

# Investigation of the equilibrium form of helium crystals in the vicinity of faceting phase transitions

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The equilibrium form of  $\text{He}^4$  crystals does not exhibit critical behavior, as predicted by the existing theory of two-dimensional phase transitions.

The notion of a phase transition of the surface of a crystal from an atomically rough to an atomically smooth state was introduced by Burton and Cabrera.<sup>1</sup> Such a transition is manifested microscopically as the appearance of a smooth, equilibrium section of the surface at  $T < T_c$  determined by the crystallographic orientation. A large number of theoretical studies concerning faceting phase transitions have appeared, especially in recent years<sup>2–6</sup>; however, systematic experimental studies of this phenomenon have thus far not been performed.

A helium crystal, whose equilibrium form is characteristically established over very short times due to the large amplitude of zero-point vibrations, is the most convenient object for making quantitative studies of faceting transitions. At present, three such transitions in the hcp phase of  $\text{He}^4$  are known<sup>7–9</sup>: 1) on the surface oriented parallel to the basal plane (0001),  $T_{c1} = 1.2$  K; 2) on the family of faces parallel to the  $c$  axis  $\{10\bar{1}0\}$  (or  $\{11\bar{2}0\}$ ),  $T_{c2} = 0.9$  K; and 3) on the family of faces  $\{10\bar{1}1\}$  (or, correspondingly,  $\{11\bar{2}2\}$ ),  $T_{c3} = 0.36$  K. In this letter we are analyzing the equilibrium form of helium crystals near transitions 1 and 2.

The apparatus used for the measurements and the method for growing the crystals with the required orientation are similar to those used previously.<sup>8</sup> The changes concerned only the construction of the experimental cell, which in this case consisted of a cylindrical container 25 mm long with an inside diameter of 15 mm. The axis of the cylinder was oriented horizontally, and the flat glass windows in the bases of the cylinder remained open in order to observe the entire volume of the cylinder; the flat copper bottom (25 mm long and 8 mm wide) formed a thermal contact between the container and an  $\text{He}^3$  bath. An electronic stabilizer<sup>10</sup> made it possible to vary the temperature of the container to a fixed law or to maintain the temperature constant to within  $10^{-4}$  K.

In equilibrium, this crystal fills the required part of the container and its surface forms a convex meniscus. The crystals are 25 mm long ( $y$  axis), 10–14 mm wide ( $x$  axis), and 2.5–5 mm high ( $z$  axis). With this geometry, the curvature of the meniscus in the longitudinal direction is much smaller than in the transverse direction (the capillary constant  $\sim 1$  mm), and the profile of the surface is described with sufficient accuracy by the one-dimensional equation of equilibrium, so that for small values of  $z'_x$

$$\tilde{\alpha} z''_{xx} = \Delta \rho g z, \quad (1)$$

where  $\tilde{\alpha} = \alpha + (\partial^2 \alpha / \partial z_x^2)$  is the rigidity of the surface,  $\Delta\rho$  is the difference between the densities of the crystal and of the liquid, and  $g$  is the acceleration of gravity. The slope angles of the particular facet investigated [i.e., for example, the (0001) face in the case of transition 1] to the horizontal  $\varphi_{ox}$  and  $\varphi_{oy}$  were measured to within  $1 \times 10^{-3}$  rad. For the ten crystals investigated the angle  $\varphi_{oy}$  did not exceed  $5 \times 10^{-3}$  rad, and the angle  $\varphi_{ox}$  was chosen in the range from 0 to  $1.3 \times 10^{-2}$  rad. Equation (1) can be used to determine  $\tilde{\alpha}$  as a function of temperature and of the angular variable  $\varphi = z' - \varphi_{ox}$ .

The profile of the surface  $z(x)$  was measured with the help of a narrow He-Ne laser beam incident on the surface under investigation at a small angle ( $3-5^\circ$ ). The deflection angle [in the  $(x,y)$  plane]  $\theta$  of the reflected beam was measured as a function of  $x$  (Fig. 1); in this manner the derivative  $z'(x)$ , taken along the extremal section of the surface, was measured directly, i.e., at  $z'_y = 0$ . The width of the beam ( $\sim 0.3$  mm) was chosen so as to minimize the total angular broadening of the reflected beam (due to the curvature of the surface as well as diffraction). When a flat section is present on the surface, its size along the  $y$  axis could be measured, as done previously in Ref. 8, by the diffraction method.

Typical experimental curves are shown in Fig. 1. Curve 1 corresponds to  $T > T_c$ , when the surface is completely curved; curves 2 and 3 correspond to  $T < T_c$  and, in this case, the surface contains a flat section (the region where  $\varphi = 0$ ), whose size decreases continuously to 0 if  $T \rightarrow T_c$ . At a fixed temperature the angle  $\varphi$  on the boundary of the flat and curved sections varies continuously, i.e., there are no edges on

