Conductivity of metals and semiconductors with defects with long- and short-range potentials in a magnetic field

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The transverse conductivity $\sigma_\perp$ of a metallic system with defects of two types in a classically strong magnetic field H is examined. The range a of the potential of one of the defects is much greater than the Larmor radius of an electron r; the range of the other defects is shorter than r. It is shown that the contributions to the conductivity from scattering by different defects are not additive. A dependence of the form $\sigma_\perp \sim H^{-4/3}$ is possible.

The transverse conductivity $\sigma_\perp$ of metals with a closed Fermi surface and of semiconductors in a strong magnetic field $r \ll l$ (l is the mean free path of electrons) arises due to the displacement of the centers of electronic orbits as a result of scattering. The conductivity $\sigma_\perp$ is related to the coefficient of diffusion across the magnetic field $D_\perp$ by Einstein's relation, which for a degenerate electron is $\sigma_\perp = e^2(\partial n/\partial \epsilon)D_\perp$, where $\partial n/\partial \epsilon$ is the state density at the Fermi level. If in each scattering event the center of the orbit of an electron is displaced in a random manner, then the diffusion coefficient is (see Ref. 1, p. 457)

$$D_\perp = \frac{\Sigma(\Delta x)^2}{2\delta t}, \tag{1}$$

where the summation extends over collisions experienced by an electron in a time interval δt, and Δx is the change in the x coordinate of the center of the orbit as a result of the collision.

Relation (1) is valid for scattering by phonons[2] or point defects.[3] However, as will be shown below, it is not always valid in a system with randomly distributed defects, whose potential range satisfies the inequality $r \ll a \ll l$. This condition can be satisfied, for example, for the potential of dislocations in a crystal or the potential of ionized impurities in semiconductors.

In this paper we examine the conductivity of a degenerate electron gas in a crystal with defects of two types. The range of the potential of one of the defects is $a \ll r$, and that of the other is $b \ll r$. We shall assume that the long-range potential is smooth and that it has an amplitude $u_0 \ll \epsilon_F$, where ϵ_F is the Fermi energy of electrons. The density of defects with this potential is $n \lesssim a^{-3}$. For scattering by short-range defects we introduce the momentum relaxation time τ. In the absence of long-range defects the coefficient of diffusion across the magnetic field would be $D_\perp \sim r^2/\tau$, while for diffusion along the field it would be $D_\parallel \sim v_F^2 \tau$, where v_F is the Fermi velocity. In contrast to Refs. 4 and 5, we examine the case in which $l = v_F \tau \gg a$, $(na^2)^{-1}$. Here $(na^2)^{-1}$ is the mean free path of electrons between collisions with the long-range defects. The relaxation of the longitudinal component of the momentum by defects a can be ignored, since $n_0 \ll \epsilon_F$. Scattering by defects a can be viewed as a drift in crossed fields: The field H and the field of the defect, $E \sim u_0/ea$ (see Ref. 1, p. 308). A single scattering event a displaces the center of the orbit of an electron by an amount $\Delta x \sim v_{\mathrm{dr}} \Delta t \sim n(u_0/\epsilon_F)$, where $v_{\mathrm{dr}} \sim c(u_0/eHa)$ is the drift velocity, and $\Delta t \sim a/v_F$ is the transit time of the electron in the field of the defect. The transverse displacement of the electron along its mean free path in the direction of the magnetic field l, due to the interaction with defects a, is $\Delta x_l \sim r(u_0/\epsilon_F)(na^2 l)^{1/2} \sim n(u_0/\epsilon_F)(\tau/\tau_0)^{1/2}$, where $\tau_0 = (na^2 v_F)^{-1}$. It is assumed that $\Delta x_l \ll a$.

The main feature of our analysis is that the electron, after being scattered once by a defect of size a will return during the random motion along the magnetic field many times to the defect before it is displaced across the field by a distance $\simeq a$. Moving in the field of the long-range defect under examination, the electron will be displaced each time approximately in the same direction by an amount $\simeq \Delta x$. The total displacement is $\simeq \Delta x M$ (M is the number of times the electron returns to the defect), whereas Eq. (1) yields a displacement $\simeq \Delta x M^{1/2}$ due to these distance collisions. The transverse motion of the electron by a distance shorter than a has a nondiffusive nature and becomes diffusive only a scale greater than a. The step of such diffusion is on the order of a, and the diffusion coefficient is $D_\perp \sim a^2/t_0$ where t_0 is the time over which the electron is displaced by a distance $\sim a$. To determine t_0 we must find the displacement of the electron $\Delta X(t)$ as a function of time for $\Delta X(t)$ smaller than a. We can find t_0 from the equation $\Delta X(t) \approx a$.

