Experimental search for the long-range Yang-Mills field

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The shift of the interference bands caused by an isospin gauge field has been calculated in a Bohm-Aaronov experiment with neutrons.

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The techniques of neutron interferometry have improved considerably in recent years. Its capabilities have recently been demonstrated in a Bohm-Aaronov experiment, in which slow neutrons cross a region of space with a nonvanishing static electromagnetic vector potential **A**, but with field strengths equal to zero ($\mathbf{H} = \mathbf{E} = 0$). Under these conditions the electric charge of a neutron e_n was limited $e_n < 10^{-12}\ e$, where e is the electronic charge (see Ref. 1)

In view of this, it is important to analyze quantitatively the results of an interference experiment on the search for a classical Yang-Mills field corresponding to the SU(2) isospin gauge group. Below we present the results of a calculation of the interference effect produced as a result of the motion of a neutron in a Yang-Mills field for two types of experiments—one involving a rotating solid (such an experiment, considered as a thought experiment, was mentioned in a paper of Wu and Yang[2]) and one involving a rotating superfluid He^3-A.

Two points, however, should be mentioned at the outset. First, we emphasize that the unquantized isospin gauge field produced by an *external source* (like the classical electromagnetic field) is a long-range field. The question of whether some particles (massive or massless) correspond to a quantized isospin field remains open for the present; hence, we shall not discuss it in our letter. Second, we note that the most common current working hypothesis of strong interactions (quantum chromodynamics) holds that local symmetry applies to "color," whereas "flavor" (specifically, isospin) is not related to any gauge field. In this letter, however, we analyze the effects whose origin is attributable to a non-Abelian gauge field associated with a "flavor" group. Since this possibility has not been ruled out experimentally, it would be useful to carry out such experiments.

We shall start with the equation for an isospin magnetic field $\mathbf{H}^k$ produced by an external current with a density $\mathbf{j}^k$

$$\mathrm{rot}\mathbf{H}^k + \epsilon e_{klm}\left[\mathbf{A}^l\mathbf{H}^m\right] = (4\pi/c)\mathbf{j}^k. \tag{1}$$

Here $\mathbf{A}^l$ is the vector potential of the field, ϵ is the gauge charge appearing in the covariant derivatives, and e_{klm} is the antisymmetric unit tensor; all the isospin indices k,l,m are assigned the values 1, 2, and 3. The current density $\mathbf{j}^k$ is given by

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$$\mathbf{j}^k = \epsilon\rho^k \mathbf{v}, \rho^k = \langle \hat{T}_k \rangle \rho, \tag{2}$$

in which ρ is the density of the isospin carriers (nuclei), $\hat{T}_k$ is the isospin operator, and $\mathbf{v}$ is the carrier velocity. For real nuclei the $\hat{T}_3$ operator is diagonal, whereas the $\hat{T}_1$ and $\hat{T}_2$ operators are not diagonal and their average values are equal to zero. We have

$$\langle \hat{T}_3 \rangle = \frac{Z-N}{2}, \langle \hat{T}_1 \rangle = \langle \hat{T}_2 \rangle = 0. \tag{3}$$

Hence,

$$\mathbf{j}_3 = \epsilon\rho \frac{Z-N}{2} \mathbf{v}, \mathbf{j}^1 = \mathbf{j}^2 = 0 \tag{4}$$

(Z is the number of protons in a nucleus and N is the number of neutrons in a nucleus). Since Eq. (4) is equivalent to Eq. (1), a gauge of the vector potential for which $\mathbf{A}^1 = \mathbf{A}^2 = 0$ is reconcilable; as a result, $\mathbf{H}^1 = \mathbf{H}^2 = 0$, and we obtain an ordinary Maxwell's equation for $\mathbf{H}^3$. The field $\mathbf{H}^3$ produced by an infinitely long, rotating cylindrical block of matter and its flux F across the surface that intersects the cylinder can therefore be easily calculated. Since $\mathbf{H}^3 = 0$ outside the cylinder and since it is parallel to the rotation axis inside it, we find

$$F = \frac{\epsilon}{c} \pi^3 R^4 (Z-N)\rho n, \tag{5}$$

where R is the cylinder radius, and n is the number of revolutions per second.

