Effect of magnetic induction on the Kosterlitz–Thouless transition in layered superconductors

A. Yu. Martynovich
Donetsk Phystech, 340114 Donetsk, Ukraine

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The effect of external magnetic field on the resistive transition in a decoupled, layered, Josephson superconductor is investigated. The critical induction B_{cr}, which is quite small with respect to the lower critical field H_{cl}, is obtained. At a small induction, $B < B_{cr}$, the second-order resistive transition occurs as a Kosterlitz–Thouless transition. At a large induction, $B > B_{cr}$, the second-order phase transition changes to a first-order transition and the vortex pair dissociation has a hysteretic behavior in the temperature interval which is close to and below the Kosterlitz–Thouless temperature T_{KT}. The lowest boundary of this interval decreases with increasing induction and coincides with the temperature of the resistive transition.

The appearance of resistivity in a layered Josephson-decoupled superconductor is stipulated by a two-dimensional (2D) vortex pair dissociation. The resistive transition occurs at the temperature

$$T_{KT}=\frac{\phi_0^2}{16\pi^2\Lambda(T_{KT})} \tag{1}$$

in zero magnetic induction and is fully described in the framework of the Kosterlitz–Thouless model as a second-order phase transition.[1-3] Here ϕ_0 is a flux quantum, $\Lambda=2\lambda/d$, λ is the magnetic penetration depth, and d is the interlayer distance. Recent experimental studies of the effect of the magnetic field on this transition have revealed a set of unusual properties. The temperature of the resistive transition T^* coincides with T_{KT} when the external field increases from zero to a critical value B_{cr} and when T^* begins to slowly decrease in a field larger than B_{cr}.[4] The rate of reduction of $T(B)$ noticeably increases with increasing insulating layer thickness of the superconductor.[5] A hysteretic behavior of the resistive transition and of the first-order phase transition was observed in Ref. 6. The hysteresis width of the temperature increases with applied field. In this paper I will attempt to explain these peculiarities on the basis of the Kosterlitz–Thouless model by taking into consideration the magnetic field which penetrates into a layered superconductor as a flux line lattice.

We consider a system of identical superconducting layers with a spacing d between them, a layer thickness much less than d, and a penetration depth λ. The distribution of the vector potential $\mathbf{A}(\mathbf{x})$ in all the space and the screening currents in the superconducting kth layer will be described on the basis of the Lawrence–Doniach model with zero Josephson coupling:

$$\text{curlcurl}\ \mathbf{A}(\mathbf{x})=\frac{2}{\Lambda}\left[\frac{\phi_0}{2\pi}\sum_{i,k}\ \nabla\theta(\mathbf{x}-\mathbf{x}^0_{i,k})-\mathbf{A}(\mathbf{x})\right]\delta(x_3-kd). \tag{2}$$

Here the phase gradient $\nabla\theta(\mathbf{x}-\mathbf{x}^0_{i,k})$ is a magnetic field source of a 2D vortex placed in the kth layer. The 2D vortices are divided into two main parts:

$$\sum_{i,k}\nabla\theta(\mathbf{x}-\mathbf{x}^0_{i,k})=\sum_{\mathbf{R}}\ \nabla(\mathbf{x}-\mathbf{R})+\sum_k\ \int d^2\,\mathbf{x}'[n_k^+(\mathbf{x}')-n_k^-(\mathbf{x}')]\nabla\theta(\mathbf{x}-\mathbf{x}'). \tag{3}$$

One of them forms a flux line lattice, i.e., stacks of 2D vortices whose centers R are the same in each layer. The second one is a gas of free excitations. Averaging densities of positive excitations $n_k^+(\mathbf{x})$ and negative excitations $n_k^-(\mathbf{x})$, we see that they coincide with each other, which follows from the condition of vortex-antivortex pair dissociation. Interaction of free excitations with the flux lines leads to the space distribution of gas density:

$$n^{\pm}(x)=n_0^{\pm}(\mathbf{x}')\exp[-\beta U(\mathbf{x}-\mathbf{x}')], \tag{4}$$

$$\beta=\frac{1}{T}.$$

$U(\mathbf{x})$ is the energy of interaction of the 2D excitation and the flux line lattice. To find n_0^- we consider an extreme case of 2D antivortex on the 3D vortex axis. The 2D antivortex and 2D vortex from a stack of vortices annihilate each other and their place is occupied by a pair from the free dipole gas, whose density is equal to n_{dip}. Thus,

$$n_0^-=n_{\text{dip}}\ \exp[-\beta U(0)-\beta F_0]. \tag{5}$$

Here F_0 is the 2D vortex self-energy, and $U(0)$ is the energy of a 3D vortex link interaction with all other flux lines of the lattice.

