# Čerenkov radiation of superconducting cosmic strings

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A collision of rectilinear superconducting cosmic strings is accompanied by the emission of electromagnetic radiation if the velocity of the intersection point exceeds the velocity of light. This effect stems from a gravitational interaction of the strings. It can be thought of as a realization of Ginzburg's mechanism of radiation from a superluminous reflection of a light beam.

Linear topological defects which form in the course of cosmological phase transitions associated with grand unified models with a partially broken $U(1)\times U(1)$ symmetry, known as "superconducting cosmic strings,"[1] can be the cause of some unusual electromagnetic phenomena. Superconducting strings, which are capable of carrying currents up to $10^{20}$ A, may be the source of intense electromagnetic radiation in various parts of the spectrum and may generate several plasma effects, including an acceleration of relativistic particles.[2–7] Two mechanisms for the generation of electromagnetic waves have been discussed previously: an emission which occurs as strings pass through magnetic fields and an emission by closed oscillating loops. In the typical cosmological scenarios, the strings form predominantly in the form of a random network of rectilinear moving segments. The typical length of these segments is determined by causality. The loops, on the other hand, arise primarily during the evolution of this network, as a result of reconnections. It is assumed here that rectilinear moving strings do not radiate in the absence of a external electromagnetic fields. In the present letter we wish to show that, when the gravitational interaction is taken into account, colliding rectilinear segments of superconducting strings also become sources of intense electromagnetic radiation. This new emission mechanism can be thought of as a realization of Ginzburg's idea of Čerenkov radiation from a superluminous reflection of a light beam.[8]

A solitary rectilinear string, either an ordinary one or one carrying a current, is stable for topological reasons. Less obvious is the fact that a system of moving rectilinear nonsuperconducting strings does not lose energy due to gravitational radiation.[9,10] Two strings which make an angle $\alpha$ and which are moving parallel to each other with a relative velocity $v$ form a cluster of gravitational tensions which is localized near the point of the minimum distance between strings. This point is moving at a superluminous velocity when the condition $v > \sin\alpha$ holds. In this situation, kinematic considerations would lead us to expect Čerenkov gravitational radiation.[8] However, as was shown in Ref. 9, such emission actually cannot occur, because of the symmetries of the Einstein–Nambu action, and because of the absence of massless excitations in a $(1+2)$ gravitation.

In the case of superconducting strings, the corresponding symmetries do not impose a dynamic prohibition against Ĉerenkov radiation by massless particles. As is shown below, electromagnetic radiation of this type does indeed occur. For simplicity we consider a collision of nonintersecting, nonparallel, rectilinear strings, one "ordinary" and one superconducting. The total action of the system is

$$S=S_{\rm gr}+S_N+S_{\rm NO}+S_{\rm em}+S_{\rm int}, \tag{1}$$

where $S_{\rm gr}=-(16\pi G)^{-1}\int R\sqrt{-g}d^4x$ is the Einstein action, $S_{\rm em}=-(16\pi)^{-1}\int F^2\sqrt{-g}d^4x$ ($F_{\mu\nu}$ is Maxwell's tensor),

$$S_N=-\frac{\mu_1}{2}\int \sqrt{-\gamma_1}\gamma_1^{AB}\frac{\partial x_1^\mu}{\partial\zeta_1^A}\frac{\partial x_1^\nu}{\partial\zeta_1^B}g_{\mu\nu}[x_1(\zeta_1)]d^2\zeta_1 \tag{2}$$

is the Nambu action of an ordinary string in a gravitational field $g_{\mu\nu}$,

$$S_{\rm NO}=\int \sqrt{-\gamma_2}\gamma_2^{AB}(\zeta_2)\left[-\frac{\mu_2}{2}\frac{\partial x_2^\mu}{\partial\zeta_2^A}\frac{\partial x_2^\nu}{\partial\zeta_2^B}g_{\mu\nu}[x_2(\zeta_2)]+\frac{1}{2}\frac{\partial\phi}{\partial\zeta_2^A}\frac{\partial\phi}{\partial\zeta_2^B}\right]d^2\zeta_2 \tag{3}$$

is the action for the superconducting string in Nielsen–Olesen form ($\phi$ is a scalar field on the world sheet of the string), and, finally, the term

$$S_{\rm int}=-e\int \sqrt{-\gamma_2}\epsilon^{AB}\frac{\partial x_2^\mu}{\partial\zeta_2^A}\frac{\partial\phi}{\partial\zeta_2^B}A_\mu[x_2(\zeta_2)]d^2\zeta_2 \tag{4}$$

describes the interaction of the superconducting string with the electromagnetic field ($\epsilon^{AB}=e^{AB}/\sqrt{-\gamma}$ is the 2D Levi–Civita tensor, with $e^{01}=-e^{10}=-1$). The action in (1) is invariant under a reparametrization of the world surfaces of each of the strings, under space–time diffeomorphisms, and also under gauge transformations of the field $A_\mu$. In the case under consideration here, the strings interact with each other only gravitationally.

