Coherently precessing spin structure in a normal Fermi liquid

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A solution of the spin-dynamics equations is derived for a normal Fermi liquid in the collisionless region. This solution describes a spin structure which is precessing coherently in a slightly nonuniform magnetic field. The structure arises because of a Fermi-liquid interaction. It consists of two domains. In one of these domains, the magnetization is parallel to the field, while in the other it is antiparallel. The stability of the structure is demonstrated numerically. The evolution of the solution from various initial states is traced.

1. INTRODUCTION

It has been shown previously that the possibility of a reversible transfer of spin by a condensate of Cooper pairs renders the spin dynamics nonlocal in superfluid phases of helium-3. In other words, the motion of the spin in some part of the volume of the helium depends on the distribution of the magnetic field over the entire volume and on the boundary conditions. In particular, even a slight nonuniformity of the field under pulsed NMR conditions gives rise to the formation of a two-domain, coherently precessing structure with an anomalously long lifetime in ^{3}He-B (Refs. 1 and 2).

An anomalously long-lived induction signal has also been observed[3] recently in an experimental study of polarized solutions of ^{3}He in ^{4}He, by a pulsed NMR method. The signal was generated by a precession of the spins of the ^{3}He atoms, which form a normal Fermi liquid in the solution. The spin current in a normal Fermi liquid, in the collisionless limit, has a nondissipative component. As a result, several effects which are specific to a Fermi liquid arise: Silin waves,[4] the Leggett–Rice effect,[5,6] and a multiple spin echo.[7]

The anomaly mentioned above was observed in Ref. 3 in the collisionless region. A numerical simulation based on the Leggett equations[6] was carried out in the same study in an effort to explain this anomaly. The results of the simulation indicate that a long-lived precessing structure consisting of two domains can form in a solution in a slightly nonuniform magnetic field. That conclusion would explain the result found. A formation of domains is also indicated by a numerical simulation of experiments[8] carried out to observe nonlinear spin waves in solutions of ^{3}He in ^{4}He.

In an effort to carry out a more comprehensive study of the conditions for the formation of a coherently precessing structure, of the properties of this structure, and of their dependence on the external parameters, we have combined analytic and numerical solutions of the Leggett equations. In the limit of an infinite time τ between collisions of quasiparticles, for an isolated volume of helium in a slightly nonuniform magnetic field, we analytically derived a steady-state solution which has the form of a two-domain spin structure. This structure precesses with a phase which is constant throughout the volume. At large but finite values of τ, we found the relaxation law for this structure. Through a numerical solution of the Leggett equations, in which the time dependence was taken into account, we traced the evolution to a steady-state solution from a given initial state. Adopting specific values of the parameters, we analytically tested the behavior found.

2. STEADY-STATE SOLUTION

We assume (and this assumption is justified by the results) that the length scale of the structure which we are seeking, λ, is large in comparison with the length scale $l_\omega = v_F/\omega$, where v_F is the Fermi velocity. Under such conditions the equations of motion for the spin density $\mathbf{S}$ and for the spin current density $\mathbf{J}_i$ form a closed system.[6] The index i specifies the spatial component of the spin current, and the boldface type specifies a vector in spin space. We introduce the variables $\boldsymbol{\sigma} = (\gamma^2/\chi)\mathbf{S}$ and $\mathbf{j}_i = (\gamma^2/\chi)\mathbf{J}_i$, where γ is the gyromagnetic ratio, and χ is the magnetic susceptibility. Defined this way, $\boldsymbol{\sigma}$ has the dimensionality of an angular frequency, and $\mathbf{j}_i$ that of an angular frequency multiplied by a velocity. The Larmor frequency $\boldsymbol{\omega}_L = \gamma \mathbf{H}$ may depend on the coordinates. We also introduce $w^2 = v_F^2(1+F_0^a)(1+F_1^a/3)$, $\kappa = -(F_0^a - F_1^a/3)/(1+F_0^a)$, and $\tau_1 = \tau/(1+F_1^a/3)$, where F_0^a and F_1^a are the first two harmonics of the exchange part of the Fermi-liquid interaction. In this notation, the equations of motion—the Leggett equations—are

