Limits of applicability of the macroscopic theory of transition radiation

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The conditions of applicability of macroscopic electrodynamics for the description of transition radiation are shown to be more stringent than those for the propagation of fields in matter. This can be seen in the case of transition radiation of nonrelativistic particles or grazing incidence of a charge on the interface between two media.

1. Transition radiation, which arises when a charged particle moving at a constant velocity crosses the interface between two media, is usually studied with the help of macroscopic electrodynamics.[1–3] The purpose of this letter is to call attention to the fact that the small value of the wave vector **k** of the emitted wave in comparison with the inverse interatomic spacing b^{-1},

$$kb \ll 1. \tag{1}$$

is not sufficient for such an analysis. For fixed **k**, the condition of applicability of the macroscopic description for the intrinsic field of a uniformly moving charge

$$\mathbf{E}_0(\mathbf{r},t) = \int d^3q \int d\omega\, \mathbf{E}_0(\mathbf{q})\,\delta(\omega - \mathbf{qv})\exp(i\mathbf{qr} - i\omega t) \tag{2}$$

may turn out to be more stringent than (1). It is not difficult to obtain such a condition from the following considerations. Transition radiation arises as a result of reflection and refraction of the intrinsic field of a charge from the interface between two media $z=0$. The conservation of the field frequency and of the wave-vector components tangential to the plane of the interface with reflection and refraction leads to the relations

$$q_x = k_x; \quad q_y = k_y; \quad \mathbf{qv} = \omega; \quad q_z = (\omega - k_x v_x - k_y v_y)/v_z. \tag{3}$$

These relations determine the Fourier component of the intrinsic field which accounts for the emission of a quantum and for which the macroscopic description must be used. From (1) and (3) it follows that $bq_x \ll 1$ and $bq_y \ll 1$, so that we need only to require that $bq_z \ll 1$, i.e.,

$$(\omega - k_x v_x - k_y v_y)(b/v_z) \ll 1. \tag{4}$$

The macroscopic electrodynamics can be used for transition radiation because both (1) and (4) are valid. The macroscopic analysis is incorrect when one of these conditions does not hold.

2. Condition (4) will break down when condition (1) is satisfied if the values of v_z are small, i.e., for nonrelativistic particles or for grazing incidence of the charge on the interface between two media (the velocity of the charge is almost parallel to the surface). We note that in many papers the transition radiation with grazing incidence was examined macroscopically, under the assumption that condition (1) was satisfied, but without a formulation of the complete conditions of applicability of macroscopic theory (1) and (4). Condition (4) must also be used in determining the region of applicability of such results. It is interesting to note that the disagreement between the results of the macroscopic theory and experimental data for transition radiation of nonrelativistic (from 100 to 40 keV) electrons with grazing incidence of the electrons on the surface of silver was discovered more than 10 years ago.[4] The disagreement with the macroscopic theory was observed outside the region of applicability of this theory, when condition (4) is violated.

3. If condition (1) is satisfied and condition (4) is violated, then the transition radiation must be studied microscopically, in spite of the large wavelength of the emitted field. In this case the density of microcurrents $\mathbf{j}_m$ induced in the matter by the field of the moving charge must be taken into account in Maxwell's macroscopic equations. At frequencies larger than the atomic frequencies the value of $\mathbf{j}_m$ can be calculated in a way similar to the method used to calculate it in the theory of diffraction of x rays[5]:

$$\mathbf{j}_M(\mathbf{r},\omega) = (ie^2/m\omega)n(\mathbf{r})\mathbf{E}(\mathbf{r},\omega), \tag{5}$$

where $n(\mathbf{r})$ is the electron density in the matter, averaged over the quantum-mechanical electronic state and over the thermal motion of the atoms. The averaging over a physically infinitesimal volume, which is characteristic of macroscopic electrodynamics, is not performed here, and the nonuniformity associated with the density of electrons in the atoms is taken into account.