The free excitation density can be written in the form

$$n^-(\mathbf{x})=n_0^-\exp(\beta\langle U\rangle)\exp(\beta\ [U(\mathbf{x})-\langle U\rangle]).$$

For the linearization of $n^-(x)$ we expand the second exponent only, because the value $\beta U(\mathbf{x})$ can be in excess of 1 in the case of a dense flux line lattice.

The equality of total numbers of free 2D vortices and antivortices from excitation gas leads to the condition

$$n_0^+\exp(-\beta\langle U\rangle)=n_0^-\exp(\beta\langle U\rangle)\equiv n_0.$$

The main results, which are obtained by solving linearizing Eqs. (2)–(4) following Ref. 7, are the self-energy of the 2D excitation,

$$F_0=\frac{\phi_0^2}{8\pi^2\Lambda}K_0\left(\frac{\xi}{l}\right),\quad l(n_0)=\frac{2\pi\Lambda}{n_0\beta\phi_0^2},$$

and the equation for the unknown equilibrium "density" n_0 of excitations,

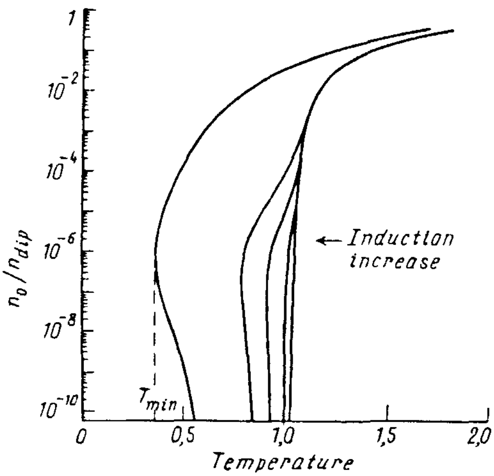

FIG. 1. Temperature dependence of the free excitation density for $T_{KT}=1$, $b_{cr}=1$, $\kappa=200$. $b=0$; 1; 10; 10; 10.

$$n_0=n_{\rm dip}\left(1+\frac{b}{1+(\lambda/l)^2}\right)^{\beta T_{KT}}\exp\left[-2\beta T_{KT}K_0\left(\frac{\xi}{l}\right)\right]. \tag{6}$$

Here the dimensionless induction is

$$b=B\left(\frac{4\pi\lambda^2}{\phi_0}\right)\frac{2\sqrt{3}}{\pi}.$$

It is clear that at a finite value of $l(n_0)$ the logarithmic singularity of F_0 is absent and the self-energy of a 2D vortex in the excitation gas is finite.

Let us solve Eq. (6). One of the solutions of Eq. (6) is $n_0=0$, which is valid in the entire temperature range. Nonzero solution of $n_0(T)$ defines the nonzero resistivity, so the temperature minimum at which $n_0\neq 0$ is a first-order resistivity transition one T^*.

In the case of zero induction $b=0$, the nonzero density

$$n_0=n_{\rm dip}\,p^z\ll 1, \tag{7}$$

$$p=n_{\rm dip}\xi^2 2\pi\,\exp\left(2\gamma\,\frac{T_{KT}}{T}\right),\quad z=\frac{T_{KT}}{T-T_{KT}},$$

appears at $T>T_{KT}$ only. Here we assume that $p<1$ at $T=T_{KT}$. An analysis of all known experimental studies of superlattices and strong anisotropic superconductors confirms this assumption. We note, however, that the value of p is very close to unity[8] and that any induction $b\neq 0$ leads to an increase of this value. It is worth noting that the critical value of the induction at which $p=1$ exists is quite small, $b_{cr}\ll 1$, and we find $p>1$ at $b>b_{cr}$.