$$\frac{\partial \boldsymbol{\sigma}}{\partial t} + \frac{\partial \mathbf{j}_i}{\partial x_i} = \boldsymbol{\sigma} \times \boldsymbol{\omega}_L, \tag{1}$$

$$\frac{\partial j_i}{\partial t} + \frac{w^2}{3} \frac{\partial}{\partial x_i} (\boldsymbol{\sigma} - \boldsymbol{\omega}_L) = \mathbf{j}_i \times \boldsymbol{\omega}_L + \kappa\, \mathbf{j}_i \times \boldsymbol{\sigma} - \frac{\mathbf{j}_i}{\tau_1}. \tag{2}$$

Here $\hat{\mathbf{z}}$ is a unit vector along the direction of $\boldsymbol{\omega}_L$. For a closed volume, we find conservation of the total projection of the spin onto $\hat{\mathbf{z}}$ from Eq. (1):

$$\hat{\mathbf{z}} \cdot \int \sigma dV = \text{const.} \tag{3}$$

Yet another conservation law is found by combining Eqs. (1) and (2):

$$\frac{\partial}{\partial t}\left[\frac{(\boldsymbol{\sigma} - \boldsymbol{\omega}_L)^2}{2} + \frac{3}{2}\frac{\mathbf{j}_i \cdot \mathbf{j}_i}{w^2}\right] + \frac{\partial}{\partial x_i}[(\boldsymbol{\sigma} - \boldsymbol{\omega}_L) \cdot \mathbf{j}_i] = -\frac{3\mathbf{j}_i \cdot \mathbf{j}_i}{w^2 \tau_1}. \tag{4}$$

The energy density is inside the time derivative here. It follows from Eq. (4) that for an isolated volume we would have

$$\frac{d}{dt}\int \left[\frac{(\boldsymbol{\sigma} - \boldsymbol{\omega}_L)^2}{2} + \frac{3\mathbf{j}_i \cdot \mathbf{j}_i}{2w^2}\right] dV = -\frac{3}{w^2 \tau_1}\int \mathbf{j}_i \cdot \mathbf{j}_i dV \leqslant 0. \tag{5}$$

Under the condition $\tau^{-1}\neq 0$, only a state without currents can be a steady state. According to inequality (5), a stable state corresponds to a minimum of the energy for a given value of the total longitudinal component of the spin. Introducing a Lagrange multiplier $\boldsymbol{\omega}_p$ (also parallel to $\hat{\mathbf{z}}$) to incorporate the conservation of this component, we find that a minimum is reached at $\boldsymbol{\sigma}=\boldsymbol{\omega}_L-\boldsymbol{\omega}_p$.

In the limit $\tau \to \infty$, there is no relaxation, and Eqs. (1) and (2) can have non-equilibrium steady-state solutions. We seek a solution of Eqs. (1) and (2) corresponding to a precession of the spin and of the spin current in a closed volume with a frequency $\boldsymbol{\omega}_p$ which is constant throughout the volume. In other words, we seek a solution such that we have $\partial\sigma/\partial t=\boldsymbol{\sigma}\times\boldsymbol{\omega}_p$ and $\partial\mathbf{j}_i/\partial t=\mathbf{j}_i\times\boldsymbol{\omega}_p$ at each point. We substitute these equalities into Eqs. (1) and (2). We also assume that the vessel holding the liquid is a cylinder with an axis parallel to $\boldsymbol{\omega}_L$ and that the walls of the vessel do not transmit a spin current. In other words, we adopt the boundary condition

$$\mathbf{j}_i\cdot\mathbf{n}_i=0, \tag{6}$$

where $\mathbf{n}_i$ is the normal to the wall. Under these conditions, we can seek a solution which depends on only the one coordinate z, which is longitudinal with respect to $\boldsymbol{\omega}_L$. For such a solution, there is no transverse spin transport, and the longitudinal component of the spin current, $\mathbf{j}_3$, and the spin $\boldsymbol{\sigma}$ satisfy the equations

$$\partial\mathbf{j}_3/\partial z=(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)\times(\boldsymbol{\omega}_L-\boldsymbol{\omega}_p), \tag{7}$$

$$\frac{w^2}{3}\frac{\partial(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)}{\partial z}=\mathbf{j}_3\times(\boldsymbol{\omega}_L-\boldsymbol{\omega}_p+\kappa\boldsymbol{\sigma})-\frac{\mathbf{j}_3}{\tau_1}. \tag{8}$$

