An unusual superconductivity in UBe_{13}

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(Submitted 10 May 1984)

Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 12, 550–553 (25 June 1984)

The most symmetric superconducting phases in cubic crystals are found. Some of their thermodynamic and magnetic properties are discussed.

The compound UBe_{13}, which goes superconducting at $T_c = 0.85$ K, exhibits several exotic properties.[1] Above 100 K its spin susceptibility obeys a Curie law, becoming independent of the temperature only at $T_F \sim 10$ K. The state density extracted from this behavior and from data on the electron specific heat corresponds to an effective mass $m^* \simeq 200 m_e$. A similar inference comes from the anomalously high critical fields [more precisely, $(dH_{c2}/dT)_{T_c} = -257$ kOe/K; Ref. 1]. It has therefore been suggested that for this compound [and also $CeCu_2Si_2$ (Ref. 2) and U_6Fe (Ref. 3)] it would be better to speak in terms of a Fermi liquid of heavy fermions ($5f$ electrons) with a degeneracy temperature $T_F \sim 10$ K. Correspondingly, we would expect that the superconducting pairing would not be of a phonon nature but would instead result from magnetic mechanisms (the exchange of paramagnons) and would correspond to a triplet pairing. More briefly, it is expected that in these compounds the superconductivity might in many respects be more reminiscent of the superfluidity in ^{3}He than the ordinary Bardeen-Cooper-Schrieffer superconductivity. Recent measurements[4] of the electron specific heat of UBe_{13} below T_c have shown that its low-temperature behavior corresponds to $C_{se} \simeq \gamma T (T/T_c)^2$, instead of the exponential dependence $\exp(-\Delta/T)$ for BCS superconductivity with a gap. Such a dependence is characteristic of ^{3}He-A, where the gap in the quasiparticle spectrum varies along the Fermi surface, vanishing at two points on this surface.

 0021-3640/84/120674-04$01.00

Nevertheless, a direct analogy with the superfluid phases of ^3He (in particular, the triplet pairing mechanism) does not follow from these results. The primary distinction between a superconductor and ^3He stems from its discrete crystal symmetry. For example, the compound UBe_{13} belongs to the cubic group.[5] In the present paper we are basically listing the types of superconducting states (the superconducting symmetry cases) which are possible for cubic crystals. According to Ott *et al.*,[4] the UBe_{13} ground state differs from the ordinary type, but these results do not rule out the possibility of a singlet pairing. We will also briefly mention certain phenomena which might help resolve the question.

To list the substantially different symmetry classes for an equilibrium superconducting state is only slightly more complicated than to construct the magnetic classes.[6] This assertion also applies to the case of triplet pairing, provided that the spin-orbit effects are sufficiently strong (i.e., provided that rotations of the lattice simultaneously rotate the spins pinned to it). Treating the singlet and triplet states separately, we can split off an inversion transformation. The complete group whose discrete subgroups are to be found contains the spatial rotation group O, time reversal R, and Abelian gauge-rotation group U(1). In order to construct expanded subgroups [whose elements contain transformations from O in combination with elements from U(1) and R] we need to construct factor groups on one of the invariant subgroups (there are two in O: $D_2 \equiv V$ and T). From the elements $(e^{2\pi i/3}, e^{4\pi i/3})$ or $(e^{\pi i/2}, e^{\pi i}, e^{-\pi i/2})$, and an element of R we in turn construct a group which is isomorphic to the factor group. Invariant combinations can be constructed on any basis of representations of the group O whose characters are the same as those for the factor group.

The superconducting order parameter is[7]

$$\hat{\Delta}(\mathbf{k}) = \psi(\mathbf{k}) i\hat{\sigma}_y \quad (S = 0);$$
$$\hat{\Delta}(\mathbf{k}) = i(\hat{\sigma}\mathbf{d}(\mathbf{k}))\hat{\sigma}_y \quad (S = 1).$$

The coordinate part [$\psi(\mathbf{k})$ and $\mathbf{d}(\mathbf{k})$] is correspondingly either even or odd under the replacement $\mathbf{k} \to -\mathbf{k}$. Table I lists only those superconducting phases in a crystal of group O_h (UBe_{13}) which have the highest symmetries (in the theory of the superfluidity of ^3He, these phases are called "inert").

The superconducting class $O \times R$ has an analog either an ordinary superconductor ($S = 0$) or the B phase of ^3He ($S = 1$). In other cases the rotation elements are linked up with elements of the gauge group (this is a violation of relative gauge-rotation invariance). As a result, at the points at which the Fermi surface intersects certain axes, or on lines where this surface intersects certain symmetry planes, the superconducting gap in the excitation spectrum vanishes [if this condition is written as $\det\hat{\Delta}(\mathbf{k}) = 0$, it would correspond in the case $S = 1$ to $\mathbf{d}^2(\mathbf{k}) = 0$]. The behavior $C_{se} \propto T^3$ in UBe_{13} (Ref. 4) is satisfied by four phases: a singlet phase $O(D_2)$ and three triplet phases : $O(D_2)$, $O(T) \times R$, and $D_4(E)$.