Solution of $n_0(T)$ has a significant peculiarity at $b>b_{cr}$, namely, a nonsingle-valued dependence on the temperature. Numerical solutions of Eq. (7) at different inductions are shown in Fig. 1. We see that in the case $b>b_{cr}=1$ a nonzero density n_0

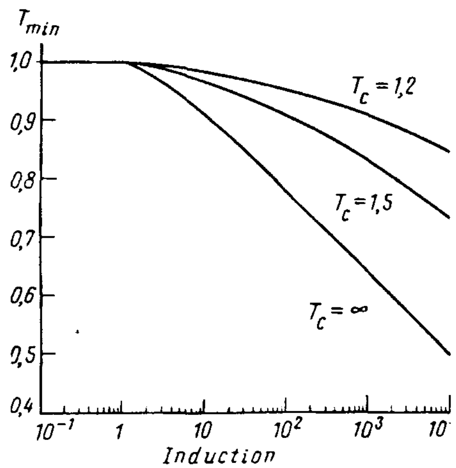

FIG. 2. Dependence of the low limit of $T_{\min}$ of the hysteresis region of $n_0(T)$ on the induction for $T_{KT}=1$, $b_{cr}=1$, and $\kappa=200$.

exists below the critical temperature $T_{KT}=1$. The lowest value of $T_{\min}$ on the temperature scale occurs when $n_0(T)\neq 0$. The dependence $T_{\min}(b)$ is shown in Fig. 2. The lowest curve with the label $T=\infty$ is the solution of Eq. (7) with constant values of λ and ξ. This condition is valid when T_{KT} is very small with respect to the superconducting transition temperature T_c. The difference between T_c and T_{KT} of the layered superconductor is roughly equal to several degrees, and we must take into account the dependence of λ and ξ on T. In Fig. 2 we also show the dependences $T_{\min}(b)$ for two small values of T_c which slightly exceed T_{KT}.

The discovery of critical induction B_{cr} at which the type of phase transition changes is the main result of my paper. At a small induction $B<B_{cr}$ the 2D pair dissociation occurs following the KT scenario. In this case the free excitations are absent upon an increase in temperature to T_{KT} and their density slowly increases at a temperature greater than T_{KT}. Another picture is observed at a large induction $B>B_{cr}$, when the disruption of the vortex pairs is caused by a first-order phase transition. There is a certain temperature interval

$$[T_{\min}, T_{KT}], \tag{8}$$

in which the function $n_0(T)$ is nonsingle-valued. It is easy to understand that the two solutions, namely, $n_0(T)=0$ and the maximum value of n_0, correspond to the local minimum of free energy in the hysteretic region (8), and the middle of $n_0(T)$ describes a potential well between them. We note that stable solutions of $n_0(T)$ do not come in contact with each other and the transition from one solution to the other must be accomplished by a jump.

The hysteretic behavior of $n_0(T)$, which consists of density jumps, is more probable in the case of temperature cycling. The resistivity of the superconductor should increase abruptly upon appearance of free excitation. Hysteresis of $n_0(T)$ therefore manifests itself in the form of irreversible and step-like behavior of the current-voltage characteristic. At the same time, an increase in induction leads to an expansion of the temperature region (8), in which the resistivity is irreversible. Such an $R(T)$ dependence was described in Ref. 6. A nonlinear widening of the hysteretic temperature interval with increasing induction is in agreement with the numerical solution shown in Fig. 2. In addition, the experimentally observed hysteretic interval is very narrow, <0.1 degree. Such a narrow range (8) can be obtained from the theory when T_{KT} and T_c differ from each other by only about 1 degree, which was confirmed by many experiments.

For the first-order phase transition a sample divided into domains is more typical with respect to a uniform phase transition. In our case the domains are regions with $n_0=0$ and nonzero n_0. A domain can exist only in the temperature region (8). The relative volume of the domain with $n_0\neq0$ increases from zero to unity at $T=T_{KT}$ with increasing temperature. The resistivity in a sample divided into domains should occur at $T\sim T_{\min}(B)$. A decrease in the temperature of the resistive transition as a result of increasing the induction has been established experimentally (see, for example, Refs. 4 and 5) and it qualitatively coincides with the dependence $T_{\min}(B)$ obtained by us. We note, in particular, Ref. 4 in which the resistive transition was studied at small inductions and it was shown that $T_{\min}$ deviates from T_{KT} at $B>15$ Oe. I believe that future experiments will confirm my model in detail.

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