Cyclotron instability in the presence of a cold plasma

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The injection of a stream of cold plasma into an open confinement system does not suppress the cyclotron instability of a high-pressure plasma. An explanation is offered for experimental results obtained in the $2X\,\mathrm{II}B$ open system.

1. Loss-cone instabilities pose a serious danger to plasma confinement in open systems. A cold plasma is injected to suppress them,[1,2] but experiments in the $2X\,\mathrm{II}B$ device have shown[2] that this method does not stabilize a high-pressure plasma ($\beta \sim 1$). In this case the injection of cold plasma simply changes the nature of the instability: Specifically, the spectrum of unstable waves becomes narrower, "contracting" to the minimum value of the cyclotron frequency in the system, ω_{i0}; in addition to the waves traveling in the "ion" sense in the azimuthal direction, waves propagating in the opposite direction appear; and the time dependence of the wave amplitude becomes a series of "peaks." Neither these properties of the waves nor the very fact that they are excited in the presence of a cold plasma has been explained in the conventional interpretation, which is limited to an analysis of the drift-cone instability.[3] We believe that extraordinary waves with a negative energy are growing in these experiments. This interpretation leads to an explanation of the characteristic properties of the waves which are observed.

To describe the nonlinear evolution of negative-energy waves we introduce a new type of phenomenological equation, which leads to "peaked" solutions.

2. We start from the circumstance that in the experiments of Ref. 2 the hot and cold plasmas occupied regions with different magnetic fields. The hot ions, with a highly anisotropic distribution function, occupied a region of a minimum magnetic field, over which the field changed by $\sim 1\%$. The cold plasma, incident from the exterior along magnetic lines of force, was reflected from a "hill" of the ambipolar potential. Consequently, only a small fraction, $\sim 1\%$, of the stream of cold plasma penetrated to the center of the confinement system.

In this situation the waves which are stretched out along the magnetic field ($k_\parallel \ll k_\perp$; Ref. 2), with a frequency $\omega > \omega_{i0} + \delta\omega_i$, are stabilized by the cold plasma. Here $\delta\omega_i$ is the change in ω_i in the region occupied by the hot plasma. The waves with $|\omega - \omega_{i0}| < \delta\omega_i$ (the waves which were observed in Ref. 2) are at resonance with the hot ions. The excitation of these waves cannot be attributed to a drift-cone instability mechanism, since the electrons have only a slight effect on these waves.

The ion waves of a high-pressure plasma with $\omega \approx n\omega_i$, $k_\perp \gg k_\parallel$ are commonly called "extraordinary ion cyclotron waves." They are described by the dispersion relation

$$N^2\epsilon_{xx} - \epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}^2 = 0. \tag{1}$$

As usual, we adopt a local Cartesian coordinate system to analyze the small-scale waves. The X axis is directed along the azimuthal direction in the system (parallel to the wave vector), and the Y axis is directed along the radius. We use the approximation $f(\mathbf{v}) = (1/2\pi v_\perp)\delta(v_\perp - v_{\perp 0})\delta(v_\parallel)$ of the highly anisotropic ion velocity distribution typical of the experiments of Ref. 2. We then find the following expression for the components of the dielectric tensor:

$$\epsilon_{ik} \approx -\frac{q_i}{\Delta^2 \xi^2}(\Delta \xi \alpha'_{ik} + 2\Omega \alpha_{ik}),$$

where $q_i = (\omega_{pi}/\omega_i)^2$, ω_{pi} is the ion plasma frequency, $\xi = k\rho_i$, $\rho_i = v_{\perp 0}/\omega_i$, $\Omega = kV^*/\omega$, $V^* = (1/2)\omega_i \rho_i^2(1/B)(\partial B/\partial r)$ is the azimuthal ion drift velocity caused by the radial nonuniformity of the magnetic field, $\Delta = (\omega - \omega_{i0} - kV^*)/\omega$, $\alpha_{xx} = J_n^2(\xi)$, $\alpha_{xy} = -(i\xi/2n)[J_n^2(\xi)]'_\xi$, and $\alpha_{yy} = (\xi/n)^2[J'_n(\xi)]^2$.

From (1) we find

$$\Delta = \frac{1}{\xi J_n^{2\prime}}\left(-2J_n^2\Omega + \beta\left(J_n^2\left(1 - \frac{n^2}{\xi^2}\right) + J_n^{\prime 2}\right)^2\right), \tag{2}$$

where $\beta = 4\pi n m_i v_{\perp 0}^2/B^2$.

The energy density of these waves is

$$W = \frac{1}{8\pi}\frac{N^2 q_i}{\xi\Delta^2} J_n^{2\prime} \frac{\epsilon_{xx}}{|\epsilon_{xy}|^2}|E_x|^2 . \tag{3}$$

This energy density is negative if $J_n J'_n > 0$.

