Squeezed quantum state of a disoriented chiral condensate

I. I. Kogan[1)]
Physics Department, Princeton University, Jadwin Hall, Princeton, NJ 08544 USA

(Submitted 28 January 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **59**, No. 5, 289–296 (10 March 1994)

The quantum state describing the disoriented chiral condensate (DCC) which may be produced in high-energy collisions is considered. Using the approach suggested by Rajagopal and Wilczek to describe the amplification of the long-wavelength classical pion modes, we consider the quantum-mechanical evolution of the initial vacuum state into the final squeezed state that describes the DCC. The wave function has several interesting properties which are discussed.

The possibility of producing disoriented chiral condensate (DCC) in high-energy hadronic or heavy-ion collisions has attracted considerable attention in the last few years.[1–14]

It is well known that a QCD Lagrangian is invariant (approximately if nonzero masses for the light N_f quarks are taken into account) under global chiral $SU(N_f)_L \times SU(N_f)_R$, where N_f is the number of the light flavors. This symmetry is spontaneously broken down to the vector $SU(N_f)_V$ which leads to N_f^2-1 (quasi-) Goldstone bosons—pions (if $N_f=2$) or pions, kaons, and η meson (if $N_f=3$). The order parameter for this breaking is the vacuum expectation value of the quark condensate $\langle\bar{\psi}\psi\rangle$. One can imagine, however, that under certain special conditions, for example, after a high-energy collision, there is a "cool" region surrounded by a relatively thin, "hot" expanding shell, which separates the internal region from the outer space. This picture was suggested by Bjorken, Kowalski, and Taylor.[6] It is called now the "Baked Alaska" scenario. As a result, the quark condensate may be disoriented in isotopic space—one obtains the DCC. After hadronization the interior disoriented vacuum collapses as it decays into pions. The interesting signature of this process is the coherent production of either charged or neutral pions. There are some arguments that the DCC was observed in the Centauro cosmic-ray events.[15]

It is convenient to consider the toy model describing the chiral dynamics—the linear sigma-model with a four-component field $\phi^a=(\sigma,\vec{\pi})$, where σ and $\vec{\pi}$ are an isoscalar field and an isovector field (here we use the same notation as in Ref. 5).

$$S=\int d^4x\left[\frac{1}{2}\partial_\mu\phi^a\partial_\mu\phi^a-\frac{\lambda}{4}(\phi^a\phi^a-v^2)^2+H\sigma\right], \tag{1}$$

where $H\sim m_q$ describes the small, explicit, chiral symmetry breaking. The pion mass is $m_\pi^2=H/f_\pi=\lambda(f_\pi^2-v^2)$, where $f_\pi=\langle\sigma\rangle$. The sigma meson mass is $m_\sigma^2=3\lambda f_\pi^2-\lambda v^2\approx 2\lambda f_\pi^2$. With $m_\pi=135$ MeV, $m_\sigma=600$ MeV, and $f_\pi=92.5$ MeV we have[5] $\lambda=20$ and $v=87.4$ MeV. In the usual vacuum one has $\langle\sigma\rangle=f_\pi$, $\langle\pi\rangle=0$ and since σ is an isoscalar, there is an isoscalar condensate $\langle\bar{\psi}\psi\rangle$ only. However, one can consider another configuration—$\langle\sigma\rangle = f_\pi \cos\theta$ and $\langle\pi\rangle = f_\pi \mathbf{n} \sin\theta$; here $\mathbf{n}$ is a unit vector in the isospace, which describes DCC, i.e., some classical pion field configuration which is metastable and which decays after some time into pions—the signature for this event is the large number of either neutral pions (π^0) or charged pions ($\pi^\pm$). Using the classical picture of DCC which predicts equal probability for all isotopic orientations of the condensate, we obtain[1–4] the probability $1/\sqrt{f}$ for the fraction of neutral pions $f = N_{\pi^0}/(N_{pi^+} + N_{\pi^-} + N_{\pi^0})$.

