Instability at a nonequilibrium superconducting junction

A. I. Larkin and Yu. N. Ovchinnikov
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 10 May 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 12, 556–558 (25 June 1984)

The tunnel injection of electrons from an auxiliary junction at a Josephson junction is accompanied by an instability. The threshold for the instability and the state that arises are determined by both the normal current and the dissipative part of the Josephson current. The average value of the voltage that arises is found, as is the amplitude of the alternating voltage.

At low temperatures the distribution function of the excitations in one of the electrodes of a superconducting junction can easily be driven from equilibrium. If a deviation from equilibrium is induced in an electrode with a large value of the order parameter Δ, then the resistance of the Josephson junction will become negative even if the deviation from equilibrium is comparatively small [greater than $\exp(-\Delta/T)$]. As a result, a static solution with a direct current J becomes unstable. This phenomenon was first pointed out by Aronov and Spivak[1] in a study of the photoelectric effect at a Josephson junction. In the present letter we wish to call attention to two important circumstances. The first is related to a simpler possibility for experimentally observing this phenomenon, by using an additional tunnel contact to create a nonequilibrium distribution function. We derive a condition for the occurrence of an instability for this case. The second circumstance is related to the fact that at a Josephson junction there is, in addition to the one-particle tunnel current, yet another dissipative current, which is proportional to $\cos 2\phi$ (Refs. 2 and 3). An instability in this current sets *in before that in the one-particle current, with important consequences for the* qualitative picture of the resulting effect. The $\cos 2\phi$ dependence of the dissipative current is very nonlinear, determining the amplitude of the steady-state oscillations. Upon a slow change in the voltage across the junction, the current J through the junction is

$$\frac{J}{e} = \frac{C}{e^2}\frac{\partial^2\varphi}{\partial t^2} + I_1(\omega)\sin 2\varphi + I_2(\omega)\cos 2\varphi + I_3(\omega), \tag{1}$$

where $\omega = eV = \partial\phi/\partial t$, and C is the capacitance of the junction.

The coefficients $I_{1,2,3}$ at equilibrium were found in Ref. 2. Generalizing those results to the nonequilibrium case, we find

$$I_2(\omega) = \frac{1}{2R_N e^2}\int d\epsilon\,\{F_L^-(\epsilon)F_R^-(\epsilon-\omega)(f_L^{(1)}(\epsilon) - f_R^{(1)}(\epsilon-\omega))\ ;$$

$$I_3(\omega) = \frac{1}{2R_N e^2}\int d\epsilon\,\rho_L(\epsilon+\omega)\rho_R(\epsilon)[f_L^{(1)}(\epsilon+\omega) + f_L^{(2)}(\epsilon+\omega) - f_R^{(1)}(\epsilon) - f_R^{(2)}(\epsilon)], \tag{2}$$

where

$$\rho(\epsilon) = \frac{|\epsilon|\,\theta(|\epsilon| - \Delta)}{(\epsilon^2 - \Delta^2)^{1/2}}\ ; \qquad F^-(\epsilon) = \frac{\Delta}{\epsilon}\rho(\epsilon), \tag{3}$$

$f^{(1)}(\epsilon)$ is the odd part of the distribution function [at equilibrium, we would have $f^{(1)}(\epsilon) = \tanh(\epsilon/2T)$], and $f^{(2)}(\epsilon)$ is the even part.

The coefficient I_1 contains no dissipative terms. At a zero frequency we would have $I_1 = J_c/e$, where J_c is the critical current of the junction. At a zero voltage a nonequilibrium current $I_3(0)$, proportional to the distribution function $f^{(2)}$ arises. The current $I_3(0)$ causes the critical current of the junction to become dependent on the voltage.

The stability of the static solution of Eq. (1) is determined by the sign of the coefficient of $\partial\phi/\partial t$. Let us assume that a deviation from equilibrium has been induced in the superconductor on the right with a large order parameter Δ_R. For a voltage $eV \ll \Delta_R - \Delta_L$ we then find

$$R^{-1} = (J(V) - J(-V))/2V = \frac{e^2}{\omega}[I_2(\omega)\cos 2\varphi + (I_3(\omega) - I_3(-\omega))/2\,]$$

$$= \frac{\Delta_R}{R_N(\Delta_R^2 - \Delta_L^2)^{1/2}}$$

$$\times \left\{ \frac{(2\pi T\Delta_R)^{1/2}}{\omega} \exp\left(-\frac{\Delta_R}{T}\right)\sinh\left(\frac{\omega}{T}\right)\left(1 + \frac{\Delta_L}{\Delta_R}\cos 2\varphi\right) - \frac{\Delta_L N}{\Delta_R^2 - \Delta_L^2}\left(\cos 2\varphi + \frac{\Delta_L}{\Delta_R}\right)\right\}. \tag{4}$$