To take the limit $\tau_1 \to \infty$ we write $\mathbf{j}_3$ and $\boldsymbol{\sigma}$ as power series in τ_1: $\mathbf{j}_3=\tau_1\mathbf{i}+\mathbf{j}_0+\dots$. In the series for σ, we need retain only the leading term, which is on the order of τ^0. Collecting terms on the order of τ_1 in Eq. (8), we find

$$\mathbf{i}=-\frac{w^2}{3u^2}\mathbf{u}\left(\mathbf{u}\cdot\frac{\partial(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)}{\partial z}\right), \tag{9}$$

where $\mathbf{u}=\boldsymbol{\omega}_L-\boldsymbol{\omega}_p+\kappa\boldsymbol{\sigma}$. In the leading order in τ_1, Eq. (7) becomes $d\mathbf{i}/dz=0$. Using boundary conditions (6), we find that this result means that we also have $\mathbf{i}=0$; i.e., the current does not contain terms which increase with τ_1. In zeroth order in τ_1, Eq. (8) leads to the following expression for the current:

$$\mathbf{j}_3=\frac{w^2}{3u^2}\mathbf{u}\cdot\frac{\partial(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)}{\partial z}. \tag{10}$$

Using the condition $(\mathbf{j}_3\cdot\boldsymbol{\omega}_p)=0$, we verify that the derivative $\partial\sigma/\partial z$ lies in the $\omega_L\sigma$ plane. Consequently, the spin $\boldsymbol{\sigma}$ varies in the coordinate system rotating at the frequency $\boldsymbol{\omega}_p$, remaining in a single plane. We assume that this is the yz plane, and we denote by θ the angle between $\boldsymbol{\sigma}$ and the z axis. The vector $\boldsymbol{\sigma}$ then has the following components: $(0, \sigma\sin\theta, \sigma\cos\theta)$. The current has only a single, nonvanishing component:

$$j_3^x=-\frac{w^2}{3u^2}\left[(\boldsymbol{\sigma}\cdot\mathbf{u})\frac{d\theta}{dz}+\kappa\sigma\sin\theta\frac{d\omega_L}{dz}+(\omega_L-\omega_p)\sin\theta\frac{d\sigma}{dz}\right]. \tag{11}$$

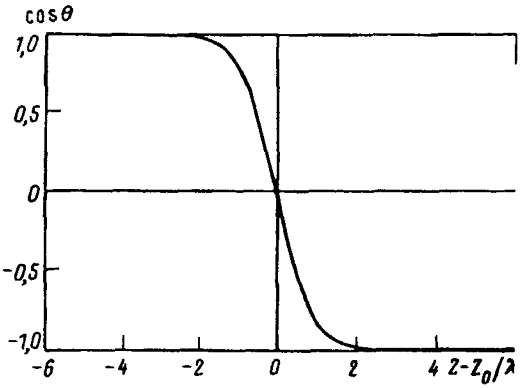

FIG. 1.

The current and the spin density are thus governed completely by the two functions $\sigma(z)$ and $\theta(z)$. To determine them we have two equations. One is found by equating expression (9) for the current **i** to zero, and the other is found by substituting (11) into (7). We consider the most interesting case, in which the Fermi-liquid interaction is dominant, i.e., the case $|\kappa\sigma| \gg |\omega_L - \omega_p|$. For the spin current we find the simple expression $j_3^x = -(w^2/3\kappa)(d\theta/dz)$ in this limit. Substituting it into Eq. (8), and using $\omega_L - \omega_p = (d\omega_L/dz)(z - z_0)$, where z_0 is determined by the condition $\omega_L(z_0) = \omega_p$, we find

