New finite model in $d = 2$

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(Submitted 16 April 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 12, 573–576 (25 June 1984)

An $N = 4$ supersymmetry model for scalars and spinors in dimensionality two has been derived through a dimensional reduction of an $N = 2$ real chiral superfield to $d = 4$. All the ultraviolet divergences cancel out in all orders of perturbation theory.

The recent appearance of finite quantum field theories is a consequence of the development of supersymmetry. The best-known examples are the $N = 4$ supersymmetry generalization of the Yang-Mills theory[1,2] to $d = 4$ and the $N = 4\sigma$ model of a special type[3] to $d = 2$. Theories with an expanded supersymmetry can usually be sim-

 0021-3640/84/120703-04$01.00

plified by reformulating them in a space-time with a higher dimensionality. An $N=4$ super-Yang-Mills in model $d=4$, for example, can be formulated as an $N=1$ super-Yang-Mills model[4] in $d=10$, and an $N=4$ σ model in $d=2$ can be formulated as an $N=1$ σ model[5] in $d=6$. In the present letter we describe a dimensional reduction of an $N=2$ real chiral superfield[6] in $d=4$ to $d=2$. As a result, we find an $N=4$ theory in $d=2$ for interacting fields of spins 0 and 1/2.

Supersymmetry σ models in a two-dimensional space-time are well known. The general theory is described by

$$L = \frac{1}{2} g_{ab}(A)\,\partial_\mu A^a\,\partial_\mu A^b + \text{supersymmetrization}, \tag{1}$$

where g_{ab} is the metric of the Riemann manifold W, whose coordinates are $\{A^d\}$. The expanded supersymmetry presupposes couplings on the metric. In particular, an $N=2$ supersymmetry requires the Kähler[7] metric. Our model may be thought of as a special case of a theory with

$$L = \frac{1}{2} g_{ab}(A)\,\partial_\mu A^a\,\partial_\mu A^b + \frac{1}{2} g_{ab}(A)\,\partial_\mu B^a\,\partial_\mu B^b$$

$$+ \frac{1}{2} f_{ab}(A)\,\epsilon_{\mu\nu}\,\partial_\mu B^a\,\partial_\nu B^b + \text{supersymmetrization}, \tag{2}$$

where the antisymmetric coefficients f_{ab} along with g_{ab} are fixed by a single function. In particular, the manifold W in our case is a Kähler manifold but not Ricci-planar manifold. The model is nevertheless finite.

The $N=2$ real chiral superfield satisfies the following equations in superspace[8,9]:

$$\bar{D}_i^{\dot{\alpha}}\,\Phi = 0, \quad \frac{1}{12} D_i^\alpha D_{\alpha j} D^{\beta i} D_\beta^j\,\Phi = \Box\bar{\Phi}. \tag{3}$$

Solving (3), we find the following supermultiplet in $d=4$:

$$(\Phi,\ \psi^i,\ C_m,\ \tilde{F}_{\mu\nu}), \tag{4}$$

where Φ is a complex scalar, ψ^i is an SU(2) Majorana isodoublet, C_m is a real isovector, and the real antisymmetric tensor $\tilde{F}_{\mu\nu}$ satisfies the couplings

$$\epsilon_{\mu\nu\lambda\rho}\,\partial_\nu \tilde{F}_{\lambda\rho} = 0. \tag{5}$$

After a dimensional reduction to $d=2$, the couplings (5) can immediately be separated:

$$F_{01} = n = \text{const}, \quad F_{23} = D,$$

$$F_{\mu 2} = \frac{1}{2}\epsilon_{\mu\nu}\,\partial_\nu (F+\bar{F}), \qquad F_{\mu 3} = \frac{1}{2i}\epsilon_{\mu\nu}\,\partial_\nu (F-\bar{F}), \tag{6}$$

where the tensor $F_{\mu\nu}$ is the dual of $\tilde{F}_{\mu\nu}$. We thus obtain an $N=4$ supermultiplet in $d=2$:

$$(\Phi,\ \psi^i,\ C_m,\ F,\ D), \tag{7}$$

where Φ and F are complex scalars, ψ^i is a Dirac isodoublet, C_m is a real isovector, and D is a real scalar $(8+8$ components).

The corresponding transformation laws of the supersymmetry are

$$\delta\Phi = \bar{\epsilon}_{iR}\,\psi^i_R + \bar{\psi}^i_R\,\epsilon_{iR}\,,$$

$$\delta\psi^i = -\tau^i_{mj} C_m \gamma_5 \epsilon^j - i\partial\!\!\!/\Phi\epsilon^i_L + i\partial\!\!\!/\bar{\Phi}\epsilon^i_R - iD\epsilon^i + - 2n\gamma_5\epsilon^i + i\partial\!\!\!/\tilde{\epsilon}^i\bar{F},$$

$$\delta C_m = -\frac{1}{2}\bar{\epsilon}_i i\partial\!\!\!/(\tau_m)^i_j\,\psi^j + h.\,c., \tag{8}$$

$$\delta F = -\,\bar{\epsilon}_i\gamma_5\tilde{\psi}^i,$$

$$\delta D = -\frac{1}{2}\,\bar{\epsilon}_i\gamma_5\,\partial\!\!\!/\psi^i + h.\,c.,$$

where $\tilde{\psi}^i$ is the Majorana-adjoint spinor.