Let us assume[8] $\det\hat{\Delta}(\mathbf{k}) = |\det\hat{\Delta}(\mathbf{k})| \exp(i\varphi(\mathbf{k}))$. Near a point where we have $\det\hat{\Delta}(\mathbf{k}) = 0$, either the condition $(\mathbf{k}) = 0$ holds [e.g., $O(T) \times R$], or the phase $\varphi(\mathbf{k})$ acquires an increment of $2\pi N$ when this point is circumvented (a boojum). In the A phase of ^3He, this singularity is associated with an orbital angular momentum. In a super-

TABLE I. The most symmetric superconducting phases in a cubic crystal

Superconducting class	Degeneracy	Positions of zeros in spectral gap Type of order parameter	Magnetic moment	Isotropic analog
$O \times R$ $(S=0)$	1	None $\psi(\mathbf{k}) = f(\mathbf{k}) = f(-\mathbf{k})$, real cubic function	—	Superconducting in S state
$O \times R$ $(S=1)$	1	None $\mathbf{d} = (\tilde{x}k_x + \tilde{y}k_y + \tilde{z}k_z)f(\mathbf{k})$	—	B-^{3}He
$O(D_2)$ $(S=0)$	2	8 intersections of Fermi surface with threefold axes $\psi = (k_z^2 + k_x^2 e^{2\pi i/3} + k_y^2 e^{-2\pi i/3}) f(\mathbf{k})$	—	—
$O(D_2)$ $(S=1)$	2	The same zeros $\mathbf{d} = (\tilde{z}k_z + \tilde{x}k_x e^{2\pi i/3} + \tilde{y}k_y e^{-2\pi i/3}) f(\mathbf{k})$	—	α-phase ^{3}He, $S=1$
$O(T) \times R$ $(S=0)$	1	Lines where the Fermi surface intersects diagonal planes of the cube $\psi = (k_x^2 - k_y^2)(k_y^2 - k_z^2)(k_z^2 - k_x^2) f(\mathbf{k})$	—	—
$O(T) \times R$ $(S=1)$	1	Points at which the Fermi surface intersects fourfold axes $\mathbf{d} = \{\tilde{x}k_x(k_y^2 - k_z^2) + \tilde{y}k_y(k_z^2 - k_x^2) + \tilde{z}k_z(k_x^2 - k_y^2)\} f(\mathbf{k})$	—	—
$D_4(E)$ $(S=1)$	6	2 points at which the Fermi surface intersects a fourfold axis $\mathbf{d} = \{a\tilde{z}(k_x + ik_y) + bk_z(\tilde{x} + i\tilde{y})\} f(\mathbf{k})$	Along fourfold axis	A-^{3}He
$D_4(E)$ $(S=0)$	6	At the same places and also at the intersection of the Fermi surface with the perpendicular symmetry plane $\psi = k_z(k_x + ik_y) f(\mathbf{k})$	Along fourfold axis	—

Here $\tilde{x}, \tilde{y}, \tilde{z}$ are unit vectors along the coordinates. Here the groups are $O(T) = (E, 8C_3, 3C_2, 6C_2 e^{i\pi}, 6C_4 e^{i\pi})$; $D_4(E) = (E, C_2 e^{i\pi}, C_4 e^{i\pi/2}, C_4^3 e^{-i\pi/2}, U_{2x} e^{i\pi} R, U_{2y} R, 2U_2' e^{\pm \pi i/2} R)$; $O(D_2) = (E, 3C_2, 2C_4^x R, 2C_2^{x\perp} R, 2C_4^y e^{2\pi i/3} R, 2C_2^{y\perp} e^{2\pi i/3} R, 2C_4^z e^{-2\pi i/3} R, 2C_2^{z\perp} e^{-2\pi i/3} R, 4C_3 e^{-2\pi i/3}, 4C_3^2 e^{2\pi i/3})$.

conductor, it would correspond to an orbital magnetic moment (and also a spin moment in the triplet phase). It is easy to show that the corresponding moments in $O(D_2)$ are ordered antiferromagnetically (along threefold axes) because of the cubic symmetry. A slight uniaxial deformation, however, gives rise to a magnetic moment piezoelectric magnetism).

The question of the orbital angular momentum and its magnitude are most important for the $D_4(E)$ $(S=1)$ phase, which is a superconductor with an orbital and spin ferromagnetism. Even in the ground state, a current should flow along the surface of the sample and cancel the field of the current inside it. (The field can be estimated to be $H \sim \mu_B n \sim 1$ kOe.) A discrete (sixfold) degeneracy allows the formation of domain structures which offset the loss of magnetic energy.

Let us assume that the spin-orbit interaction is nonvanishing but still weak enough that the angle (θ) through which the spins rotate around a fourfold axis could vary over space. In this case the charging current would acquire an additional term[9]:

$$\mathbf{j} = \rho_s \left(\vec{\nabla}\varphi - \frac{2e}{c}\mathbf{A}\right) + \rho_1 \vec{\nabla}\theta .$$

From this term we would expect a nontrivial flux-quantization condition: $\Phi = \varphi_0[N_1 + N_2(\rho_1/\rho_s)]$. Here we have $\rho_1/\rho_s \sim 1$, and this ratio is otherwise arbitrary. The finite spin-orbit coupling determines how advantageous the formation of an additional spin structure will be. This structure might form, for example, around a vortex associated with a change in θ upon a circumvention of 2π $(N_2 = 1,2,...)$.

The magnitude of the spin-orbit coupling is also important for distinguishing the singlet pairing from the triplet pairing in the most symmetric phase, $O(D_2)$. Spin degrees of freedom might be excited at frequencies smaller than the energy gap. We have in mind the possibility that modes of this type would be related to the spin modes (ferromagnetic or antiferromagnetic resonances) in magnetic materials. Spin oscillations associated with the rotation of the system of spins with respect to the lattice of course exist in any phase with $S=1$. For a charged system, however, their description requires appealing to a model.

We wish to thank V. P. Mineev for useful discussions.

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Translated by Dave Parsons
Edited by S. J. Amoretty