Spin susceptibility of disordered Fermi systems

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The effect of interaction in a disordered Fermi system on the spin susceptibility is examined. The susceptibility is found to have a root and logarithmic dependence on temperature and on the magnetic field in three-dimensional and two-dimensional systems, respectively.

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The interaction of electrons with each other in disordered systems leads to an anomaly in the density of states at the Fermi level and to singularities in the kinetic coefficients[1,2] in three-dimensional and two-dimensional systems.[3] Al'tshuler and Aronov[2] showed that thermodynamic quantities such as specific heat and magnetic susceptibility must also have anomalous temperature dependences caused by the interference of the interaction between electrons and elastic scattering. The spin susceptibil-

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ity in two-dimensional systems was studied by Fukuyama[4] who obtained a logarithmic temperature dependence. Al'tschuler *et al.*[5] studied the nuclear spin relaxation rate in two-dimensional systems, a rate which is proportional to a single-point correlation function of electron spins, and obtained logarithmic corrections. In this letter we study the field and temperature dependences of the susceptibility in three-dimensional and two-dimensional Fermi systems. We assume that the condition $p_F l \gg 1$ is satisfied. Here $l = v_F \tau$ is the mean free path ($h \equiv 1$).

The singular contributions to the temperature and field dependences of the susceptibility arise from the diagrams a and b (see Fib. 1), which take into account the interaction between the electrons in different spin subbands. They are obtained from the Hartree correction for the density of states. Since the interaction of electrons in one spin subband, however, changes the density of states in the same way in each subband the total spin polarization remains the same. The wavy lines in diagrams a and b of Fig. 1 compare the screened-out Coulomb interaction,[1,3] and the field vertex corresponds to the quantity $g\,\mu_B \sigma_z$, where σ_z is the Pauli matrix, μ_B is the Bohr magneton, and g-g is the conduction-electron factor.

Each diagram describes both the diffusion contribution (with a small difference momentum) and the Cooper contribution (with a small total momentum), depending on the relative direction of the electronic Green's functions in the different loops.

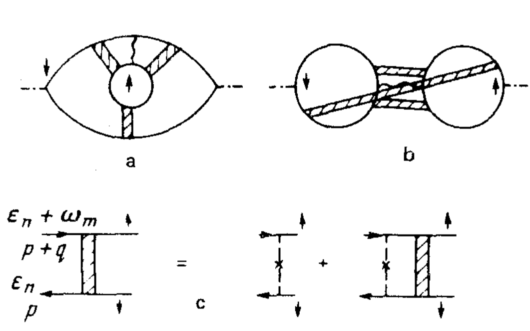
In the model for Coulomb Fermi gas with a uniform compensating charge, the Hartee diagrams contract because of the electroneutrality condition. In a disordered Fermi gas the electroneutrality breaks down in the regions smaller in magnitude or of the order of l and is restored in the regions larger than l. Hence, the main contribution to the Hartee diagrams comes from the momentum region $kl > 1$.

Figure 1c shows an equation for a diffusion in the triplet state. In the region $\omega_m \tau \ll 1$, $Dq^2\tau \ll 1$. $g\mu_B H\tau \ll 1$ this equation can be written as follows:

$$(|\omega_m| - D\nabla_r^2 + i\,\omega_s \operatorname{sign}\omega_m)\,D(\mathbf{r},\mathbf{r}';\omega_m\,\epsilon_n) = \frac{1}{2\pi\nu_0\tau^2}\,\theta[-\epsilon_n(\epsilon_n+\omega_m)]\,\delta(\mathbf{r}-\mathbf{r}'),$$

where μ_0 is the single-spin density of states, $\omega_S = g\mu_B$, and D is the diffusion coefficient.

The equation for the cooperon C ($\mathbf{rr}'$, $\omega_m \epsilon_n$) is a similar equation; however, $-i\nabla_\mathbf{r}$ must be replaced by $-i\nabla_\mathbf{r} - \frac{re}{c}\mathbf{A}$ in the magnetic field.[6] The corresponding



characteristic frequency in the magnetic field is $\omega_H = \frac{4D|e|H}{c} \gg \omega_s$. Detailed calculations will be published separately. In this letter, however, we present the final results.