The question we would like to discuss here is connected with the quantum description of the DCC. A simple way to describe it is in terms of the usual coherent state. The usual coherent state wave function, however, was criticized in Ref. 4, because such a description leads to creation of a pion system with arbitrarily large charge fluctuations. Instead, it was suggested in Ref. 4 that the quantum state must be an isosinglet state. For the state with $2n$ neutral pions and total number of pions $2N$ it was suggested that

$$|\Psi\rangle \sim [2a_+^\dagger a_-^\dagger - (a_0^\dagger)^2]^N |0\rangle. \tag{2}$$

The probability of having 2n neutral pions will then be[4,16]

$$P(n,N) = \frac{(N!)^2 2^{2N}}{(2N+1)!} \frac{(2n)!}{(n!2^n)^2} \sim \sqrt{N/n}, \quad n, N \gg 1, \tag{3}$$

which corresponds to $1/\sqrt{f}$ distribution in the classical picture.

We note, however, that the same distribution will be obtained in the case of an arbitrary relative factor between charged and neutral creation operators $[2a_+^\dagger a_-^\dagger - \exp(i\theta)(a_0^\dagger)^2]^N$. We see that from this point of view the zero isospin conditiion is not so important—what is important is that the wave function is constructed by the operators quadratic in $a^\dagger$ and according to the construction, there is an equal number of π^+ and π^-.

It would be interesting to determine what is the most natural class of these functions and what are the dynamic mechanisms leading to the generation of these functions. As we shall demonstrate here, the quantum state of the DCC is the squeezed state. These quantum states were known for a long time in quantum optics and measurement theory (for a review of squeezed states see, for example, Refs. 17–20). The simplest one-mode squeezed state is parametrized by the two parameters r and ϕ and can be obtained by acting on the vacuum by the squeezed operators $S(r,\phi)$:

$$S(r,\phi)|0\rangle = \exp\left[\frac{r}{2}(e^{-2i\phi}a^2 - e^{2i\phi}a^{\dagger 2})\right]|0\rangle. \tag{4}$$

These states are minimum uncertainty states with $\Delta X_1 \Delta X_2 = 1/4$, where $a(a^\dagger) = X_1 \pm X_2$, as well as coherent states $\exp(\alpha a^\dagger)|0\rangle$. However, coherent states have a minimal quantum noise, and $\Delta X_1 = \Delta X_2 = 1/2$. For the squeezed states one can reduce the quantum noise for one variable (increasing it for the conjugate one) and $\Delta Y_1 = e^{-r}/2$, $\Delta Y_2 = e^r/2$, where $Y_1 + iY_2 = (X_1 + iX_2)e^{-i\phi}$. The mean particle number is $\langle N\rangle = \sinh^2 r$.

To obtain the squeezed quantum state for the DCC, let us consider the mechanism for the amplification of the long-wavelength pion modes, suggested by Rajagopal and Wilczek in Ref. 5, where the dynamics of the $O(4)$ linear sigma model after quenching was considered. The amplification of the long-wavelength pion modes was found in the period immediately after quenching. This amplification leads to the coherent pion oscillations, i.e., to the creation of the DCC. Such a behavior can be understood if one considers the equation of motion for the pion field[5]

$$\frac{\partial^2}{\partial t^2}\vec{\pi}(\mathbf{k},t)+[\mathbf{k}^2+\lambda(\langle\phi^2\rangle(t)-v^2)]\vec{\pi}(\mathbf{k},t)=0, \tag{5}$$

where we replaced (as in Ref. 5) the $\phi^a\phi^a$ in the nonlinear term in (5) by its spatial average $\langle\phi^2\rangle(t)$—this is the Hartree–Fock or the mean-field approximation. In the initial conditions one has $\langle\phi^2\rangle < v^2$ and the long-wavelength modes of the pion field with $\mathbf{k}^2 < \lambda(v^2-\langle\phi^2\rangle)$ begin to grow exponentially. The $\langle\phi^2\rangle(t)$ starts to oscillate near the vacuum expectation value $\langle\sigma\rangle$ and after some time the oscillations will be damped enough so that all the modes will be stable. We thus see that at the classical level each long-wavelength mode is described by the equation for a parametrically excited oscillator and one obtains the DCC as a result of amplification of the zero-point quantum fluctuations of the pion field.

This picture is similar to one which was discussed in Ref. 21, where the relict graviton production from zero-point quantum fluctuations during the cosmological expansion was considered. For graviton mode with a momentum n the equation $y''+[n^2-(R''/R)]y=0$ was obtained. Here $R(\eta)$ is the scale factor of the metric $ds^2=R^2(\eta)(d\eta^2-dx^2)$ and a prime represents $d/d\eta$. One can see that this equation is equivalent to the pion equation (5) if the scale factor R is connected with $\langle\phi^2\rangle(t)$ as $\lambda[v^2-\langle\phi^2\rangle(t)]=R''/R$.