In deriving (4) we assume that the nonequilibrium correction to the distribution function is concentrated near the threshold Δ_R. The nonequilibrium distribution function created by tunnel injection from an auxiliary junction was studied in Refs. 4–6. At a voltage eV_u on the auxiliary contact slightly above the gap size Δ_R, the difference between the distribution function $f^{(1)}$ and unity is a step. In this case we can write

$$N = \int_{\Delta_R}^{\infty} d\epsilon \rho(\epsilon)(1 - f^{(1)}(\epsilon)) = -\delta f^{(1)}(2\Delta_R)^{1/2}(eV_u - \Delta_R)^{1/2}. \tag{5}$$

The size of the step, $\delta f^{(1)}$, can be expressed in terms of the resistance of the auxiliary junction, R_u, the energy relaxation time τ_ϵ, and the volume of the superconducting film[6] ϑ:

$$-\delta f^{(1)} = (w\tau_\epsilon)^{1/2} / (2(eV_u - \Delta_R))^{1/4};$$

$$w = \frac{1}{8e^2 R_u \vartheta \nu}; \qquad \tau_\epsilon^{-1} = g\Delta_R^3 / \omega_D^2; \qquad \nu = \frac{mp_0}{2\pi^2}. \tag{6}$$

It follows from (4) and (5) that an instability occurs at an exponentially small value of $\delta f^{(1)}$:

$$N_{\rm cr} = N_{\rm cr}^{(0)} \frac{1 + \frac{\Delta_L}{\Delta_R}(1 - (J/J_c)^2)^{1/2}}{(1 - (J/J_c)^2)^{1/2} + \Delta_L/\Delta_R}; \quad N_{\rm cr}^{(0)} = \left(\frac{2\pi\Delta_R}{T}\right)^{1/2} \frac{\Delta_R^2 - \Delta_L^2}{\Delta_L} \exp\left(-\frac{\Delta_R}{T}\right). \tag{7}$$

The instability threshold increases with increasing external current J.

The nature of the steady state will depend on the extent to which the threshold is exceeded. If the threshold is exceeded only slightly, the steady state will be oscillatory. The period and amplitude of the oscillations can be found from the condition

$$\oint dt \left(\frac{\partial\varphi}{\partial t}\right)^2 R^{-1}(\varphi, \omega) = 0. \tag{8}$$

The integral in (8) is evaluated over the oscillation period.

If the threshold is exceeded only slightly, the oscillation amplitude ϕ_1 is small:

$$\varphi(t) = \varphi_0 + \varphi_1 \sin\Omega t \quad ; \quad \Omega^2 = \frac{2eJ_c}{C}(1-(J/J_c)^2)^{1/2}; \quad \cos 2\varphi_0 = (1-(J/J_c)^2)^{1/2} \; ;$$

$$\varphi_1^2 = \frac{e\delta V_u}{2(eV_u - \Delta_R)}\left[1 + \frac{\Delta_L}{\Delta_R \cos 2\varphi_0}\right]\left[1 + \frac{\Delta_L}{\Delta_R}\cos 2\varphi_0\right](1 - (\Delta_L/\Delta_R)^2). \qquad (9)$$

There exists a certain pump level above which there will be an abrupt change to a growing solution $\phi(t)$. For the current $J=0$ this pump level N is $N = N_{\rm cr}^{(0)}(3\Delta_R + \Delta_L)/(\Delta_R + 3\Delta_L)$. For higher pump levels, an average voltage appears across the junction. This average voltage is given by

$$J/e = \frac{1}{\pi}\int_0^{\pi} d\varphi\,[I_2(\omega)\cos 2\varphi + I_3(\omega)]; \qquad \omega = \frac{\partial\varphi}{\partial t}, \qquad (10)$$

where $I_{2,3}$ are given by Eq. (2). If the resulting voltage eV is large in comparison with Ω, then we would have

$$J/e = I_3(eV).$$

If the current is not too low, $J < J_c$, the voltage eV is approximately equal to the sum $\Delta_R + \Delta_L$.

[1]A. G. Aronov and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 218 (1975) [JETP Lett. **22**, 101 (1975)].
[2]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **51**, 1535 (1966) [Sov. Phys. JETP **24**, 1035 (1966)].
[3]N. R. Werthamer, Phys. Rev. **147**, 255 (1966).
[4]J. Clarke, Phys. Rev. Lett. **28**, 1363 (1972).
[5]M. Tinkham, Phys. Rev. B **6**, 1747 (1972).
[6]I. É. Bulyzhenkov and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **74**, 224 (1978) [JETP **47**, 115 (1978)].

Translated by Dave Parsons
Edited by S. J. Amoretty