4. At high frequencies the value of $\mathbf{j}_m$ may be assumed to be small and the method of successive approximations can be used to solve Maxwell's equations. The solution in a zeroth approximation ($\mathbf{j}_m = 0$) is then the intrinsic field (2) of a charge moving with uniform velocity in a vacuum, where

$$\mathbf{E}_0(\mathbf{q}) = (ie/2\pi^2)(\mathbf{v}(\mathbf{qv}) - \mathbf{q})(q^2 - (\mathbf{qv})^2)^{-1}. \tag{6}$$

The equations for the field in a first approximation, $\mathbf{E}_1 = \mathbf{E} - \mathbf{E}_0$, corresponds to Maxwell's equations in a vacuum with a fixed current density

$$\mathbf{j}_M(\mathbf{r},\omega) = (ie^2/m\omega)n(\mathbf{r})\mathbf{E}_0(\mathbf{r},\omega),$$

which vanishes in the half-space $z > 0$ outside the matter. The transition-radiation energy in the frequency range $d\omega$ in the element of solid angle $d\Omega$ (oriented along the unit vector n) is given by

$$d^2E(\mathbf{n},\omega) = (e^4/m^2)d\omega d\Omega \int d^3q' \int d^3q'' [\mathbf{n}\mathbf{E}_0(\mathbf{q}')][\mathbf{n}\mathbf{E}_0^*(\mathbf{q}'')]\delta(\omega - \mathbf{q}'\mathbf{v}) \cdot$$

$$\cdot\, \delta(\omega - \mathbf{q}''\mathbf{v}) \int_{z'<0} d^3r' \int_{z''<0} d^3r'' n(\mathbf{r}')n(\mathbf{r}'')\exp\{-i(\mathbf{k}-\mathbf{q}')\mathbf{r}' + i(\mathbf{k}-\mathbf{q}'')\mathbf{r}''\}. \tag{7}$$

This expression must be averaged over the fluctuations of the electronic density. To perform this averaging we will use the well-known equation ($n_0 = \langle n \rangle$)

$$\langle n(\mathbf{r}')n(\mathbf{r}'')\rangle = n_0^2 w(\mathbf{r}' - \mathbf{r}'') + n_0\,\delta(\mathbf{r}' - \mathbf{r}''), \tag{8}$$

where

$$w(\mathbf{r}' - \mathbf{r}'') = \int d^3 s g(\mathbf{s}) \exp(i\mathbf{s}\mathbf{r}' - i\mathbf{s}\mathbf{r}'') \tag{9}$$

is the probability that one electron is located at the point $\mathbf{r}'$ and a second electron is located at the point $\mathbf{r}''$. We might note that the term which is linear in n_0 in (8), after substitution into (7), gives an expression for transition radiation induced by inhomogeneities in the bulk of the matter. This transition radiation, which is not related to the presence of the interface, was investigated theoretically in Refs. 2 and 6. We will therefore take into account below only the terms in (8) which are quadratic in n_e and which are directly related to the case of interest to us—the transition radiation which occurs when a charged particle crosses the interface between matter and the vacuum. After averaging (7) with use of (8), we find

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{n_0^2 e^4}{m^2 v_z^2} \int \frac{d^3 s g(\mathbf{s})}{(Q + s_z - k_z)^2} \left| [\mathbf{n}\mathbf{E}_0(\mathbf{k}_t - \mathbf{s}_t + \mathbf{Q})] \right|^2 (2\pi)^4. \tag{10}$$

In this expression $\mathbf{k}_t$ and $\mathbf{s}_t$ denote the components of the vector $\mathbf{k}$ and $\mathbf{s}$ which are tangential to the surface, the vector $\mathbf{Q}$ is oriented along the z axis, and $Q = (\omega - k_x v_x - k_y v_y)/v_z$. The formal use of the macroscopic theory in this problem, which corresponds to the substitution $g(\mathbf{s}) \rightarrow \delta(\mathbf{s})$, gives instead of (10) the relation

$$\left(\frac{d^2E(\mathbf{n},\omega)}{d\omega\, d\Omega}\right)_M = \frac{n_0^2 e^4}{m^2 v_z^2} \frac{1}{(Q - k_z)^2} \left| [\mathbf{n}\mathbf{E}_0(\mathbf{k}_t + \mathbf{Q})] \right|^2 (2\pi)^4. \tag{11}$$

The function $w(\mathbf{r})$ effectively varies over distances on the order of the average interatomic distance b. The effective values of $\mathbf{s}$ in $g(\mathbf{s})$ are therefore on the order of b^{-1}. It follows that the maximum deviation of microscopic result (10) from macroscopic result (11) occurs when $Qb \simeq 1$.

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