$$\frac{d^2\theta}{dz^2} = -\frac{1}{\lambda^3}(z - z_0)\sin\theta. \tag{12}$$

The combination of parameters $\lambda = \{w^2[3\kappa\sigma(d\omega_L/dz)]^{-1}\}^{1/3}$ plays the role of a length scale here. The equation which determines the spatial variation of σ in the same approximation is

$$\frac{d\sigma}{dz} = \frac{d\omega}{dz}\left[\cos\theta - \frac{1}{\kappa}(z - z_0)\frac{d\cos\theta}{dz}\right]. \tag{13}$$

As a boundary condition on Eqs. (12) and (13) we require that the current vanish at the upper and lower walls of the vessel. If a domain wall is at a distance far greater than λ from these vessel walls, we can replace the boundary condition at the walls by a condition at infinity; i.e., we can assume $d\theta/dz \to 0$ as $z \to \pm\infty$. Figure 1 shows a solution of Eq. (12) which satisfies these conditions. This solution has the form of two domains, which are separated by a wall with a thickness on the order of λ. The orientation of the equilibrium domain with respect to $d\omega_L/dz$ with $\kappa > 0$ is opposite that which has been observed[1] in ^{3}He-B. By virtue of our assumption that $d\omega_L/dz$ is small, the change in σ must also be small. From Eq. (13) we easily find the asymptotic behavior of σ far from the wall: $\sigma \simeq \sigma_0 \pm [\omega_p - \omega_L(z)]$ as $(z - z_0) \to \pm\infty$. The constant σ_0 is determined by the initial conditions.

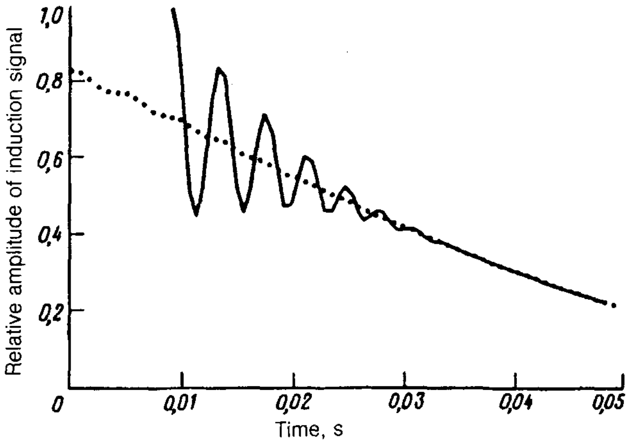

FIG. 2. Numerical simulation of the time evolution of the induction signal for two different initial conditions. Solid line—The magnetization makes an angle of $\pi/2$ with the equilibrium orientation; dotted line—the standard solution (in the limit $\tau \to \infty$) is adopted as the initial condition. In each case, the following parameter values were used in the simulatipn: a cell length $L=0.4$ cm, a field $H=284$ Oe, a field gradient 0.3 Oe/cm, and a temperature $T=0.7$ mK.

3. STABILITY AND RELAXATION OF A TWO-DOMAIN STRUCTURE

The steady-state solution found in Sec. 2 is determined by the specification of two integrals of motion: the longitudinal component of the spin and the energy. In their place it is convenient to use respectively the precession frequency ω_p and the constant σ_0, which determines the asymptotic behavior of σ. At large but finite values of τ, the frequency ω_p remains constant, while σ_0 begins to depend on the time. This dependence can be found by substituting the steady-state solution into both sides of Eq. (5). If the chamber height L is large in comparison with λ, we can ignore terms on the order of λ/L in the expression for the energy. As a result of the substitution, we then find the equation

$$\frac{d}{dt}[\sigma_0(t)]^{5/3} = -\frac{w^{4/3}(\nabla \cdot \omega_L)^{1/3}}{3^{2/3}\tau_1 \kappa^{5/3}} J, \tag{14}$$

where

$$J = \lambda \int_{-\infty}^{+\infty} \left(\frac{d\theta}{dz}\right)^2 dz \simeq 2{,}3.$$