Our problem now is to present the quantum mechanical formulation in terms of pion creation and annihilation operators and to determine the wave function of the DCC. In the mean-field approximation the wave function $|\Psi\rangle=\Pi_{i,\mathbf{k}}|\psi\rangle_{i,\mathbf{k}}$ is the product of the wave functions for each mode with a momentum $\mathbf{k}$ and isotopic index i. Below we shall omit i. The equation of motion (5) for each mode $\pi(\mathbf{k},t)$ can be derived from the Lagrangian

$$\begin{aligned}L_k&=\frac{1}{2}\dot{\pi}^2(\mathbf{k},t)-\frac{1}{2}\Omega^2(\mathbf{k},t)\pi^2(\mathbf{k},t),\\ \Omega^2(\mathbf{k},t)&=\mathbf{k}^2+\lambda[\langle\phi^2\rangle(t)-v^2].\end{aligned} \tag{6}$$

The wave function $|\psi\rangle_{\mathbf{k}}$ obeys the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi\rangle_{\mathbf{k}}=H_k(t)|\psi\rangle_{\mathbf{k}}=\left[\frac{1}{2}\mathscr{P}_\pi^2+\frac{1}{2}\Omega^2(\mathbf{k},t)\pi^2(\mathbf{k})\right]|\psi\rangle_{\mathbf{k}}, \tag{7}$$

where $\pi(\mathbf{k})$ and $\mathscr{P}_\pi=-id/d\pi(\mathbf{k})$ are the quantum-mechanical coordinate and the momentum for the mode with a spatial momentum $\mathbf{k}$. We can rewrite the Hamiltonian in (7) in terms of the creation and annihilation operators which make it diagonal at any given moment. It is evident that we are interested in obtaining the wave function in terms of the creation and annihilation operators of ordinary pions. We must therefore diagonalize the Hamiltonian in the limit $t\rightarrow\infty$, where the oscillation of the $\langle\phi^2\rangle(t)$ is damped. We thus define

$$a(\mathbf{k})=\frac{\mathscr{P}_\pi+i\omega(\mathbf{k})\pi(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}},\quad a^\dagger(\mathbf{k})=\frac{\mathscr{P}_\pi-i\omega(\mathbf{k})\pi(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}},\tag{8}$$

where $\omega(\mathbf{k})=\Omega(\mathbf{k},\infty)=\sqrt{\mathbf{k}^2+m_\pi^2}$. It is easy to see that the Hamiltonian is

$$H_k=\frac{1}{2}\,\omega(\mathbf{k})\left[1+\frac{\Omega^2(\mathbf{k},t)}{\omega^2(\mathbf{k})}\right]a^\dagger(\mathbf{k})a(\mathbf{k})+\frac{\omega^2(\mathbf{k})-\Omega^2(\mathbf{k},t)}{4\omega(\mathbf{k})}\,[a^2(\mathbf{k})+a^{\dagger 2}(\mathbf{k})].\tag{9}$$

The $a^2(\mathbf{k})$ and $a^{\dagger 2}(\mathbf{k})$ terms in the Hamiltonian are the reason that H transforms the initial vacuum state $|0\rangle$ into a squeezed state $S(r,\phi)|0\rangle$ (4).

To calculate the squeezing and the phase parameters r and ϕ, we will analyze the solution of the Schrödinger equation (7) in the coordinate (π in our case) representation (we shall omit the label $\mathbf{k}$ for moment)

$$\langle\pi|\psi\rangle(t)=C(t)\exp[-B(t)\pi^2].\tag{10}$$

For $B=\omega/2$ this wave function describes the vacuum state. For all other values, this wave function describes the squeezed state (4), where the parameters r and ϕ are related to B by the relation[17] (see also Refs. 19 and 21):

$$B=\frac{\omega}{2}\frac{\cosh r+\exp(2i\phi)\sinh r}{\cosh r-\exp(2i\phi)\sinh r},$$
$$\cosh 2r=\frac{\omega^2+4|B|^2}{4\omega\,\mathrm{Re}\,B};\quad \sin 2\phi=\frac{1}{\sinh 2r}\frac{\mathrm{Im}\,B}{\mathrm{Re}\,B}.\tag{11}$$

