Effect of the reaction $d\mu(n) + t \rightarrow d + t\mu(n)$ on the kinetics of muon-catalysis processes in a $D_2 + T_2$ mixture

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The isotope exchange reaction $(d\mu)_n + t \rightarrow d + (t\mu)_n$ from excited states with $n \sim 2$–5 of mesic atoms of deuterium leads to optimum conditions for muon-catalysis reactions in a deuterium-tritium mixture which are significantly different from those found previously without allowance for this isotope exchange.

1. A description of the kinetics of muon catalysis in a deuterium-tritium mixture usually starts from the premise that negatively charged muons with an energy $E_\mu \sim 3$ keV in the mixture $D_2 + T_2$ are captured in a time $\sim 0.8 \times 10^{-12} \rho^{-1}$ s to highly excited ($n \gtrsim 14$) states of $d\mu$ and $t\mu$ mesic atoms. They then reach mesic-atom states with

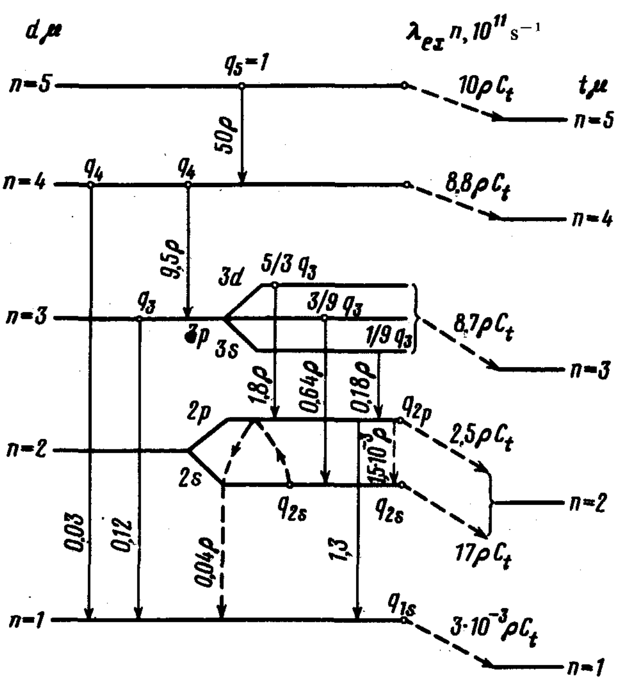

FIG. 1. Transitions in the $d\mu$ mesic atom. The transition rates are expressed in units of 10^{11} s^{-1} and correspond to a $d\mu$ energy $E = 0.05$ eV.

$n = 6$–7 in a time $\sim 0.5\times 10^{-12}\rho^{-1}$ s, and finally, in a time $\sim 1.4\times 10^{-11}\rho^{-1}$ s, they reach the 1s ground state of the $d\mu$ and $t\mu$ mesic atoms.[4,5] It is usually assumed without discussion that their relative concentrations are the same as those of the deuterium and tritium, i.e.,[1,2] $C_{d\mu}:C_{t\mu} = C_d:C_t$. This assumption has been considered so obvious that it has never been questioned. In the present letter we show that the quasiresonant charge exchange[3]

$$(d\mu)_n + t \to d + (t\mu)_n \tag{1}$$

from excited states with $n \lesssim 5$ of $d\mu$ atoms to the same states of $t\mu$ atoms violates this assumption and has a significant effect on the sequence of muon-catalysis reactions.

