Superluminous propagation of light in a resonant gain medium in connection with dynamic chaos

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As a light pulse with an exponential leading edge propagates through a resonant gain medium, this pulse may undergo a dynamic transition to chaos.

1. Kryukov and Letokhov[1] have shown that the interaction of electromagnetic radiation with a gain medium may result in a steady-state, gain-free propagation of the radiation pulse, with the pulse velocity exceeding the group velocity of the light, v_g. This circumstance, paradoxical at first glance, has a simple explanation: The "superluminous" propagation of the pulse is associated not with a transport of energy at a velocity v_g but with a gain-generated rapid motion of the pulse peak along the leading edge of the pulse.

The analysis in Ref. 1 was based on the energy-balance equations for the difference between the populations of a two-level active medium and the intensity of the electromagnetic radiation. Those equations do not contain phase relations between the electromagnetic field and the polarization of the medium, since it is assumed that the polarization rapidly tracks all changes in the electromagnetic field. That approximation, valid for many active media, is frequently called the "incoherent-interaction approximation."

In the present letter we analyze the problem of the steady-state propagation of a light pulse in an active medium in the case of a *coherent* interaction, in which the phase relations between the polarization and the field are important. As we will see, a dynamic chaos can arise in this case.

2. In our analysis we take an approach which is widely used in the study of resonant interactions of electromagnetic radiation with matter.[1-10] This approach starts with a description of the material of the active medium on the basis of the model of a two-level atom, while the field is modeled by a plane wave $E(z,t)=E(z,t)e^{i(kz-\omega t)}$, where $E(z,t)$ varies slowly in comparison with the exponential function as a function of both variables $(k=\omega/c)$. In this case, the system of equations describing the propagation of the light pulse in the medium is[1]

$$\frac{\partial E}{\partial t}+v_g\frac{\partial E}{\partial z}+\gamma E=i2\pi\omega P, \tag{1a}$$

$$\frac{dP}{dt}+\gamma_2 P=-i(\mu^2/\hbar)NE, \tag{1b}$$

$$\frac{dN}{dt}+\gamma_1 N=J+(i/2\hbar)(EP^*-E^*P). \quad (1c)$$

In these equations, P is the polarization of the active medium, N is the difference between the populations of the upper and lower resonant levels, J is the pump intensity [in units of photons/$(\mathrm{cm}^3\cdot \mathrm{s})$], v_g is the group velocity of the electromagnetic wave, $\gamma=\kappa v_g$, κ is the coefficient of linear (unsaturated) absorption, $\gamma_{1,2}$ are the relaxation constants of the population difference and the polarization, respectively, and μ is the dipole moment of the transition between the resonant levels.

Let us assume that a pulse of the form $E(z,t)=E(t-z/v)$, $P(z,t)=P(t-z/v)$, $N(z,t)=N(t-z/v)$ can undergo steady-state propagation; the pulse propagation velocity v is unknown at this point. Equations (1) then become

$$\frac{dX}{d\xi}=-\sigma(X-Y), \quad (2a)$$

$$\frac{dY}{d\xi}=-Y+XZ, \quad (2b)$$

$$\frac{dZ}{d\xi}=b(r-Z)-b(r-1)XY. \quad (2c)$$

Here X, Y, and Z are the field, the polarization, and the population difference. These variables are normalized in such a way that in a steady-state wave, defined by the condition

$$\frac{dX}{d\xi}=\frac{dY}{d\xi}=\frac{dZ}{d\xi}=0,$$

where $X=Y=Z=1$, we have

$$r=2\pi\mu^2\gamma J/h\gamma_1\gamma_2, \quad \sigma=\gamma/\gamma_2\nu, \quad \nu=1-v_g/v, \quad b=\gamma_1/\gamma_2, \quad \xi=\gamma_2(t-z/v).$$

Equations (2) are of exactly the same form as the equations for a single-mode laser oscillator.[5,6] The sole distinction is that in an oscillator we would have $\sigma=\gamma/\gamma_2$, while in the case at hand, of a freely traveling wave, we have $\sigma=\gamma/\gamma_2\nu$. The presence of the parameter ν introduces some new possibilities for the formulation of experiments.