Substituting (10) into (7), we obtain the equations for $B(t)$ and $C(t)$

$$i\dot{B}(t)=2B^2(t)-\frac{1}{2}\,\Omega^2(t),\quad i\frac{\dot{C}(t)}{C(t)}=B(t),\tag{12}$$

which means that $B(t)$ is related to the solution of the classical equation (5)

$$B(t)=-\frac{i}{2}\frac{\dot{\psi}(t)}{\psi(t)},\quad \ddot{\psi}(t)+\Omega^2(t)\psi(t)=0\tag{13}$$

and $C(t)=C(0)\exp[i\int_0^t B(\tau)d\tau]$ is a phase factor. The last equation can be viewed as a Schrödinger equation describing the wave function $\psi(t)$ of a "particle" with a mass $m=1/2$ on a line t which has the energy $\omega^2(k)=k^2+m_\pi^2$ and which moves through the potential barrier $V(t)=-\lambda[\langle\phi^2\rangle(t)-f_\pi^2]$:

$$-\frac{d^2\psi(t)}{dt^2}+\lambda[f_\pi^2-\langle\phi^2\rangle(t)]\psi(t)=\omega^2(k)\psi(t).\tag{14}$$

Far from the barrier, i.e., in the limit $t\rightarrow\pm\infty$ we have $V(\pm\infty)=0$ and the general solution of the Schrödinger equation, as $t\rightarrow\pm\infty$, is the superposition of the left and right moving waves

$$\psi^+(t)=S_R^+e^{-i\omega(k)t}+S_L^+e^{+i\omega(k)t},\quad t\to+\infty,$$
$$\psi(t)=S_R^-e^{-i\omega(k)t}+S_L^-e^{+i\omega(k)t},\quad t\to-\infty. \tag{15}$$

Because of the unitarity, the total fluxes for $t\to\pm\infty$ must be equal: $|S_L^-|^2-|S_R^+|^2=|S_L^-|^2-|S_R^-|^2$. We find

$$S_R^+=\cosh rS_L^- - e^{2i\theta}\sinh rS_L^-,$$
$$S_L^+=-e^{-2i\theta}\sinh rS_R^- + \cosh rS_L^-, \tag{16}$$

where θ is the scattering phase, and the factor r is defined by the probability of the transition through the barrier.

Let us remember that we are starting from the vacuum for $t\to-\infty$, i.e., from $B=\omega/2$, so one must have $S_R^-=0$. This means that at the left (large negative t) we have only a left moving outcoming wave $S_L^-e^{+i\omega(k)t}$. At the right (large positive t)one has both the left and the right moving waves, i.e, the incoming $S_L^+e^{+i\omega(k)t}$ and the reflected $S_R^+e^{-i\omega(k)t}$ waves. The transition coefficient can be obtained from (16) by setting $S_R^-=0$:

$$\frac{|S_L^-|^2}{|S_L^+|^2}=\frac{1}{\cosh^2 r}. \tag{17}$$

Now let us calculate $B(t)=-(i/2)(\dot{\psi}/\psi)$ for large, positive t. Using (15), we find after simple calculations (which restore the k dependence) the relation

$$B(k)=\frac{\omega(k)}{2}\,\frac{\cosh r(k)+\exp[2i(\theta-\omega(k)t)]\sinh r(k)}{\cosh r(k)-\exp[2i(\theta-\omega(k)t)]\sinh r(k)}, \tag{18}$$

in complete agreement with (11). Here the phase factor $\phi(t)$ depends on time as $\phi=\theta-\omega(k)t$.

The squeezing parameter $r(k)$ depends on the absolute value k of the mode spatial momentum $\mathbf{k}$ and is determined by the probability of tunneling through the potential barrier $V(t)=\lambda[f_\pi^2-\langle\phi^2\rangle(t)]$. We note that tunneling takes place exactly when $\omega^2(k)-V(t)<0$, i.e., when the classical long-wavelength modes are exponentially amplified. We see again that squeezing is ultimately connected with the exponential growth of the classical long-wavelength modes, and that the squeezing for each mode k is determined by the function $\langle\phi^2\rangle(t)$—this is the only input information we must know to calculate the DCC wave function.