As the action for the interacting theory we consider the following expression in superspace:

$$S = \int d^2x\, d^2\theta_R\, d^2\tilde{\theta}_L\, V(\Phi) + h.\,c. \tag{9}$$

with the analytic function $V(\Phi) = -\frac{1}{4}\Phi^2 + V_{\rm int}(\Phi)$, where Φ is the superfield (3) reduced to $d=2$ and with a generally exterior index. After some rather lengthy calculations, we find from (9) the following Lagrangian in component form:

$$\begin{aligned} L = {} & g_{ab}\,\partial_\mu\Phi^a\partial_\mu\bar{\Phi}^b - g_{ab}\,\bar{\psi}^{ia}_R\, i\partial\!\!\!/\psi^b_{iL} - g_{ab}\,\bar{\psi}^{ia}_L\, i\overleftarrow{\partial\!\!\!/}\psi^b_{iR} + g_{ab}\,C^a_m C^b_m \\ & - g_{ab}\,n^a n^b + g_{ab}\,D^a D^b + g_{ab}\,\partial_\mu F^a\partial_\mu\bar{F}^b + 2ig_{ab}\,n^a D^b \\ & + g_{ab}\,\epsilon_{\mu\nu}\,\partial_\mu\bar{F}^a\partial_\nu F^b - g_{abc}\,(\bar{\psi}^a_R\,\tau_m\,\psi^b_R)\,C^c_m + 2g_{abc}\,\bar{\psi}^a_{iR}\,\psi^{bi}_R\; n^c \\ & + ig_{abc}\,\bar{\psi}^a_{iR}\,\psi^{bi}_R D^c - \frac{i}{2}g_{abc}\,[\,\bar{\psi}^a_{iR}\,\gamma_\mu\,\tilde{\psi}^{bi}_L\;\partial_\mu\bar{F}^c + \bar{\tilde{\psi}}^a_{iL}\,\gamma_\mu\,\psi^{bi}\,\partial_\mu F^c\,] \\ & - \frac{1}{6}g_{abcd}\,(\bar{\psi}^{ia}_R\;\psi^{jb}_R)(\bar{\psi}^c_{iR}\;\psi^d_{jR} + \bar{\psi}^c_{jR}\,\psi^d_{iR}) + h.\,c., \end{aligned} \tag{10}$$

where

$$g_{a_1\cdots a_n} = -\left.\frac{\partial^n V(\Phi)}{\partial\Phi^{a_1}\ldots\partial\Phi^{a_n}}\right|_{\Phi(x,\,\theta)\,=\,\Phi(x)} \tag{11}$$

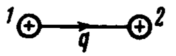
The fields C_m and D are auxiliary fields and can be eliminated through the use of the equations of motion. We see that for the case $n^a = 0$ Lagrangian (10) actually has the form of a theory (2) with a Kähler metric. We may assume that the theory (10) is fixed by two forms and their derivatives, which in our case are determined by the single function $V(\Phi)$:

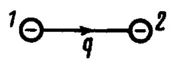
$$G = (g_{ab} + \bar{g}_{ab})\, d\Phi^a d\bar{\Phi}^b \, , \qquad E = (\bar{g}_{ab} - g_{ab})\, dF^a \wedge d\bar{F}^b \, . \tag{12}$$

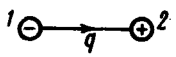
In a free theory, there would be no E.

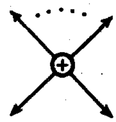
The constant vector n^a generates a spontaneous breaking of the supersymmetry. This breaking corresponds to a Goldstone behavior in the transformation law for ψ^i for $n \neq 0$ in (8).

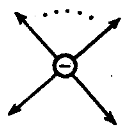
To analyze possible divergences of the Green's functions of this model, we use a Feynman technique used directly in the superspace.[10,11] As in the case of a simple supersymmetry, many of the chiral exponential factors cancel out,[10] so that we find the following Feynman rules:

 $= i\delta^4(\theta_1 - \theta_2)$

 $= i\delta^4(\bar{\theta}_1 - \bar{\theta}_2)$

 $= \dfrac{-i}{q^2} \exp(-\theta_2 \not{q}\, \bar{\theta}_1)$

 $= i\int d^4\theta\, V_{int}(\Phi)$

 $= i\int d^4\bar{\theta}\, V_{int}(\bar{\Phi})$.

Using these rules, we can estimate the divergence index of an arbitrary diagram:

$$\omega \leqslant 2 - 2(V_- + V_+) + 4[\min(V_+, V_-) - 1] \leqslant -2, \tag{13}$$

where $V_\pm$ is the number of vertices of the corresponding chirality. The model is thus finite in the ultraviolet region. We have tested this result by explicitly analyzing all possible supergraphs with an accuracy to within three loops.

We wish to thank O. V. Ogievetskiĭ and M. A. Vasil'ev for useful discussions.

The detailed calculations will be published separately.

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Translated by Dave Parsons
Edited by S. J. Amoretty