# Confinement in macroscopic electrodynamics

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Despite the complete screening of the charges, a distance-independent attractive force acts between two unlike charges in an isotropic ferroelectric. The confinement mechanism is the surface tension at the boundary of a 180° domain which binds the charges.

The emergence of compact field configurations such as "strings" and "bags" and of attractive forces which do not fall off over distance (confinement) in the physics of strong interactions is frequently linked with a "superdiamagnetism," $\mu = 0$ (or "superdielectricity," $\epsilon = 0$), of an ordered quantum-chromodynamics vacuum. Here $\mu$ (or $\epsilon$) is the long-wavelength static permeability (or permittivity) of the vacuum.

Can compact configurations and confinement occur in an ordered electrodynamic (Abelian) medium? Could other mechanisms for these phenomena be manifested in such a medium? Let us discuss these questions, which are pertinent to both macroscopic physics and chromodynamics itself.

**1.** Well-known examples of compact configurations in electrodynamics are the vortex filaments in a superconductor (or a superdiamagnet). Only by incorporating magnetic monopoles, however, do we obtain a confinement: for monopoles in an ordinary superconductor (the Nambu mechanism) and for charges in a dual system with a monopole condensate (the Toft mechanism).

An equilibrium superdielectric electrodynamic medium cannot exist ($\epsilon \geqslant 1$; Ref. 1). A disappearance of $\epsilon$ is possible in a *nonequilibrium* ferroelectric in the absence of external charges (an electric displacement $D = 0$ and a field $E \neq 0$), where compact configurations also arise. Although unlike charges are connected by a string, they experience an anticonfinement: They are repelled with a constant force (we will go into this matter in more detail in a separate paper). We might note that a medium of the nature of a ferroelectric with both compact configurations and confinement, which has been analyzed previously by one of the present authors,[2] has properties that simulate asymptotic freedom which are "wrong" from the standpoint of electrodynamics.

**2.** Another mechanism for the appearance of compact configurations and confinement, which corresponds to $\epsilon \to \infty$, acts in an *equilibrium* ferroelectric ($E = 0$ and a spontaneous electric displacement $D = D_0$) bracketed by grounded capacitor plates (Fig. 1). The constitutive equation of an isotropic ferroelectric,[1)]

$$\mathbf{D} = D_0 \mathbf{E}/E + \epsilon \mathbf{E}, \quad D \geqslant D_0, \tag{1}$$

supplements Maxwell's equations ($\rho_e$ is the density of the external charges, with a characteristic value $Q$):

$$\text{rot}\,\mathbf{E} = 0 \quad \text{(a)}, \qquad \text{div}\,\mathbf{D} = 4\pi \rho_e \quad \text{(b)} \tag{2}$$

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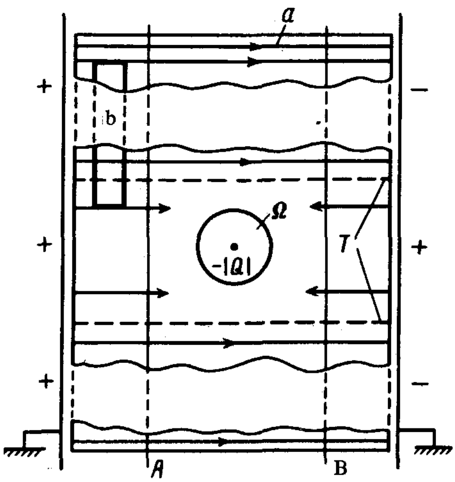


FIG. 1.

Equations (1) and (2) contain a characteristic parameter with the dimensionality of a length: $r_0 = (|Q|/D_0)^{1/2}$.

A charge placed in the ferroelectric creates a compact configuration of the field $E$ which is localized in a finite region $\Omega$ of volume $\sim r_0^3$ ($E = 0$, $D = D_0$ outside $\Omega$). According to Gauss's law, Eq. (2b), and the inequality in (1), the lines of force coming from the charge do not fill the entire space but are instead enclosed in a tube $T$ of radius $\sim r_0$. Outside this tube the integral $\int d\mathbf{s}\mathbf{E}$ along a line of force connecting capacitor plates with a zero potential difference (Fig. 1a) is zero, so that we have $E = 0$. The same is true of the regions inside the tube which are separated from the charge by distances greater than $r_0$, as can be seen by applying Stokes's theorem and Eq. (2a) to the circuit in Fig. 1b formed by two lines of force and two equipotentials. The complete screening of the charge in a nonconducting (!) ferroelectric stems from Eq. (1): The decrease in $D$ to the value $D_0$ with distance from the charge corresponds to a decrease in $E$ to zero.

**3.** It would seem that we have completely ruled out the possibility of confinement: Charges separated by distances greater than $r_0$ do not interact with each other at all. In fact, however, unlike charges are bound by a distance-independent attractive force. This can be seen by applying Gauss's law to equipotential surfaces $A$ and $B$ in Fig. 1, which have identical areas and on which we have $D = D_0$. By virtue of inequality (1), the required flux deficit $4\pi|Q|$ cannot be obtained without reversing the directions of lines of force inside tube $T$. As a result, a 180° domain with a cross-sectional area of $2\pi r_0^2$ arises, connecting the charge with the like plate.

In a real ferroelectric, however, there is a surface tension $\sigma \sim D_0^2$ at a domain wall because of a gradient of the electric displacement. The corresponding force, $f \sim \sigma r_0$, tends to push a charge toward the like plate or to pull two unlike charges toward each other (see the discussion below).

**4.** Figure 2 is a pattern of lines of force for a very simple configuration of charges separated by a distance greater than $r_0$ (in other words, it is a bag, that arises rather than a string. The region $\Omega$ is a sphere of radius of $r_0$; inside this sphere we have

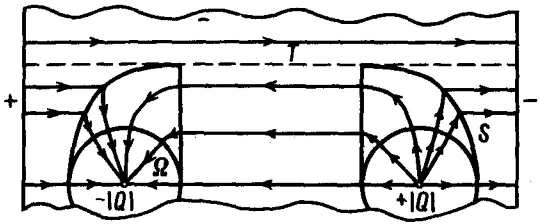


FIG. 2.

$D = |Q|/r^2$ and $E = (D/\epsilon)(1 - r^2/r_0^2)$. At the boundary of $\Omega$ and on the surface $S$ of the force-line discontinuity we have the boundary condition that the values of $D_n$ are equal. The domain radius is $\sqrt{2}r_0$, and the confinement force is $f = 2\pi\sqrt{2}r_0\sigma$.

We do not have room for the corresponding analytic expressions here. They will appear in a separate detailed paper, where we will also discuss the general configuration of charges, their excitation spectrum, the case of an anisotropic ferroelectric, and the very similar problem of magnetic monopoles in a magnetic medium.

[1)]This case is far simpler than that of a real, anisotropic ferroelectric (although they have several qualitative features in common). It is possible that a ferroelectric similar to a dispersive ferromagnet (a "liquid magnet") may be isotropic.


[1]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred, Nauka, Moscow, 1982 (Electrodynamics of Continuous Media, Pergamon, New York); O. V. Dolgov, D. A. Kirzhnits, and V. V. Losyakov, Zh. Eksp. Teor. Fiz. **83**, 1894 (1982) [Sov. Phys. JETP **56**, 1095 (1982)].

[2]D. A. Kirzhnits, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 624 (1979) [JETP Lett. **30**, 587 (1979)].



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