2. Figure 1 shows the transitions from the $n = 5$ initial state of the $d\mu$ mesic atom. The rates of isotropic exchange reaction (1), $\lambda_{\rm ex}(n)$, were calculated in Ref. 3 to be 10^{11} s^{-1}. The rates for cascade transitions between the n and n' states of the $d\mu$ atom, $\lambda_{nn'}$, were calculated in Refs. 4–6 to be 10^{11} s^{-1}. The rate $\lambda_{2p,2s} = 1.5\times 10^{-3}\rho$ for the $2p\to 2s$ transition and the rate $\lambda_{2s,1s} = 0.04\rho$ of the induced transition $2s\to 2p\to 1s$ were found in Refs. 3 and 7. In contrast with the rates of the radiative transitions $\lambda_{41} = 0.03$, $\lambda_{31} = 0.12$, and $\lambda_{21} = 1.3$, the rates $(\lambda_{nn'})$ of the Auger transitions $\lambda_{54} = 50\rho$, $\lambda_{43} = 9.5\rho$, and $\lambda_{3l,2l'}$ are proportional to the mixture density ρ, since they are caused by collisions of excited mesic atoms of hydrogen with atoms. These rates depend weakly on the kinetic energy of the mesic atoms over the interval $0.04 \lesssim E \lesssim 1$ eV. The high rates[3,4] of the Stark mixing of states in such collisions, $\lambda_{\rm st} \sim 10^{13}\, n\rho$ s^{-1},

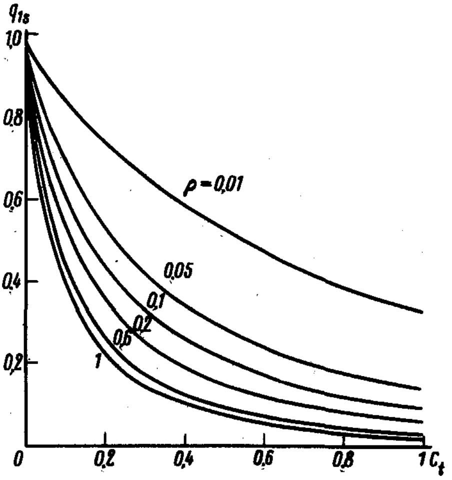

FIG. 2. The populations (q_{1s}) of the ground state of the $d\mu$ atom as functions of the tritium concentration C_t for various densities ρ of the $D_2 + T_2$ mixture.

lead to a statistical population $(2l + 1/n^2)q_n$ of the nl sublevels of the state n, where q_n is the total population of level n.

Calculations use the assmumption $q_5 = 1$, which is equivalent to ignoring charge exchange (1) from states with $n > 5$, since for such states the condition $\lambda_{nn'} \gg \lambda_{ex}(n)$ holds. The steady-state populations q_n are calculated from[5]

$$q_n = \sum_{n' > n} q_{n'} \frac{\lambda_{n'n}}{\lambda_{n'}}, \qquad q_5 = 1$$

$$\lambda_{n'} = \lambda_{ex}(n') + \sum_{n'' < n'} \lambda_{n'n''} . \tag{2}$$

Figure 2 shows the populations q_{1s} of the $1s$ state of the $d\mu$ mesic atom as functions of the concentration C_t of tritium nuclei in the mixture $(C_d + C_t = 1)$ for various densities (ρ) of the $D_2 + T_2$ mixture. We see that these populations are much smaller than unity, in contradiction of the assumption in all previous calculations. At $\rho \gtrsim 0.3$ and $C_t \gtrsim 0.3$, the population q_{1s} depends rather weakly on ρ and C_t.

3. The kinetics of muon-catalysis processes and, in particular, the number (X_c) of catalysis cycles executed by one muon in the $D_2 + T_2$ mixture of the chain $\mu^- \to d\mu \to t\mu \to dt\mu \to {}^4\text{He} + n + \mu^-$ depends on the fraction $(P_{1s} = C_d q_{1s})$ of $d\mu$ mesic atoms that reach the $1s$ state in the course of the cascade. At a deuterium concentration C_d and mixture densities $\rho \gtrsim 10^{-2}$ (Refs. 1 and 2) we have

$$X_c^{-1} \approx \omega_s + \frac{\lambda_0}{\overset{\circ}{\lambda}_{dt\mu}\,\rho\, C_d} + C_d q_{1s} \frac{\lambda_0 + \frac{\beta}{1+\beta}\,\omega_d\,\overset{\circ}{\lambda}_{dd\mu}\,\rho\, C_d}{\lambda_0 + \overset{\circ}{\lambda}_{dd\mu}\,\rho\, C_d\,(1 - C_d q_{1s}) + \overset{\circ}{\lambda}_{dt}\,\rho\, C_d}$$