In the quasiclassical approximation it is easy to calculate the squeezing parameter $r(k)$:

$$r(k)=2\,\mathrm{Re}\int dt\sqrt{\lambda[v^2-\langle\phi^2\rangle(t)]-k^2}, \tag{19}$$

which is valid for small k when $r(k)\gg 1$. The average number of particles in each mode is $\langle N_k\rangle=\sinh^2 r(k)$. We see that $\langle N_k\rangle$ sharply decreases with increasing k. To obtain a rough estimate, let us consider the simple model for $\langle\phi^2\rangle(t)$ assuming that $\langle\phi^2\rangle(t)=0$ in the interval $t\in(0,\tau)$ and equal to its usual v.e.v. $\langle\phi^2\rangle(t)=f_\pi^2$ outside this interval. Such a behavior of $\langle\phi^2\rangle(t)$ is a very rough approximation of a more realistic behavior obtained in Ref. 5 in a numerical experiment. Nevertheless one can use it for some preliminary estimates. The transmission coefficient in this case is well known (see, for example, Ref. 22) and in our notation takes the form

$$\frac{1}{\cosh^2 r(k)}=\frac{4\omega^2(k)[m_\sigma^2/2-\omega^2(k)]}{4\omega^2(k)[m_\sigma^2/2-\omega^2(k)]+(m_\sigma^2/2)^2\sinh^2\tau\sqrt{m_\sigma^2/2-\omega^2(k)}}, \quad (20)$$

where we used the relation $m_\sigma^2=2\lambda f_\pi^2$. The average number of particles with a momentum k will then be

$$\langle N(k)\rangle=\sinh^2 r(k)=\left(\frac{m_\sigma^2}{2}\right)^2\frac{\sinh^2\tau\sqrt{m_\sigma^2/2-\omega^2(k)}}{4\omega^2(k)[m_\sigma^2/2-\omega^2(k)]}. \quad (21)$$

One can estimate the average number of particles $\langle N(k)\rangle$ for different k. For τ we shall use an estimate $\tau=3$–$6\ m_\sigma^{-1}$, which comes from the results[5] $\tau=1$–2 (200 MeV)$^{-1}$. For $k=0$ we then obtain $\langle N(0)>\approx 10^2$–$10^3$. At $k=m_\pi$ the $\langle N(m_\pi)\rangle$ is approximately twice as small, but at $k=3m_\pi\approx 400$ MeV it is two orders of magnitude smaller $\langle N(3m_\pi)\rangle\approx 1$–$10$. Of course, this is a very crude estimate; however, it demonstrates the qualitative features of the phenomenon—the strong exponential dependence on k and the large amplification factor in r which is on the order of $m_\sigma\tau$, where τ is the characteristic time of damping of the $\langle\phi^2\rangle(t)$ oscillations.

In modeling a realistic "Baked Alaska" scenario one should include the effect of the expansion. In Ref. 5 it was suggested that it be described as an inclusion of the term $\dot{a}\pi/a$ in the equations of motion, where $a(t)$ is a scale factor for the expanding plasma. This means that we are considering the problem in the space-time with the metric $ds^2=dt^2-a^2(t)dx^2$. It is easy to show that choosing the new time $\tilde{t}(t)=\int dt/a^3(t)$, we obtain the same Schrödinger equation as in (7) but with a new $\tilde{\Omega}^2(k,\tilde{t})=a^6(\tilde{t})\Omega^2(k,\tilde{t})$. Thus, taking into account the expansion, we obtain a squeezed state again but with the parameters which depend on the particular features of the expansion—the scale factor $a(t)$.

Another interesting feature is the phenomenon of bunching and super-Poissonian statistics.[18,20] We can consider the second-order correlation function

$$g^2(t)=\frac{\langle N(t)N(0)\rangle}{\langle N\rangle^2}, \quad (22)$$

which gives us the relative probability of measuring two particles in an interval t, and $g^2(0)$ measures the probability of simultaneous detection. For the coherent state we have $g^2(t)=1$, which means that for a coherent state the detection events are not statistically dependent. For a squeezed state with large r we have[2)] (Ref. 18) $g^2(0)=2+\coth^2 r>3$, and $g^2(0)$ is increasing with increasing k. The experimental measurement of $g^2(0)$ for pions with different k will be extremely interesting.

We note that the picture we have considered here may have applications not only to DCC, but also to the gluon condensate. In a recent paper[23] it was shown that a high-frequency standing wave in SU(2) gauge theory is unstable against decay into long-wavelength modes, which gives the mechanism for energy transfer from initial high-momentum modes to final states with low-momentum excitations. The picture described here is similar to the amplification of pion soft modes, and it is possible to obtain the same squeezed description of the final state for the soft gluon modes. It is interesting that because the squeezed final state is quadratic in the field, one can find the gluon condensate $\langle G_{\mu\nu}^2 \rangle$.

