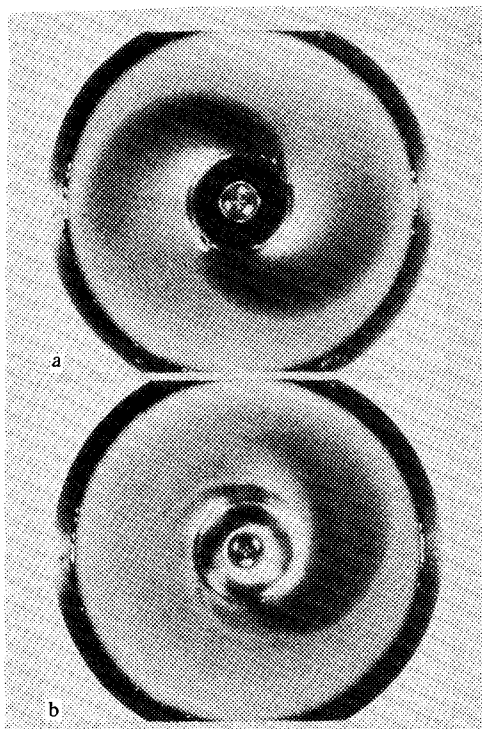


FIG. 2. Some typical spirals: the $m = 2$ and 1 modes.

thin liquid layer of depth $H_0 = 2, 3$, or 4 mm (shallow water) along the surface of a rotating cone 1 (the "core") and a fixed disk 2 (the "periphery"). The radius of this discontinuity is $R = 4$ cm, and the diameter of the fixed disk is $D = 28$ cm. The working liquid is water colored with a dye. In contrast photography¹¹ a wave crest (a

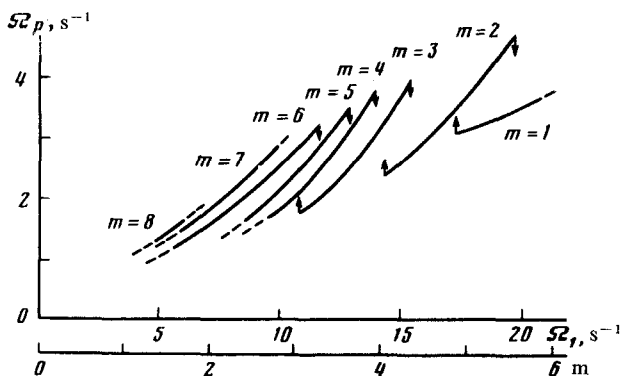


FIG. 3. Rotation velocity of the spiral pattern versus the core rotation frequency at a fixed depth of the liquid, $H_0 = 2$ mm. The lower abscissa scale shows the Mach number M . The arrows pointing downward indicate transitions from higher to lower modes as the core rotation velocity is increased, while the arrows pointing upward indicate the opposite transitions as the rotation velocity is reduced.

region where the water layer is relatively thick) appears darker than the trough between crests against the background of the white bottom. Let us examine the experimental results (Figs. 2 and 3).

1) The shallow water in the apparatus shown in Fig. 1 experiences a centrifugal instability⁴⁻⁶ due to the more rapid rotation of the core. (Another instability—the classical instability of a tangential discontinuity, which does not depend on the sign of the velocity discontinuity—does not occur at the large velocity jumps seen here.¹²) The instability generates surface-density waves in the form of lagging spirals (Fig. 2; the crests of the spirals correspond to regions in the galactic disk where the gas density builds up). 2) The number of arms in the spiral (the mode index) decreases with increasing core rotation velocity, i.e., with increasing Mach number [for shallow water, we would have $M = R\Omega_1/(g^*H_0)^{1/2}$], and also with increasing H_0 . The highest value found for M in these experiments was 7–8. 3) At a fixed value of m , the rotation frequency of the spiral pattern is approximately or exactly (at small and large values of H_0 , respectively) proportional to the core rotation frequency (Fig. 3). The coefficient of the proportionality between Ω_p and Ω_1 is several times smaller than the theoretical value, and the discrepancy increases with decreasing m . This discrepancy between experiment and the linear theory¹⁾ may be due to the nonlinearity of the processes involved. One indication of a nonlinearity is the hysteresis in the transitions between modes with different values of m (Fig. 3). A second nonlinearity factor is revealed by the experiments: The observed perturbations are vortices at the base of the spirals (these vortices are generated by the instability under discussion here, which occurs at the discontinuity in the rotation velocity). These vortices have a rather high amplitude—high enough that their boundaries are impenetrable to the liquid particles. The motion of these vortices, which are reminiscent of the Rossby vortices studied experimentally in Refs. 11 and 13, excites “ship waves,” in this case large spirals on the shallow water. The velocity at which these spirals rotate around the center of the system is equal to the velocity of the vortices, so that it is not surprising to find that this velocity differs from that predicted by the linear theory. 4) Let us compare the experimental values of the Mach number M at which, with increasing core rotation frequency and under the conditions in Fig. 3, the $(m+1)$ st mode converts into the m th mode, on the one hand, with the theoretical values of the Mach number at which, according to Ref. 6, the growth rate of the m th mode begins to exceed that of the $(m+1)$ st, on the other. For the mode transitions $6 \rightarrow 5$, $5 \rightarrow 4$, $4 \rightarrow 3$, and $3 \rightarrow 2$ the experiments yield $M = 3.3, 3.7, 4.0$, and 4.4 , while the theory predicts $M = 2.5, 2.7, 3.3$, and 5.2 , respectively. We see that the experimental results are in reasonable agreement with the theory.

This comparison of the results of the linear theory with experiment was made for $q = \Omega_2/\Omega_1 = 0$, where Ω_1 and Ω_2 are the rotation velocities respectively before and after the discontinuity. The results of this theory enable us to approximate the experimental results for arbitrary q . The experiments carried out under the conditions in Fig. 3 accordingly yield qualitatively the same results when the periphery is rotated, up to values $q \simeq 0.1$, not far from those which are characteristic of the galaxies of interest here. The maximum rotation frequency of the periphery was $\Omega_2 \simeq 2 \text{ s}^{-1}$.

There is one final experimental fact which we wish to point out. Under time-

varying conditions, i.e., as the core rotation velocity is gradually changed, the change in mode occurs more rapidly near the core; the mode change at the periphery lags behind in time. We thus observe a "two-level" structure in the spiral pattern: The number of spiral arms at the periphery is greater than in the interior when the core is being speeded up, or smaller when the core is being slowed down. This fact suggests that the two types of two-level galaxies which are observed are in a very unsteady state.

In summary, we have experimentally observed a theoretically predicted new instability of rotating shallow water with a tangential velocity "discontinuity" which, in particular, exceeds the scale wave velocity. This instability occurs if the radial gradient of the angular velocity is negative at the discontinuity. This instability is apparently responsible for the formation of the spiral structure of galaxies which have a similar rotation-velocity profile.

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¹⁾Approximately the same quantitative discrepancy is found between the predictions of this theory and the observational data on real galaxies (see the discussion above).

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