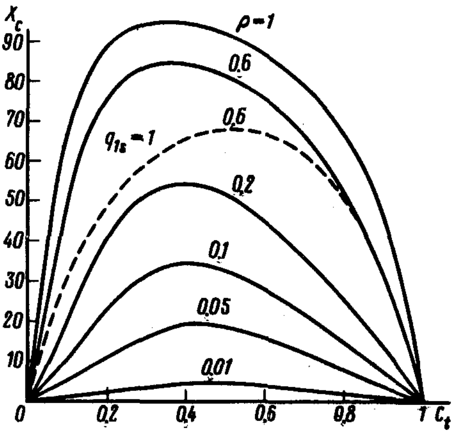

FIG. 3. The number (X_c) of catalysis cycles executed by one muon in the mixture $D_2 + T_2$ for various values of ρ and C_t. Shown for comparison by the dashed curve is the function $X_c(C_t)$ with $\rho = 0.6$ and $q_{1s} = 1$, i.e., without consideration of reaction (1).

$$+ \ \omega_t \frac{\lambda^{\circ}_{tt\mu} C_t}{\lambda^{\circ}_{dt\mu} C_d} \tag{3}$$

$$\lambda^{\circ}_{dt\mu} = \lambda^{(a)}_{dt\mu} C_d + \lambda^{(b)}_{dt\mu} C_t ,$$

$$\lambda^{\circ}_{dd\mu} = \lambda^{(a)}_{dd\mu} C_d + \lambda^{(b)}_{dd\mu} C_t ,$$

where $\lambda_0 = 0.46 \times 10^6 \text{ s}^{-1}$ is the muon decay rate, and the values $\omega_s = 0.9 \times 10^{-2}$, $\omega_d = 0.126$, $\omega_t \approx 0.1$, $\beta = 1.4$, $\lambda^{(a)}_{dt\mu} = 7 \times 10^8 \text{ s}^{-1}$, $\lambda^{(b)}_{dt\mu} = 3 \times 10^8 \text{ s}^{-1}$, and $\lambda^{0}_{tt\mu} = 3 \times 10^6 \text{ s}^{-1}$ are taken from Refs. 1 and 8–10. Figure 3 shows X_c as a function of C_t for various values of ρ; the dependence of q_{1s} on ρ and C_t was taken into account in these calculations. Shown for comparison is the function $X_c(C_t)$ for $q_{1s} = 1$ and $\rho = 0.6$ We see from this figure that when reaction (1) is taken into account the value of X_c increases, and the maximum on the curve $X_c(C_t)$ shifts toward lower C_t and increases with increasing ρ. These curves should not be regarded as definitive, since they are based on the approximation $\lambda^{(a)}_{dd\mu} \approx \lambda^{(b)}_{dd\mu} \sim 10^6 \text{ s}^{-1}$; furthermore, we have used values of $\lambda^{(a)}_{dt\mu}$ and $\lambda^{(b)}_{dt\mu}$ which were found in Ref. 10 from measurements of X_c for various values of C_t through equations like (3), but without consideration of exchange (1), i.e., with $q_{1s} = 1$.

4. Because reaction (1) is important for the kinetics of muon catalysis, we need an experimental test of its effectiveness. One possible test would be to measure the yield of γ rays from the decay $\pi^0 \rightarrow 2\gamma$ in the capture of π^- mesons into excited orbits of n mesic atoms:

$$\pi^- + H_2 \rightarrow p\pi \rightarrow n + \underset{\hookrightarrow 2\gamma}{\pi^{\circ}} \ . \tag{4}$$

The rates of the isotope exchange reactions[11]

$$(p\pi)_n + d \to p + (d\pi)_n \tag{5}$$

can be measured by accurately measuring the decrease in the yield of γ rays in reaction (4) with increasing deuterium concentration in an $H_2 + D_2$ mixture. This exchange is analogous to exchange (1) and can thus serve as a test of the calculations.[3]

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