We shall examine the Cooper contribution $\delta\chi_c$ and the diffusion contribution $\delta\chi_d$ in a three-dimensional system. In the region of weak magnetic fields ω_H/T 1 but $T\tau \ll 1$ the Cooper and diffusion contributions are equal to each other

$$\delta\chi_c = \delta\chi_d = \frac{(g\mu_B)^2 T^{1/2}}{2\sqrt{2}\pi^2 D^{3/2}} F\left(\frac{\kappa}{2p_F}\right) \int_0^\infty \frac{dx}{x^{1/2}} \frac{d}{dx} \frac{x}{e^x - 1},$$

where $F(y) = y^2 \ln\left(1 + \frac{1}{y^2}\right)$, and κ is the reciprocal screening length.

In the region of the fields $\omega_H/T \ll 1$ the Cooper contribution approaches the asymptotic form

$$\delta\chi_c = -\frac{3}{8\pi^2} \frac{(g\mu_B)^2}{D^{3/2}} \omega_H^{1/2} \int_0^\infty dx x^{1/2} \frac{d}{dx} x^{3/2} \zeta\left(5/2, \frac{1}{2} + x\right) \tag{1}$$

where ζ (u, v) is a generalized zeta function. The diffusion contribution to the susceptibility approaches the asymptotic form $H^{1/2}$ much later in the fields $\omega_S/T \ll 1$, where it has the form

$$\delta\chi_d = -\frac{(g\mu_B)^2}{2\sqrt{2}\pi^2} \frac{\omega_S^{1/2}}{D^{3/2}} F\left(\frac{\kappa}{2p_F}\right). \tag{2}$$

Hence, at $\omega_S/T \gg$ the magnetic susceptibility, which is proportional to $H^{1/2}$, is equal to the sum of the contributions determined by Eqs. (1) and (2).

The diffusion and Cooper contributions can also be isolated in a two-dimensional system in a perpendicular magnetic field

$$\delta\chi_d = -\frac{(g\mu_B)^2}{2\pi^2 D} F \begin{cases} \ln \tau\omega_s & \omega_s/T >> 4, \quad \tau\omega_s << 1 \\ \ln \tau T & \omega_s/T << 1, \quad \tau\omega_s << 1 \end{cases}$$

$$\delta\chi_c = -\frac{(g\mu_B)^2}{2\pi^2 D} F \begin{cases} \ln \tau\omega_H & \omega_H/T >> 1, \quad \tau\omega_H << 1 \\ \ln \tau T & \omega_H/T << 1, \quad \tau T << 1 \end{cases} \tag{3}$$

where

$$F = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1}{1 + \frac{2p_F}{\kappa} \sin\frac{\phi}{2}}.$$

Thus the magnetic susceptibility decreases with increasing temperature in the two-dimensional and three-dimensional systems at $\tau \ll 1$. Since the condition $\omega_H/T > 1$ is satisfied much sooner than $\omega_S/T > 1$, the logarithmic dependence on the magnetic field in weak fields is determined solely by the Cooper contribution. In strong fields $\omega_S/T > 1$ the logarithmic field dependence, as in the three-dimensional case, is determined by the sum of the Cooper and diffusion contributions. Expression (3) differs in sign from the result of Ref. 4 when $\omega_H/T \ll 1$.

Finally, we shall analyze the spin susceptibility in a nonuniform magnetic field. The characteristic size of a diffusion is estimated to be $r_d = \sqrt{D/T} = e/\sqrt{\tau T}$, which may reach a large value at low temperatures $\tau T \ll 1$. It is clear that when the diffusion is larger than the characteristic scale along which the magnetic field changes, the H and T dependence of the susceptibility vanishes.

In the three-dimensional case in the limit $Dk^2/T \gg 1$

$$\delta\chi(k) = -\frac{(g\mu_B)^2}{8\pi} F\left(\frac{\kappa}{2p_F}\right)\frac{k}{D} .$$

Note that the susceptibility depends on the wave-vector modulus, i.e., the response is substantially nonlocal in nature.

In the two-dimensional case for $Dk^2/T \gg 1$

$$\delta\chi = -\frac{(g\mu_B)^2}{\pi^2 D} F \ln Dk^2\tau .$$

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