System of equations (2) is known[5–9] to have chaotic solutions if

$$\sigma>1+b, \quad r>r_*. \quad (3)$$

In the case of model (2) (a uniformly broadened gain line) we have

$$r_*=\sigma(\sigma+3+b)/(\sigma-1-b). \quad (4)$$

For a nonuniformity broadened line, r_* depends on the ratio $\delta=\Delta\omega/\gamma_2$, where $\Delta\omega$ is the nonuniform linewidth. In the very simple model of nonuniform broadening, the line is represented as the sum of two uniformly broadened lines separated by a frequency $\Delta\omega$. In this model we have[5,10]

$$r_*=\sigma[\sigma(1-\delta^2)+3+b]/(\sigma-1-b). \tag{5}$$

Conditions (3) are generally conflicting for an oscillator. The first of these conditions (the "poor-oscillator condition") requires an increase in the losses in the system. As the losses increase, however, there is a decrease in the excitation parameter r; this decrease may make it difficult to satisfy the second condition in (3). It also accounts for the few experimental laser studies in which a chaos corresponding to model (2) (Ref. 11) or to the modification of this model involving a Doppler line broadening[12] has been observed. In the case of a wave which is propagating freely in an active medium, the first condition in (3) can be satisfied by choosing the parameter v; this approach has no effect on r. From this point of view, values $v<1$ are clearly the values of greatest interest. On the other hand, condition (3) requires $v>0$, which, under the condition $v<1$, means a "superluminous" propagation of the light pulse.

To control the parameter v, we need to find its relationship with other parameters of the pulse. Let us assume, following Ref. 1, that the low-intensity leading edge of the pulse has an exponential shape:

$$X(\xi)=X_0e^{\xi/\xi_0}, \quad Y(\xi)=Y_0e^{\xi/\xi_0}, \quad X_0,\, Y_0\ll 1. \tag{6}$$

Substituting (6) into (3), and using the linear approximation, in which we have $Z=r$, we find

$$(1+\sigma\xi_0)X_0-\sigma\xi_0Y_0=0, \tag{7a}$$

$$-r\xi_0X_0+(1+\xi_0)Y_0=0. \tag{7b}$$

System of equations (7) can have a nontrivial solution if

$$\sigma=(1+\xi_0)/[(r-1)\xi_0^2-\xi_0]. \tag{8}$$

Since we have $\sigma=\gamma/\gamma_2 v$, for a given ratio γ/γ_2 this solution determines v (and thus the pulse velocity υ) as a function of the parameter ξ_0. From (8) and the condition $1>v>0$ we find some further requirements on the gain and length of the pulse:

$$(r-1)\xi_0>1, \quad \xi_0<\xi_{\max}, \tag{9}$$

where $\xi_{\max}$ is the positive root of the equation

$$\sigma_0(r-1)\xi_0^2-(\sigma_0-1)\xi_0-1=0. \tag{10}$$

Let us plug in some numbers. In one operating regime of a He–Xe laser, random pulsations in the emission have been observed at the following parameter values:[12] $r=2.3$, $\gamma=3.5\times10^8\ \mathrm{s}^{-1}$, and $\gamma_1=\gamma_2=6.1\times10^7\ \mathrm{s}^{-1}$. For such a medium we have the value $\sigma=5.7$. If we use a He–Xe mixture to produce a gain medium with the same value of the parameter σ as in Ref. 12, then we must introduce in this medium a linearly absorbing substance with an absorption coefficient $\kappa=\gamma\nu/v_g$. Let us assume $\nu=0.5$, which corresponds to a pulse velocity twice the velocity of light. We then have $\kappa=5\times10^{-3}$ cm^{-1} and $r=4.6$ at the same population inversion as in Ref. 12. According to (8), for this experiment we would need an input pulse with a rise time $t_0=\xi_0/\gamma_2=7\times10^{-9}$ s. The length of the active medium would have to be at least $v_g t_0=2$ m.

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