

Dynamic model of the spatial development of turbulence

A. V. Gaponov-Grekhov, M. I. Rabinovich, and I. M. Starobinets

Institute of Applied Physics, Academy of Sciences of the USSR

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An infinite chain of sequentially coupled oscillators, which simulates a nonequilibrium dissipative medium with a flow, is used as an example to show that in the absence of perturbations at the boundary a turbulence, which develops “downstream along the flow,” can be established in a non-mutually coupled nonlinear medium. In a steady-state regime the characteristics of the turbulence are determined solely by the degree of departure of the system from equilibrium and by the coordinate along the chain. The results obtained make it possible to judge the relationship between turbulence in semi-bounded systems and stochasticity in finite-dimensional dynamic systems.

1. It has been found in recent years that turbulence in dissipative media can be represented in terms of random pulsations of a nonlinear field, corresponding to the motion of a finite-dimensional dynamic system on a strange attractor. Recent remarkable experiments^{1,2} have established as fact the proposed relationship between strange attractors and hydrodynamic turbulence in so-called internal flows—Couette-Taylor flow and thermoconvection in a cell. The finite-dimensional turbulence in bounded systems (resonators) can be explained graphically: The discrete infinite-dimensional system of interacting modes which describe the field in a resonator is “cut off” by

high-frequency dissipation (viscosity), and is thereby transformed into a finite-dimensional system (see, for example, Ref. 3).

Whether there is a relationship between turbulence in infinite systems, in particular, hydrodynamic shear flows, and stochasticity in finite-dimensional dynamic systems has not been clarified yet. Furthermore, it is not clear that turbulence can develop spontaneously in unbounded or semi-bounded media in the absence of global feedback. In this letter we show that at $u_0(t) \equiv 0$ there can be a turbulence along j in semi-bounded media which are not coupled with each other and in which the field is described by a discrete model such as

$$\frac{du_j}{dt} + \alpha \Delta u_j = u_j(1 - \delta |u_j|^2), \quad \Delta u_j = u_j - u_{j-1}, \quad j = 1, 2, \dots, \quad (1)$$

where u_j , α , and δ are complex values.

2. The system (1) with $\alpha = \alpha'(1 - i\alpha'')$ and $\delta = 1 - i\beta$, where α' , α'' , and $\beta > 0$, was investigated in a numerical experiment. The power spectra of the amplitude and phase of $u_j(t)$ were analyzed and the dimensionality of $u_j(t)$ ^{4,5} which occurs "downstream along the flow" was calculated as a function of j .

Some examples of the physical model of (1) are a chain of sequentially coupled van der Pol oscillators and, under certain simplifying assumptions, a "one-dimensional" flow coupled with a periodic system of cavities, and some others.

Collective excitations such as stationary travelling waves $u_j(t) = A \exp[i\omega(A^2)t - kj]$, similar to those observed in the discrete Ginzburg-Landau model⁶, can occur in an infinite or ring-shaped system like (1). If the chain is semi-bounded, then single-frequency excitations of this type can also appear in it, but they are not spatially nonuniform.

3. The numerical experiment with semi-bounded chain (2) with weak coupling (α' is small) showed that a regime of independent local (at each j) excitations at the frequency $|\omega| = \beta$, which with an increase in coupling leads to the regime of complete synchronization of all oscillators, beginning with the l th one, is established in the chain. In the presence of strong coupling the uniform synchronized regime is unstable and a spatially inhomogeneous single-frequency regime of collective excitations with $\omega \in (0, \beta)$ is established in the chain. An intermediate regime—the beat regime, which represents an interaction of local and collective excitations, is established in system (1) over a broad range of parameters α and β , for sufficiently large j , this regime transforms into a modulated stationary wave (see Fig. 1).