If energy is drawn from waves with $W<0$, these waves will grow. In this case the most effective mechanism is the absorption of energy by the cold ions which penetrate into the region occupied by the hot plasma. The number of these ions is $\sim 1\%$ of the number of hot ions. The contribution of the cold ions to the components of the dielectric tensor is $\epsilon_{xx}^c \approx \epsilon_{yy}^c \approx i\epsilon_{xy}^c \approx (\omega_{pi}^c)^2/(\omega_i^2 - \omega^2)$, where ω_{pi}^c is the plasma frequency of the cold ions.

Modifying Eq. (1) to incorporate cold ions, and averaging the result over z, we find an expression for Δ_1, the correction to Δ:

$$\Delta_1^{3/2} = -i\frac{\pi}{2}\Omega^2 \frac{L}{l}\frac{q_i^c \xi}{q_i N^2}\frac{(\epsilon_{xx} - i\epsilon_{xy})^2}{J_n^{2\prime}\epsilon_{xx}}, \tag{4}$$

where l is the length of the hot plasma, $q_i^c = (\omega_{pi}^c/\omega_i)^2$, and L is the scale dimension of the nonuniformity of the magnetic field $B(z) = B_0(1 + z^2/L^2)$. Comparing (3) with (4), we find that the waves with a negative energy are unstable. The energy absorption by cold ions which is required for wave growth actually occurs under the condition $\omega = \omega_{i0}$ or, in a different form,

$$\Omega/\beta = (J_1^2(1 - \xi^{-2}) + J_1^{\prime 2})^2/(2J_1^2 - \xi J_1^{2\prime}).$$

In the experiments of Ref. 2 the plasma and the unstable waves had the parameter values $n_0 \approx 3\times 10^{13}$ cm^{-3}, $B_0 \approx 7$ kG, $\epsilon_{\perp 0} \approx 13$ keV, $\omega_i(\mathbf{r}) \approx \omega_{i0}[1 + (r/L_\perp)^2 + (z/L)^2]$, ($\mathbf{r}$

is the distance from the axis), $L_\perp \approx 55$ cm, $L \approx 75$ cm, and $k\rho_i \approx 4$. An estimate from (4) yields a growth rate $\gamma \sim 10^{-2}\ \omega_{i0}$, in accordance with the data of Ref. 2. Also in agreement with Ref. 2, the waves can travel in either the ion direction ($\Omega < 0$) or the electron direction ($\Omega > 0$). (The drift waves travel in the ion direction.)

An instability of extraordinary waves with a nonequilibrium (cone) ion velocity distribution was also analyzed in Ref. 4. This instability is driven by an interaction of the extraordinary waves with Bernstein modes, which have a positive energy. For this instability the wave vectors would be small, $k\rho_i \lesssim 1$, in disagreement with the data of Ref. 2.

3. In the presence of a cold plasma the time dependence of the wave amplitude becomes a fairly regular series of peaks. We know that the nonlinear interaction of several wave modes with energies of different signs can lead to bursts of waves.[5] (These arguments were cited in Ref. 6 in order to explain cyclotron-instability bursts.) The measurements of Ref. 2, however, revealed no waves with a frequency substantially different from ω_{i0}. The basic nonlinear effect thus appears to be the relaxation of the nonequilibrium ion distribution function. Correspondingly, we describe the nonlinear dynamics of the negative-energy waves by the system of model equations

$$\begin{cases} s\dot{E} = -aE \\ \dot{s} = bE - I \end{cases} . \qquad (5)$$

Here s and a are defined in such a manner that $sE^2/2$ is the energy density of the waves, and aE^2 is the energy density absorbed by the cold ions per unit time. The term bE in the second equation reflects the change in the energy due to the relaxation of the ion distribution in the waves, while the term $-I$ reflects the recovery of the distribution due to the injection.

Analysis of Eqs. (5) shows that the motion on the (E,s) plane is cyclic. When the wave energy crosses zero during the relaxation of the ion distribution, the growth rate $\gamma = -a/s$ assumes infinitely large values, first positive and then negative. As a result, the $E(t)$ dependence takes the form of a peak. In the limit $s \to 0$, $E \to \infty$ we find from (5) $E = (a/b)\ln(1/|s|)$.

The standard quasilinear theory can lead to Lotki–Volterre equations which, for certain initial conditions, have solutions reminiscent of peaks.[7] However, a detailed analysis of the experiments of Ref. 2 carried out[3] in the quasilinear theory for the drift-cone instability has revealed no such solutions.

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