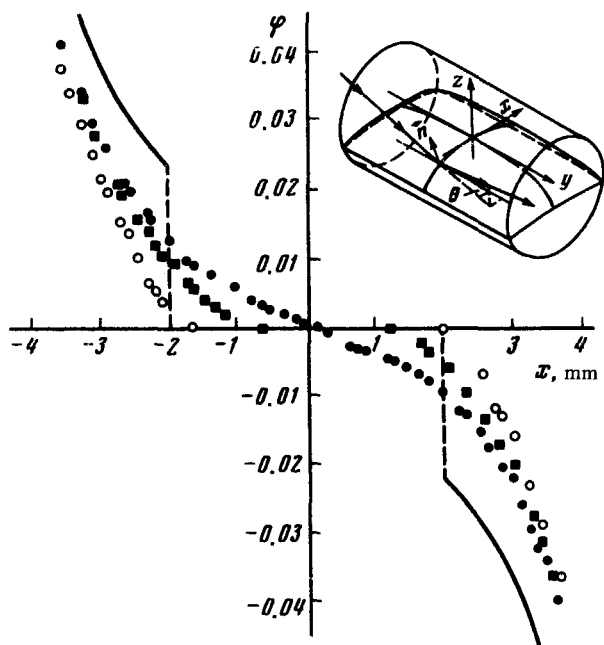


FIG. 1. Typical experimental dependences  $\varphi(x)$ : 1 —  $\bullet$  —  $T = 1.199$  K; 2 —  $\blacksquare$  —  $T = 1.194$  K; 3 —  $\circ$  —  $T = 1.88$  K. The inset shows the path of the rays with reflection from the surface of the crystal.

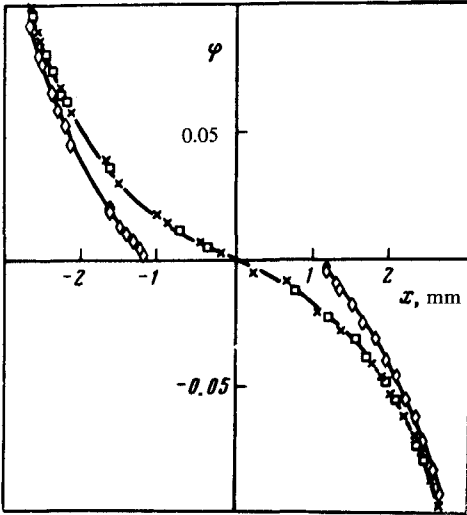


FIG. 2. Example of an analysis of the measurements:  $\square$  -  $T = 1.212$  K,  $\times$  -  $T = 1.223$  K,  $\diamond$  -  $T = 1.193$  K. The solid curves correspond to dependences of the form  $\varphi = (z_0/a)\sinh[(x - x_0)/a]$  with  $a = 1.06$  mm.

the surface. This indicates that the observed transition is not a first-order transition<sup>3,11,12</sup>; for comparison, Fig. 1 shows a solid curve calculated for a first-order transition according to Ref. 12 (for a flat, 4-mm-long section).

The results of the measurements of the surface rigidity  $\tilde{\alpha}$  for transition 1 reduce to the following. At  $T > T_{c1}$  the experimental curves for all of the samples investigated can be described by the relation  $z'_x = (z_0/a) \sinh[(x - x_0)/a]$ , which follows from Eq. (1), if  $\tilde{\alpha} = \text{const}$ , with the same value of the capillary constant  $a = 1.10 \pm 0.04$  mm, independent of temperature (in the interval from  $T_{c1}$  to 1.30 K) and independent of the angle  $\varphi_{ox}$ . In this case the values of  $z_0$  vary from sample to sample approximately by an order of magnitude. At temperatures below  $T_{c1}$  (0.01–0.02 K), when the size of the flat section is still not too large, the branches of the experimental curves corresponding to the curved sections are described by the same relations with the same value of  $a$  (see Fig. 2). Thus, everywhere in the vicinity of transition 1 (with  $0 < \varphi \leq 0.1$  rad),  $\tilde{\alpha}$  is a constant equal to  $\tilde{\alpha} = \Delta\rho g a^2 = 0.21 \pm 0.015$  erg/cm<sup>2</sup>. This result disagrees with both the predictions of the mean-field theory,<sup>2</sup> according to which  $\tilde{\alpha}$  increases indefinitely near the transition, and the predictions of the exactly solvable microscopic models of the surface,<sup>5,6</sup> according to which  $\tilde{\alpha} \rightarrow 0$  at  $T < T_c$  and  $\varphi \rightarrow 0$ . The value of  $\tilde{\alpha}$  is also at variance with the universal relation  $\tilde{\alpha}(T \rightarrow T_c, \varphi = 0) = (\pi/2)(kT_c/d^2)$ , where  $d$  is the distance between planes for the given face, as proposed by a number of authors.<sup>4,5</sup>

Analogous, although less complete, data were obtained for transition 2. The problem is that in this case the symmetry of the special face is lower than for transition 1 and the quantity  $\tilde{\alpha}$  can depend strongly on the angle between the  $x$  and  $c$  axis of the crystal. For the samples that we investigated this angle varied in the range 70–85 and the measured values of the capillary constant were approximately equal to 1.2 mm. It is significant that, as with transition 1, the flat section arises directly from the convex

surface without an intermediate (cylindrical) stage, whose existence is predicted by the mean-field theory.<sup>2</sup>

Our attempt to observe the faceting transitions in He<sup>3</sup> crystals was unsuccessful. We established that down to a temperature of 0.32 K, which corresponds to a minimum on the melting curve, where the most favorable situation for observing the equilibrium form of He<sup>3</sup> crystals is realized, the faceting phase transitions do not occur.

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