To find $\Delta X(t)$ we will make use of the fact that the probability for observing an electron a second time in the field of the effect under examination after a time t is $a/(D_\parallel t)^{1/2}$. The average drift velocity under the action of only a single defect is $v_{\mathrm{dr}} \sim v_{\mathrm{dr}}\, a/(D_\parallel t)^{1/2}$. The corresponding displacement is $\Delta x_1(t) \sim v_{\mathrm{dr}} a/(t/D_\parallel)^{1/2} \sim r v_F (u_0/\epsilon_F)(t/D_\parallel)$. The number of long-range defects with which the electron interacts within a time t is on the order of $(D_\parallel t)^{1/2}/(na^2)^{-1}$. Because of the random distribution of defects, the square of the displacement of an electron over a time t as a result of interaction with all defects, both long- and short-range, is

$$\overline{[\Delta X(t)]^2} \sim [\Delta x(t)]^2 \frac{(D_\parallel t)^{1/2}}{(na^2)^{-1}} + \frac{r^2}{\tau} t \sim \left(r v_F \frac{u_0}{\epsilon_F} \right)^2 t^{3/2} D_\parallel^{-1/2} n a^2 + r^2 \frac{t}{\tau}. \quad (2)$$

The time t_0 can be found from the approximate equality $\overline{[\Delta X(t)]^2} \approx a$. The diffusion coefficient is

$$D_\perp \sim \frac{a^2}{t_0} \sim \left(r v_F \frac{u_0}{\epsilon_F} \right)^2 (t_0/D_\parallel)^{1/2} n a^2 + r^2/\tau. \quad (3)$$

If the first term in (3) is much larger than the second term, we would have

$$D_\perp = D_1 \sim (r^4 a^2 / \tau \tau_0^2)^{1/3} (u_0/\epsilon_F)^{4/3} \propto H^{-4/3}. \quad (4)$$

We see that the contribution of scattering by long-range and short-range defects to the diffusion coefficient [and therefore to the conductivity $\sigma_\perp = e^2(\partial n/\partial \epsilon) D_\perp$] is not additive. The quantity $D_1 > r^2/\tau$, if $(a/r)(\tau/\tau_0)(u_0/\epsilon_F)^2 > 1$.

We ignored the displacement of electrons in states in which the energy of motion along the magnetic field is $\epsilon_z < u_0$. In the case of a repulsive potential u_0 an electron with $\epsilon_z < u_0$ is trapped for some time between two neighboring long-range defects, reflecting first from one defect and then from the other [under the condition that $\tau > \tau_0 (\epsilon_F/\epsilon_z)^{1/2}$]. The motion across the magnetic field is on the average driftlike. Under certain conditions the transverse diffusion is determined by the motion of electrons in states with $\epsilon_z < u_0$. Taking this into account leads to an additional restriction for expression (4), $(r/a) < (\epsilon_F/u_0)^{7/4} (\tau_0/\tau)^2$.

The mechanism for transverse diffusion of electrons which we examined above could conceivably account for the deviation from the law $\sigma_\perp \propto H^{-2}$ in bismuth, observed in magnetic fields stronger than 1 kOe (see, for example, Ref. 6). The characteristic scale of the dislocation field nonuniformity is $a \sim N^{-1/2}$, where N is the number of dislocations threading through a cross section of unit area. For $N \sim 10^6$ cm^{-2} we have $a \sim 10^{-3}$ cm. The time is $\tau_0 \sim (N a v_F)^{-1} \sim a/v_F$. In a magnetic field we have $H = 10$ kOe, in which the deviation from the law $\sigma_\perp \propto H^{-2}$, observed in Ref. 6, reaches 100%, $r \simeq 10^{-5}$ cm. Setting $u_0 \sim \Lambda (b/a) \sim 10^{-4}$ eV, where $b \simeq 10^{-7}$ cm is the modulus of Burgers' vector, $\Lambda \simeq 1$ eV is the strain potential, $\epsilon_F \simeq 10^{-2}$ eV, and $l \simeq 10^{-2}$ cm, we find that $(\tau/\tau_0)(u_0/\epsilon_F) \sim 10^{-2} \sim r/a$. This is the condition for which D_1 is comparable to r^2/τ. In stronger fields $D_1 \propto H^{-4/3}$ dominates.

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[1]E. M. Lifshitz and L. P. Pitaevskiĭ, Fizicheskaya kinetika (Physical Kinetics), Nauka, Moscow, 1979.

[2]S. Titeica, Ann. Phys. **22**, 128 (1935).

[3]B. I. Davydov and I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **9**, 1294 (1939).

[4]M. E. Erezhenov and S. I. Pekar, Fiz. Tverd. Tela (Leningrad) **5**, 1293 (1963) [Sov. Phys. Solid State **5**, 945 (1963)].

[5]C. Herring, J. Appl. Phys. **31**, 1939 (1960).

[6]Yu. A. Bogod and V. B. Krasovitskiĭ, Phys. Status Solidi B **65**, 847 (1974).

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