Toroidal spin ordering in crystals

A. A. Gorbatsevich and Yu. V. Kopaev
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

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A mechanism for the formation of ordered spin fluxes within a unit cell is proposed for the nonconservation of parity in crystals. A number of properties of the ground state are examined: the current-magnetic effect, nonuniform magnetic structures, and nonconservation of parity in atoms.

1. This paper is concerned with the ordered state of a crystal, characterized by a nonzero time-average value of the spin current density at a given point within a unit cell. The spin flux $\mathbf{J}_a^s$ is described by a tensor of rank 2 (a dyad),

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$$J_a^s = \langle \sigma_a \hat{j} \rangle \ , \tag{1}$$

where σ_a is a Pauli matrix ($a = x, y, z$), and $\hat{\mathbf{j}}$ is the current-density operator. The macroscopic symmetry of the state with the spin current density is characterized not by the local value of the tensor J_a^s, but by the symmetry of its spatial distribution in a unit cell. The quantity J_a^s is invariant under time reversal. However, the complete group of the macroscopic symmetry is not the standard group of spatial symmetry of the crystal, but a "color" group (see, for example, Ref. 1), which includes rotation operations and reflection in spin space as the "load" groups, in addition to the spatial group. If the spin component of the tensor J_a^s does not depend on the coordinates (i.e., the spins of all particles, which contribute to the spin flux, are parallel), then the symmetry of the state with $J_a^s \neq 0$ can be described simply. If the axis of the spins is oriented along the symmetry axis of the system, then the loading operation becomes a rotation in spin space through an angle π. Taking into account (1), this operation is completely equivalent to inversion of coordinates in ordinary space. We can thus describe the symmetry of spin fluxes in terms of the ordinary space symmetry groups. Using the equation of continuity for the coordinate component

$$\operatorname{div} J_a^s = 0 \ , \tag{2}$$

which is valid in the stationary case and in the absence of interactions that do not conserve spins, we factor out the toroidal and poloidal components of the spin flux. This breakdown corresponds to currents flowing along the parallels and meridians of the torus in the toroidal coordinates. In the case of the usual flux of charged particles, i.e., an electric current, the corresponding components are described by two independent families of electromagnetic multipoles: magnetic and toroidal moments.[2] A new type of ordering in solids turns out to be related to the toroidal moments.[3,4] It is convenient to represent the spin flux in the form of two equal fluxes, of particles with the same sign of the charge, but with opposite spin, flowing toward each other (Fig. 1). It is easy to show that the symmetry of the toroidal distribution of the spin flux (Fig. 1a) is described by a scalar in a uniaxial system. The corresponding invariant is $\vec{\sigma}\,\mathbf{L}$ ($\mathbf{L}$ is the orbital angular momentum). The poloidal configuration (Fig. 1b) corresponds to the symmetry group of the pseudoscalar $\vec{\sigma}\,\mathbf{T}$ ($\mathbf{T}$ the toroidal moment—is an odd polar vector under time reversal). The last type of ordering we shall call toroidal spin (TS) ordering, in contrast to toroidal current ordering.[4]

2. As a model allowing a microscopic description of the pseudoscalar TS ordering, we will examine the model of an excitonic dielectric (ED).[5] This model describes a

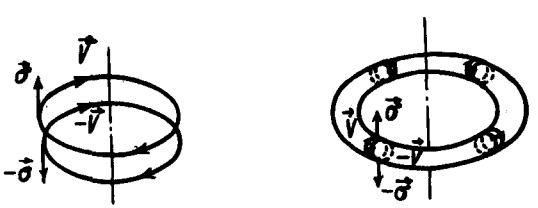

Fig. 1.

phase transition with the formation of a Bose-condensate of electron-hole pairs in terms of the order parameter Δ.

In general, the order parameter has a complex structure,

$$\Delta = \mathrm{Re}\,\Delta + i\,\mathrm{Im}\,\Delta = \Delta_R^s + \vec{\sigma}\vec{\Delta}_R^t + i(\Delta_I^s + \vec{\sigma}\vec{\Delta}_I^t) \tag{3}$$

where Δ^s and $\vec{\Delta}^t$ are the singlet and triplet components of the order parameter. Time reversal leaves Δ_R^s and $\vec{\Delta}_I^t$ invariant and changes the sign of Δ_I^s and $\vec{\Delta}_R^t$. Different components of the order parameter correspond to different physical properties. The coexistence of Δ_R^s and $\vec{\Delta}_R^t$ with finite doping ($\delta n \neq 0$) leads to the appearance of ferromagnetism.[6] The magnetic-moment density is expressed in terms of Δ_R^s and $\vec{\Delta}_R^t$,

$$\mathbf{M} \sim \delta n \Delta_R^s \vec{\Delta}_R^t . \tag{4}$$

When the extrema of the electron and hole bands coincide, we can introduce an interband matrix element of the momentum $\mathbf{P}_{12} = i\mathbf{P} = \langle 1|\hat{\mathbf{p}}|2\rangle$. The toroidal-moment density $\mathbf{T}$ is defined as