The 2D metric $\gamma^{AB}$ in (2) is, by virtue of the corresponding coupling conditions, an induced metric on the world sheet, while this is not the case in action (3) [the nature of the induced metric is conserved only for a 5D metric in the Kaluza–Klein interpretation of the action in (3)]. Nevertheless, it is possible to choose a conformally planar gauge $g_{\mu\nu}\dot{x}_a^\mu x'^\nu_a=0$, $g_{\mu\nu}(\dot{x}_a^\mu\dot{x}_a^\nu+x'^\mu_a x'^\nu_a)=0$ for both strings (the dot means the derivative with respect to $\zeta_a^0$, and the prime means the derivative with respect to $\zeta_a^1$) such that the equations of motion are

$$\ddot{x}_a^\mu-x''^\mu_a+\Gamma^\mu_{\nu\lambda}(\dot{x}_a^\nu\dot{x}_a^\lambda-x'^\nu_a x'^\lambda_a)=0. \tag{5}$$

The equations for the scalar field have a corresponding form:

$$\ddot{\phi}-\phi''=\frac{e}{2}F_{\mu\nu}e^{AB}\partial_A x_2^\mu\partial_B x_2^\nu. \tag{6}$$

We use a method of successive approximations to solve these equations along with Einstein's equations, whose right side contains the resultant energy–momentum tensor of the two strings and the Maxwell field $F_{\mu\nu}$ and Maxwell's equations with a current:

$$j^{\mu}(x)=e\int \sqrt{-\gamma_2}\epsilon^{AB}\partial_A x_2^{\mu}\partial_B\phi \frac{\delta^4\,[x-x_2\,(\zeta)]}{\sqrt{-g}}\,d^2\zeta. \tag{7}$$

As in Ref. 9, a small parameter here is the gravitational constant (see Ref. 9 for a more detailed discussion of the validity of the perturbation method for cosmic strings). We then have $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, where $\eta_{\mu\nu}=\mathrm{diag}(1,\,-1,\,-1,\,-1)$, $h_{\mu\nu}\ll 1$ ; in the discussion below, the indices are raised and lowered by means of the metric $\eta_{\mu\nu}$. Einstein's equations are then written in a quasilinear form with respect to the quantities $\psi^{\mu\nu}=h^{\mu\nu}-\frac{1}{2}\,\eta^{\mu\nu}h$ in the gauge $\partial_{\mu}\psi^{\mu\nu}=0$:

$$\eta^{\alpha\beta}\partial_{\alpha}\partial_{\beta}\psi^{\mu\nu}=-16\pi G\tau^{\mu\nu}, \tag{8}$$

with the source $\tau^{\mu\nu}=T^{\mu\nu}+S^{\mu\nu}$, where $S^{\mu\nu}$ collects all the nonlinear terms of the equation for the gravitational field.[9] Maxwell's equations can be written in a similar form:

$$\eta^{\alpha\beta}\partial_{\alpha}\partial_{\beta}A_{\mu}=4\pi i_{\mu},\quad i_{\mu}=\sqrt{-g}(j_{\mu}+S_{\mu}), \tag{9}$$

where

$$S^{\mu}=\frac{1}{4\pi\sqrt{-g}}\,\partial_{\nu}\,(\sqrt{-g}g^{\mu\lambda}g^{\nu\sigma}-\eta^{\mu\lambda}\eta^{\nu\sigma})F_{\lambda\sigma}. \tag{10}$$

We are assuming that the superconducting string is carrying a current, which can be represented as a 2D vector $I_2^A$ on the world surface. Its square, $I_2^2$, can be positive (this is the case of a time-like current), negative (a space-like current), or zero (an isotropic current). The first of these cases corresponds to a string with a nonzero electric-charge density, and the second to a current-carrying string. Radiation occurs in both cases.

The calculation is carried out in the following way. When we neglect the gravitational interaction, the world surfaces of the strings are parametrized by the equations $x_a^{\mu}=d_a^{\mu}+U_a^{\mu}\zeta^0+\Sigma_a^{\mu}\zeta^1$, $a=1,\,2$. It is convenient to carry out the calculations in the rest frame of the ordinary string ($a=1$). In this case we have

$$U_1^{\mu}=(1,0,0,0),\quad \Sigma_1^{\mu}=(0,0,0,1)$$

and

$$U_2^{\mu}=\gamma(1,\,v\cos\alpha,\,0,\,v\sin\alpha),\quad \Sigma_2^{\mu}=\epsilon(0,-\sin\alpha,\,0,\,\cos\alpha). \tag{11}$$