According to (14), $\sigma_0^{5/3}$ falls off linearly with the time, vanishing in a finite time t_f, which depends on the initial value $\sigma_0(0)$:

$$t_f = \frac{\tau_1}{5J}\left(\frac{3\kappa\sigma_0(0)}{w} L\right)^{4/3}\left(\frac{3\kappa\sigma_0(0)}{L\nabla \cdot \omega_L}\right)^{1/3}.$$

For actual experimental conditions we would have $\sigma_0(0) \simeq \omega_L$; in this case t_f would contain two large factors: $(L/l_\omega)^{4/3}$ and $(\kappa\omega_L/L\nabla\omega_L)^{1/3}$. It should be kept in mind, however, that in the case $|\kappa\sigma| \simeq |\omega_L - \omega_p|$ the condition for the applicability of the equations derived above is violated. The expression for t_f should thus be regarded as approximate. In the course of the relaxation, with decreasing σ, the thickness of the domain wall increases slowly (as $\sigma^{-1/3}$).

When surface relaxation is taken into account, the domain wall moves, causing the equilibrium domain to increase in size as time elapses. If $\kappa > 0$, the motion of the wall leads to an increase in the precession frequency as time elapses. The decay time of the induction signal is determined by bulk relaxation, while the time evolution of the precession frequency is determined by surface relaxation, so there is the possibility in principle of distinguishing these two types of relaxation experimentally.

As a test of the stability, we solved the Leggett equations numerically. We examined the evolution of the magnetization in the chamber for various initial conditions, under the assumption that all quantities depend on only a single spatial coordinate: that along the magnetic field. Figure 2 shows a typical example of the time evolution of the induction signal in the course of evolution from a state found in the case of a spatially uniform deviation of the magnetization from its equilibrium direction by an angle $\pi/2$. This simulation was carried out for realistic experimental conditions, for a 6% solution of ^{3}He in ^{4}He. After the damping of the initial oscillations, we find a monotonic decay of the induction signal. For comparison we calculated the evolution from an initial state which would be a steady state at infinite τ. In this case there are essentially no oscillations. A shift in time brings the final regions of the two curves into coincidence, just as we would expect if the system tended toward a (quasi-) steady state as time elapsed. We also tried some other states, approximately steady states, as initial states. After the damping of the initial oscillations, they, too, went into a monotonic decay corresponding to a quasisteady state. Although these calculations do not constitute rigorous proof, they leave no serious grounds for doubting that the steady-state solution is stable.

4. DISCUSSION OF RESULTS

These calculations show that an interpretation of the results of studies of pure ^{3}He and of solutions of ^{3}He in ^{4}He by the NMR method in the collisionless region should take account of the geometry of the cell and the possible formation of a nonuniform structure. Further study of the structure itself would be of independent interest. Because of the long lifetime, the precessing structure may become an extremely sensitive tool for studying magnetic relaxation and other properties of normal Fermi liquids at ultralow temperatures. The solutions remain normal down to the lowest temperatures attainable. At 1 mK, we should take account of the effects discussed here at fields of only ~ 100 Oe. In pure ^{3}He, the region in which the normal phase exists is bounded from below by the temperature of the superfluid transition, so the effects discussed here become important in normal ^{3}He only in fields on the order of 1 T.

The Fermi-liquid interaction also affects the spin current in the superfluid phases of ^{3}He. Here the condition $\omega\tau \gg 1$ holds in weaker fields than in the normal phase.

The equations derived in this letter cannot be literally extended to superfluid ^{3}He, but we would still expect that a precessing structure similar to that discussed in this letter would arise again in the superfluid case, in the collisionless region. We would expect this structure to differ from that which has been observed previously in the hydrodynamic region.[1,2] Such a structure might explain the long-lived induction signal recently observed[9] in ^{3}He-B at a temperature $\simeq 0.1$ mK. In order to draw any definite conclusions, however, it will be necessary to derive a corresponding solution of the spin-dynamics equations for ^{3}He-B.

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