Localization and superconductivity

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A system in a state of Anderson localization in its normal state can go superconducting below the critical temperature T_c. The Ginzburg-Landau coefficients are derived for the superconducting transition in the region of Anderson localization. The behavior of the upper critical magnetic field H_{c2} as a function of the degree of disorder is studied in the metallic and insulating regions.

The introduction of a sufficient amount of disorder in a metallic system gives rise to a localization of electronic states near the Fermi level, i.e., to an Anderson transition.[1,2] On the other hand, the attraction of electrons near the Fermi level gives rise to a superconducting ground state at low temperatures. We might ask about the relationship between these two types of transitions, which lead to fundamentally different ground states. This question is also of applied importance in connection with research on the superconductivity of highly amphorous metals and compounds bombarded by fast neutrons.

The effect of localization on superconductivity has recently been the subject of an extensive discussion in the literature.[3–5] In the present letter we show that a three-dimensional system in the state of an Anderson insulator in its normal state can go superconducting below a certain critical superconducting transition temperature T_c.

Assuming that there is an effective attraction of electrons at the Fermi surface, we use the Bardeen-Cooper-Schrieffer theory to calculate the coefficients of the Ginzburg-Landau functional:

$$F = A\,|\Delta|^2 + \frac{1}{2}B\,|\Delta|^4 + C\,\left|\left(\frac{\partial}{\partial \mathbf{r}} - \frac{2ie}{\hbar c}A\right)\Delta\right|^2 . \tag{1}$$

The coefficients A, B, and C are determined by the Matsubara two-particle Green's function of the system of electrons in the normal state. This function $\phi(\mathbf{q}\omega_m)$, which determines the kinetic properties of the normal state and the transition to localization, can be found in the self-consistent theory of localization.[6–8] As the degree of disorder increases (with a decrease in the "seed" electron mean free path l), the mobility threshold E_c approaches the Fermi level E_F and crosses it. At this point the conductivity vanishes, and the system goes into the insulating region ($E_F < E_c$). Near the transition point ($E_F \approx E_c$) we have

$$\phi(\mathbf{q}\omega_m) = -\frac{N(E_F)}{i|\omega_m| + iD_0\,(|\omega_m|\,\tau)^{1/3}\mathbf{q}^2}\,; \quad \omega_m = 2\pi m T, \tag{2}$$

where

$$D_0 = \frac{1}{3} v_F l , \qquad \tau = l/v_F .$$

According to the BCS model, the coefficients A and B, remain independent of the degree of disorder (the Anderson theorem) as long as there is a sufficiently large number of states near the Fermi level in an energy layer on the order of T_c in the localization region. We are thus primarily interested in the coefficient C, which describes the superconducting response of the system. For an ordinary dirty superconductor, C is proportional to the conductivity of the system, σ. This conductivity vanishes at $E_F = E_c$, and the question of the value of C near the Anderson transition and in the localization region is less trivial. Using the relation

$$C = i\pi T \sum_{\epsilon_n} \frac{\partial^2}{\partial \mathbf{q}^2} \phi(\mathbf{q}, 2\epsilon_n) \Big|_{\mathbf{q}=0} ; \qquad \epsilon_n = (2n+1)\pi T \tag{3}$$

we find the following results for the square of the correlation length:

$$\xi^2(T) = \xi_0 l \frac{\sigma}{\sigma_0} \left(1 - \frac{T}{T_c}\right)^{-1} ; \qquad R_l \ll (\xi_0 l^2)^{1/3} , \qquad E_c < E_F .$$

$$\xi^2(T) = (\xi_0 l^2)^{1/3} \left(1 - \frac{T}{T_c}\right)^{-1} ; \quad R_l > (\xi_0 l^2)^{1/3} ; \quad E_c \gtrsim E_F . \tag{4}$$

$$\xi_0 = 1.18 \hbar v_F / T_c ,$$

where $R_l = k_F^{-1} |1 - E_F/E_c|^{-1}$ is the correlation length of the Anderson transition, $\sigma = \sigma_c (k_F R_l)^{-1}$ is the static conductivity of the metal near the transition, σ_0 is the Drude conductivity of a dirty superconductor, and $\sigma_c = l^2 k_F / \pi^3 \hbar$ is the minimal metallic conductivity in the Mott sense [$\sigma_c \approx 250$ S/cm with $k_F \approx (3 \text{ Å})^{-1}$], which determines the scale of the conductivity at the metal-insulator transition. In the insulating region, R_l determines the localization radius.