The value of $\epsilon$ is found from the coupling equations. These data are used as input data for the next iterative scheme. We first construct retarded solutions for the potentials $\psi_{\mu\nu}^{(1)}$ and $A_{\lambda}^{(1)}$; the tensions $S$ on the right sides of (8) and (9) are ignored. We then calculate the corresponding deformation of the world surfaces and the corrections to the currents $j_{\mu}$ and $S_{\mu}$. The electromagnetic radiation is described by incorporating, in the expression for the current $i_{\mu}$, terms linear in $\psi_{\mu\nu}$ and $A_{\lambda}$, calculated in the first approximation. The loss of 4-momentum due to electromagnetic radiation in a collision of strings is calculated from the formula

$$\Delta P^{\mu}=\frac{1}{2\pi^2}\sum_{\lambda=\theta,\phi}\int d^4k k^{\mu}\Theta(k^0)\delta(k^2)|\ \mathbf{e}^{(\lambda)}(k)\mathbf{i}(k)|^2, \tag{12}$$

where $\lambda$ is a polarization index, and $\mathbf{i}(k)$ is the Fourier transform of the current. It turns out that the quantities $\mathbf{i}(k)$ are nonzero only under the Čerenkov condition $v>\sin\alpha$. In the rest frame of the ordinary string, which we assume to be oriented along the $z$ axis, the wave vectors of the photons form a cone with a vertex angle $\theta$:

$$\cos\theta=\frac{\sin\alpha}{v}. \tag{13}$$

The total energy loss due to radiation with a given polarization, divided by a unit length of the string, is given by

$$\frac{dE^{(\theta)}}{d\omega}=(4\pi\mu_1 I_2)^2\,\gamma^3 v^2\cos^3\alpha\,\frac{e^{-\frac{2\omega d}{\gamma v}}}{\omega},$$

$$\frac{dE^{(\phi)}}{d\omega}=\left(\frac{\tan\alpha}{\gamma v}\right)^2\frac{dE^{(\theta)}}{d\omega}, \tag{14}$$

where $I_2$ is the invariant amplitude of the current, and $d$ is an impact parameter (the shortest distance between the strings). Equations (14) correspond to a space-like current vector. For the time-like case, we should interchange the polarization indices. [The latter property can be associated with the symmetry of action (1), as in the theory derived for nonsuperconducting strings.[9] ]

The logarithmic IR divergence in the radiation spectrum can be eliminated by introducing as a parameter the length $R$, which corresponds to the distance at which collective effects become important in the system of strings. As can be seen from (14), the high-frequency cutoff occurs in an exponential fashion. The maximum frequency in the radiation spectrum is

$$\omega_{\max}=\frac{v}{2d\sqrt{1-v^2}}. \tag{15}$$

As a result, the total energy loss due to radiation, per unit length, is

$$\Delta E=(4\pi\mu_1 I_2)^2\,(v^2\gamma^3\cos^3\alpha+\gamma\cos\alpha\sin^2\alpha)\ln\frac{\gamma v R}{2d}. \tag{16}$$

The case of parallel strings moving with respect to each other corresponds to $\alpha=0$. Under this condition, Čerenkov radiation occurs at all values of $v$. In this case, the problem reduces to a (1+2) electrodynamics, and thus reveals a nontrivial correspondence between the two theories. Making use of the symmetry of action (1), we can show that the (1 + 2) interpretation can be retained in the case of nonparallel strings.[9]

In the case of parallel boson strings with $v\simeq 1$ we have

$$\frac{\Delta E}{\mu_2\gamma}\simeq 10^{-10}\left(\frac{\gamma I}{I_{\rm cr}}\right)^2\ln\left(\frac{\gamma R}{2d}\right), \tag{17}$$

where $I_{cr}=e\sqrt{\mu_2}\simeq 10^{22}$ A is the critical current.

At a high velocity (in the case of a catastrophic collision) the energy loss per pass may turn out to be comparable to the energy of the string even under the condition $I \ll I_{cr}$.

If $\alpha > 0$, a steady-state radiation source arises. As would be expected,[8] the radiation intensity is proportional to the velocity of the point of the shortest distance between the strings, $v_p=v/\sin\alpha$ ($v_p > 1$). In ordinary units we would have

$$\frac{dE}{dt} \simeq 10^{42}\left(\frac{v_p}{c}\right)\left(\frac{\gamma I}{I_{cr}}\right)^2 \gamma \ln\left(\frac{\gamma R}{2d}\right)[\text{erg/s}]. \tag{18}$$

Within the context of string cosmological scenarios, this new radiation mechanism should be taken into account in calculations on the early stage of evolution of the string network, as a fairly effective energy-loss mechanism, which has not previously been discussed in the literature. On the other hand, the characteristic features of the spectrum and angular distribution of the radiation can be utilized in searches for observational manifestations of superconducting strings. Cosmological applications will be discussed in more detail in a separate paper.


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