It is logical to assume that nonlinear interaction of collective and local excitations can establish in the system a regime with self-excited stochastic oscillations, exemplified by a strange attractor. Since the number of the degrees of freedom of the system participating in the motion increases with j , the dimensionality of the strange attractor will generally increase "downstream along the flow," i.e., there can, in principle, be an "infinite-dimensional" turbulence in the limit $j \rightarrow \infty$. This, however, does not occur even in a uniform chain ($\alpha, \delta = \text{const}$). As is evident from Fig. 2, spatial development of turbulence occurs in the chain: a quasimonochromatic regime is observed for small values of j ; which as j is increased, the quasimonochromatic regime is replaced by a beat regime with a large number of harmonics: and, finally, further "downstream

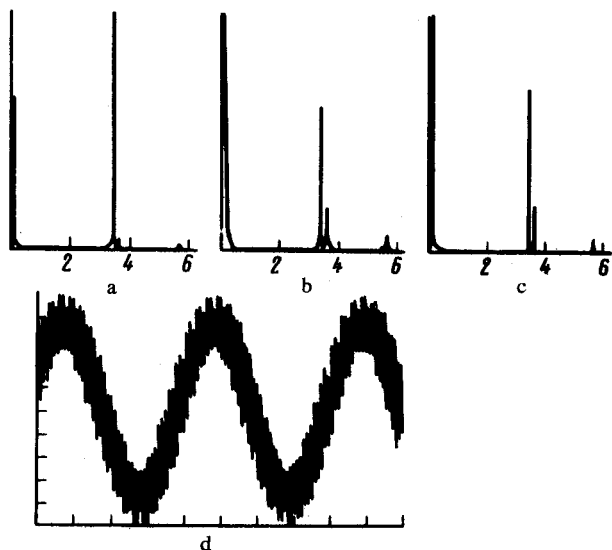


FIG. 1. Beat regime established along the chain. a, b, and c—Power spectra at the points $j = 2, 7$, and 20 , respectively; d—oscillogram of oscillations with $j = 7; 20$; $\alpha' = 0.7$, $\alpha'' = 1.71$, and $\beta = 3.42$.

along the flow” this last regime transforms into a weakly turbulent one. Turbulence develops further on and its average intensity increases, but for sufficiently large values of j it no longer changes: a regime of spatially homogeneous stationary turbulence is established. The change in the fractal dimensionality of the attractor, corresponding to the observed regime, is shown in Fig. 3. It is evident that the dimensionality of the strange attractor assumes a constant, comparatively low value already with $j = 15$.

4. Thus a turbulent regime, which is independent of external fields, can indeed be established in a one-dimensional nonequilibrium chain—a “medium” allowing the propagation of perturbations only in one direction. The independence of the dimensionality and statistical properties of the chaotic regime from the coordinates for large

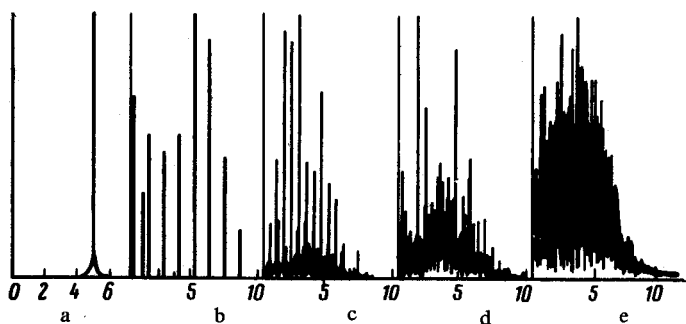


FIG. 2. Power spectra of $u_j(t)$, illustrating the spatial development of turbulence along the chain. a— $j = 2$, b— $j = 9$, c— $j = 10$, d— $j = 12$, e— $j = 20$ and 50 ; $\alpha' = 0.5$, $\alpha'' = 1.71$, and $\beta = 5.0$.

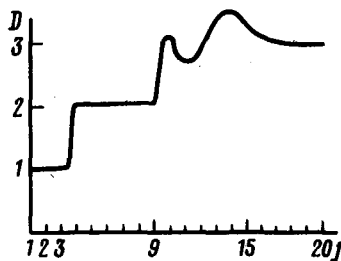


FIG. 3. Change in the dimensionality of the attractor along the chain. $\alpha' = 0.5$, $\alpha'' = 1.71$, and $\beta = 5.0$.

j indicates that the turbulence emerges into a stationary regime characterized by partial coherence. The turbulence in this case has a finite-dimensional nature even in the limit $j \rightarrow \infty$. Clearly, it would be worthwhile to explain the mechanisms responsible for the stabilization of turbulence in homogeneous nonequilibrium media like (2) and to estimate the changes in dimensionality of the strange attractor along the system.

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