We see that the superconducting response is also preserved in the localization region. It disappears only upon a violation of the inequality $R_l > (\xi_0 l^2)^{1/3}$, i.e., only for highly localized states, for which the discrete spacing of the levels in a region on the order of R_l in size is important.

We calculated the behavior of the upper critical magnetic field $H_{c2}(T)$, ignoring the effect of the magnetic field on the Anderson transition. This approximation is justified near T_c. The relationship among σ, the derivative $(dH_{c2}/dt)_{T_c}$, and the state density at the Fermi surface is

$$k = -\frac{\sigma}{8ec\hbar N(E_F)} \left(\frac{dH_{c2}}{dT}\right)_{T_c} \simeq \begin{cases} 1; \quad \sigma \gg \sigma^* & \text{(5a)} \\ \dfrac{\sigma}{16 l^2 [N(E_F) T_c]^{1/3}} ; \quad \sigma \ll \sigma^* , & \text{(5b)} \end{cases}$$

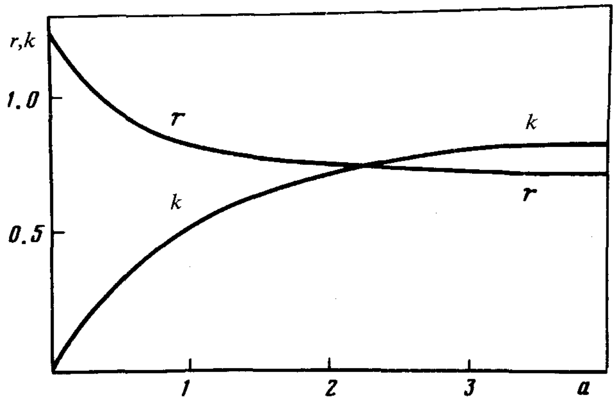

FIG. 1.

where $\sigma^* \approx \sigma_c (k_F \xi_0)^{-1/3}$ is a measure of the effect of localization on the superconductivity. This quantity is approximately equal to the minimal metallic conductivity. We see from (5) that relation (5a), which is familiar relation in the theory of dirty superconductors, is violated as we approach the Anderson transition. Figure 1 shows the complete dependence of the coefficient k on the parameter

$$a = 1.23\frac{\sigma}{\sigma^*}\left[1 + \frac{\sigma}{\sigma^*}(k_F \xi_0)^{-1/3}\right]^{-1}.$$

Also shown here is the dependence $r(a) = -H_{c2}(0)/T_c(dH_{c2}/dT)_{T_c}$. As the degree of disorder increases, this coefficient increases from the value 0.69, characteristic of ordinary dirty superconductors, to 1.24. The positive curvature on the $H_{c2}(T)$ curve gives way to a negative curvature.

We know from the work of Anderson, Muttalib, and Ramakrishnan[4] that as a system approaches the localization threshold, the critical temperature T_c falls off because of an intensification of the effective Coulomb repulsion of electrons (the attenuation of the diffusion of electrons opposes their dispersal). Our calculations show that in systems with low values of E_F (on the order of 1000 K) and a rather high initial temperature T_c (on the order of 10–15 K in the absence of disorder) the localization region can be reached while a significant value of T_c is retained.

A behavior of σ and $(dH_{c2}/dT)_{T_c}$, which agrees with (5) and Fig. 1, has been observed in some real systems: the bombarded ternary chalcogenides $SnMo_5S_6$ (Ref. 9) and $Pb_{1-x}U_xMo_6S_8$ (Ref. 10). Measurements[9] of the coefficient γ in the specific heat show that the state density $N(E_F)$ is essentially independent of the degree of disorder. We might note that compounds of both types are convenient for arranging the Anderson transition, because the values of E_F in these compounds lie near a band edge and because of the comparatively high temperatures, $T_c \approx 10$–15 K. The pronounced disordering which results from bombardment causes T_c to decrease to 1 K in these materials, causes the residual resistance to increase to values $> 10^{-3}\ \Omega$ cm, and leads to a negative coefficient of the resistance which is significant in magnitude over the entire temperature range studied. These results strongly indicate that these compounds, when subjected to neutron bombardment, are in fact near an Anderson transition while retaining superconducting properties.

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