$$\mathbf{T} \sim \mathbf{P}\,\Delta_I^s . \tag{5}$$

The triplet imaginary $\vec{\Delta}_I^t$ of the order parameter (3) is responsible for the appearance of stationary spin fluxes in the unit cell. This was pointed out by Halperin and Rice.[7] These authors, however, examined[7] the case of separated band extrema, when $\mathbf{P} \equiv 0$ and local ordering on the scales of the unit cell does not change the macroscopic symmetry of the crystal. For $\mathbf{P} \neq 0$, $\Delta = i\vec{\sigma}\vec{\Delta}_I^t, \mathbf{P} \| \vec{\Delta}_I^t \| 0z$ the spectrum of single-electron excitations in the model of ED is

$$E_\pm^2 = \epsilon^2 + \left(\frac{1}{m} P k_z \pm \Delta_I^t\right)^2 . \tag{6}$$

Here k_z is the projection of the quasimomentum along the z axis, ϵ is the seed spectrum of the phase with $\Delta_I^t = 0$ and $P = 0$, and m is the electron mass. The different signs in (6) correspond to opposite orientations of the spin. It is easy to verify that the spectrum is noninvariant under any noncharacteristic transformation and describes a system with a pseudoscalar group symmetry. According to the results of the preceding section, this is an indication of toroidal ordering of spin fluxes in the cell.

We now describe the physical manifestations of TS ordering.

3. The current-magnetic effect (CME). Spectrum (6) is split with respect to the spin (Fig. 2). However, in the equilibrium state the magnetic moment of excess charge carriers with quasimomentum $\mathbf{k}$ is exactly compensated for by the moment of particles with opposite quasimomentum. If, on the other hand, an electric current flows in the sample, then its component parallel to the z axis gives rise to an asymmetry in the distribution of charge carriers and a total magnetization $M^z \sim JP\Delta_I^t$ appears in the system. The following term in the free energy density corresponds to the CME in the pseudoscalar TS state:

$$\delta F_1 = \beta_1 j^z H^z \tag{7}$$

where the pseudoscalar is $\beta_1 \sim P\Delta_I^t$.

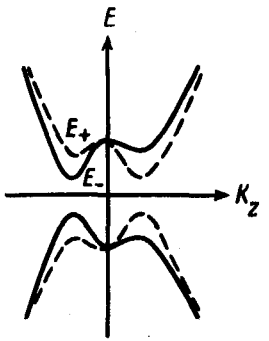

Fig. 2.

The longitudinal CME in a nonmagnetic material can serve as an experimental criterion for realization of TS ordering.

4. The pseudoscalar symmetry of the product $P\Delta_I^t$ leads to the appearance of a Lifshitz invariant in the expression for the free energy density,

$$\delta F_2 = \beta_2 P \Delta_I^t \left(M_x \frac{\partial}{\partial z} M_y - M_y \frac{\partial}{\partial z} M_x\right) \tag{8}$$

where in the limit of a low doping level in the model in Ref. 5

$$\beta_2 = \frac{2}{7}\left(\frac{c}{e}\frac{\Theta}{\Delta_R^s \delta n}\right)^2$$

(θ is the temperature). We took into account the exact relation[6] instead of (4). The coefficient in front of the Lifshitz invariant in (8), which is of a purely exchange origin, does not contain a relativistic small term. This indicates that when the coefficient is sufficiently large (high density of the pseudoscalar condensate), the derivatives of the magnetic moment are also large. The expression for the free energy density contains a term which describes the polarization of the pseudoscalar condensate by the magnetic moment and which induces the toroidal current moment,

$$\delta F_3 = \beta_3 \Delta_I^t \mathbf{MT} \tag{9}$$

In the model in Ref. 5

$$\beta_3 = \left(\frac{48}{7\zeta(3)}\right)\left(\frac{c}{e}\right)\frac{(\pi^2\Theta)^2}{v_F^3 P},$$

and we took into account (4) and (5). If the moment $\mathbf{M}_L$ is localized, then the toroidal moments $\mathbf{T}_L$ are said to be localized.

5. Amplification of weak interactions. The mechanism of nonconservation of parity in crystals described by us can lead to effects which usually arise as a result of P-odd interactions due to the exchange of Z^0 bosons in the atoms.[8,9] An atomic orbital toroidal moment can also appear as a result of a purely exchange type of interaction (9) of the pseudoscalar condensate with the spin of an atomic electron. In crystals with TS ordering we should therefore expect a large amplification of the effects associated with the nonconservation of parity in the intranuclear interactions. Exchange through the

usual toroidal current excitation, however, can be linked with a T-odd interation, which is analogous to that observed in the decay of neutral K^0 mesons.

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