Let us assume that the rotating cylinder intercepts two coherent neutron beams which collide and interfere with each other at some point beyond the cylinder. If the geometric length of the path of each beam is the same, the presence of an isospin gauge field in the vector-potential space $\mathbf{A}^3$ produces a phase difference,

$$\Delta\phi = -\frac{\epsilon_n}{\hbar c} \oint \mathbf{A}^3 dx = -\epsilon_n F/\hbar c, \tag{6}$$

where ϵ_n is the gauge neutron charge

$$\epsilon_n = -\epsilon/2. \tag{7}$$

Combining Eqs. (5)–(7) we find for the phase difference

$$\Delta\phi = \frac{\epsilon^2}{\hbar c} \frac{\pi^3}{2c} R^4 (Z-N)\rho n. \tag{8}$$

Under the conditions mentioned above the beam intensity at the collision point is equal to $\cos^2(\Delta\phi/2)$ (if the amplitude of each beam is the same and equal to 1/2). It follows from the discussion above that since the effect associated with the change in intensity is equal to $(1/4)(\Delta\phi)^2 \propto R^8$, the "wide-arm" interferometers would be more effective in the search for this effect. For a cylinder with $R = 10$ cm made from a metallic ^{238}U rotating at the velocity $n = 100$ rps, a 1% decrease in the intensity of the main peak $\Delta\phi \approx 0.2$) corresponds to $\epsilon \approx 1.3 \times 10^{-5}\, e$.

We shall now consider a rotating superfluid He^3 in the A phase. It is known (see Refs. 3 and 4) that monopole-type vortex defects ("vortices of finite extent") may appear in this quantum liquid. Volovik[3] showed that if a Cooper pair (comprised of two He^3 atoms) is charged, then a magnetic field produced by such a vortex at a sufficiently large (compared with the coherence length) distance exactly coincides with the field of a magnetic monopole with a Dirac-quantized magnetic charge.[1)] Turning from an electromagnetic field to a Yang-Mills field, we note that the gauge charge of Cooper pair is equal to ϵ and the effective "magnetic" charge of a monopole is

$$\gamma = \hbar c/2\epsilon. \tag{9}$$

Describing the monopole by a vector-potential

$$\mathbf{A}_\vartheta^3 = \mathbf{A}_r^3 = \mathbf{A}^1 = \mathbf{A}^2 = 0, \mathbf{A}_\phi^3 = \frac{\gamma}{r}\tau g\frac{\vartheta}{2}, \tag{10}$$

we determine from Eq. (6) for neutrons encompassing the vortex filament in the equatorial plane ($\theta = \pi/2$) the phase difference

$$\Delta\phi = \pi/4, \tag{11}$$

i.e., the interference effect of the order of unity for arbitrarily small ϵ. Of course, an experiment with He^3-A is much more involved than that with a rotating solid.

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[1)]It is curious that quantization in this case is governed by the point defect of the field of the director, i.e., by the homotopic group $\pi_2(S^2)$, rather than by $\pi_1(G)$, where G is the gauge group. For the electromagnetic field $G = U(1) = S^1$, and since $\pi_1(S^1) \cong \pi_2(S^2)$, the magnetic-charge quantizations turn out to be identical. The charge quantizations, which are governed by $\pi_2(S^2)$ and $\pi_1(G)$, generally differ from one another for the Yang-Mills field.

[1]D. M. Greenberger, D. K. Atwood, J. Arthur, G. Shull, and M. Schlenker, Phys. Rev. Lett. **47**, 751 (1981).
[2]T. T. Wu and C. N. Yang, Phys. Ref. **D12**, 3843 (1975).
[3]G. E. Volovik and V. P. Mineev, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 647 (1976) [JETP Lett. **23**, 593 (1976].
[4]S. Blaha, Phys. Rev. Lett. **36**, 874 (1976).

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