In conclusion, we would like to stress that in the mean-field (Hartree–Fock) approximation the wave function of DCC is the product of the squeezed-state wave functions of all three pions:

$$|\Psi\rangle_{\rm DCC}=\prod_{i=1}^{3}\left\{\prod_{\mathbf{k}} S[r(\mathbf{k}),\phi(\mathbf{k})]\,|0\rangle_i\right\}, \tag{23}$$

where the universal functions $r(\mathbf{k})$ and $\phi(\mathbf{k})$ are completely determined (in the mean-field approximation) by the only function $\langle\phi^2\rangle(t)$. This wave function leads to some interesting predictions about the k^2 dependence of the observables which would be interesting to verify experimentally. One of the most important problems is to find a way to analytically calculate $\langle\phi^2\rangle(t)$, which now is known only from numerical experiments. It is also interesting to go beyond the Hartree–Fock approximation and determine the effects of the correlations between different pion modes.

I am grateful to A. A. Anselm, D. Gross, K. Rajagopal, S. G. Matinyan, B. Müller, A. M. Polyakov, W. D. Walker, and F. Wilczek for interesting and stimulating discussions. This work was supported by the National Science Foundation grant NSF PHY90-21984.

[1)]On leave of absence from ITEP, B. Cheremyshkinskaya 25, 117259 Moscow, Russia.
[2)]For small r, i.e., when the number of particles is small, this formula has no physical meaning.

[1] A. A. Anselm, Phys. Lett. B **217**, 169 (1989); A. A. Anselm and M. G. Ryskin, Phys. Lett. B **266**, 482 (1991).
[2] J. D. Bjorken, Int. J. Mod. Phys. A **7**, 4189 (1992); Acta Physica Polonica B **23**, 561 (1992).
[3] J. -P. Blaizot and A. Krzywicki, Phys. Rev. D **46**, 246 (1992).
[4] K. L. Kowalski and C. C. Taylor, Case Western Reserve University preprint 92-6, hep-ph/9211282.
[5] K. Rajagopal and F. Wilczek, Nucl. Phys. B **309**, 395 (1993); B **404**, 577 (1993).
[6] J. D. Bjorken, K. L. Kowalski, and C. G. Taylor, SLAC-PUB-6109, March 1993.
[7] S. Yu. Khlebnikov, Mod. Phys. Lett. A **8**, 1901 (1993).
[8] I. I. Kogan, Phys. Rev. D **48**, R3971 (1993).
[9] A. Krzywicki, preprint LPTHE Orsay 93/19, hep-ph/9306260.
[10] J. -P. Blaizot and D. I. Diakonov, Saclay/PNPI preprint, hep-ph/9306301.
[11] C. Greiner, C. Gong, and B. Müller, Preprint DUKE-TH-93-53, hep-ph/9307336.
[12] P. Bedaque and A. Das, Preprint UR-1315 ER-40685-765, hep-ph/9307297.
[13] N. -P. Chang, Preprint CCNY-HEP-93-15, hep-ph/9309253.
[14] A. A. Anselm and M. Bander, Preprint UCI TR 93-32, hep-ph/9309323.
[15] C. M. G. Lattes, Y. Fujimoto, and S. Hasegawa, Phys. Rep. **65**, 151 (1980).
[16] D. Horn and R. Silver, Ann. of Phys. (N.Y.) **66** (1971) 509.
[17] H. P. Yuen, Phys. Rev. A **13**, 2226 (1976).
[18] D. F. Walls, Nature **306**, 141 (1983).
[19] B. L. Schumacher, Phys. Rep. **135**, 317 (1986).
[20] D. F. Smirnov and A. S. Troshin, Sov. Phys. Usp. **30**(10), 851 (1987).

[21] L. P. Grishchuk and Y. V. Sidorov, Class. Quantum. Grav. **6**, L161 (1989); Phys. Rev. **42** D, 3413 (1990).
[22] L. D. Landau and E. M. Lifshitz, *Quantum mechanics: Non-relativistic Theory,* Pergamon, New York (1977), p. 79.
[23] C. Gong, S. G. Matinyan, B. Mueller, and A. Trayanov, Preprint DUKE-TH-93-58, nucl-th/9310001.

Published in English in the original Russian journal. Reproduced here with the stylistic changes by the Translations Editor.