Nonperturbative equation for the infrared $\Pi_{44}(0)$ limit in the temporal axial gauge[1)]

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The nonperturbative equation for the infrared $\Pi_{44}(0)$ limit is built by using the Slavnov-Taylor identity to define the three-gluon vertex function in the temporal axial gauge. We found that all vertex corrections should be taken into account along with the standard ring graphs to keep the gauge covariance in all the calculations and to give correctly the nonperturbative g^3 term. This term is explicitly calculated and compared with the previously known results.

Recently interest has been revived (see, e.g., Refs. 1 and 2) to calculate the Debye mass beyond the leading term and to investigate its gauge dependence. This effort is strongly connected with similar calculations of the infrared $\Pi_{44}(0)$ limit, which are more simple, but today they are reliably known only up to the g^2 order. The next-to-leading term (the term of order g^3) was found many years ago (initially in Ref. 3 and then in other studies[4–6]) but until now its accuracy has not been confirmed. Unfortunately, these results are qualitatively different and they show that the g^3 term found for the infrared Π_{44} limit is a gauge-dependent quantity, and that its coefficient (which is more important) is very sensitive to maintaining the gauge covariance in all the calculations. The problem is compounded when the $\Pi_{44}(0,p)$ quantity is calculated; specifically, this limit (as it was shown in Refs. 1 and 2) is needed to define the Debye screening. However, there are many reasons for initially finding the infrared $\Pi_{44}(0)$ limit by using the temporal axial gauge, since this gauge is singled out for keeping the gauge covariance and for calculating the Debye screening.

In this paper our goal is to derive the nonperturbative equation for the infrared $\Pi_{44}(0)$ limit by using the standard Green's function method within the temporal axial gauge. The derived equation takes into account all perturbative graphs (the ring graphs as well as the vertex corrections) and its gauge covariance is guaranteed by using the exact Slavnov-Taylor identities to find the nonperturbative three-gluon vertex. This vertex is qualitatively different from the bare vertex and the derived equation is free from any divergences. The g^3 term is found to be a positive correction to the leading term. We will compare it with the previously known results.

It is well known that the temporal axial gauge is useful for building the nonperturbative schemes, since the choice of the gauge vector $\mathbf{n}_\mu$ to be parallel to the medium vector $\mathbf{u}_\mu$ considerably simplifies the Green's function method. The exact polarization tensor (in the axial gauge) is determined by only two tensor structures:[6]

$$\Pi_{\mu\nu}(k)=G\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+(F-G)B_{\mu\nu}, \tag{1}$$

and the gluon propagator has a rather simple form

$$\mathscr{D}_{ij}(k)=\frac{1}{k^2+G}\left(\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right)+\frac{1}{k^2+F}\frac{k^2}{k_4^2}\frac{k_ik_j}{\mathbf{k}^2}. \tag{2}$$

The scalar functions $F(k)$ and $G(k)$ are defined as follows:

$$G(k)=\frac{1}{2}\left(\sum_i \Pi_{ii}(k)+\frac{k_4^2}{\mathbf{k}^2}\Pi_{44}(k)\right),\quad F(k)=\frac{k^2}{\mathbf{k}^2}\Pi_{44}(k), \tag{3}$$

and they should be calculated in terms of the graph (or another) representation for Π. Because of the peculiarity of the temporal axial gauge, the functions $\mathscr{D}_{44}(k)$ and $\mathscr{D}_{4j}(k)$ are completely eliminated from the formalism, but there is a specific singularity ($k_4^2=0$) which requires a very special treatment. The exact Slavnov–Taylor identity for the three-gluon vertex function has a simple form

$$r_\mu\Gamma^{abc}_{\mu\nu\gamma}(r,p,q)=igf^{abc}[\mathscr{D}^{-1}_{\nu\gamma}(p)-\mathscr{D}^{-1}_{\nu\gamma}(q)], \tag{4}$$

which is clearly an advantage of the axial gauge. Equation (4) is our main tool which is used to build a nonperturbative vertex function for calculating the infrared $\Pi_{44}(0)$ limit. We also use Eq. (4) in a differential form, which allows us in many cases (see, e.g., Ref. 7) to define the exact infrared limit of the three-gluon vertex in a very convenient manner. For example, the infrared $\Gamma^{abc}_{4ij}(-p,0,p)$ limit can be easily found from the standard identity

$$\Gamma^{abc}_{4ij}(-p,0,p)=-igf^{abc}\frac{\partial\mathscr{D}^{-1}_{4j}(p)}{\partial p_i}, \tag{5}$$

which directly results from Eq. (4). This exact limit has a rather simple form

$$\Gamma^{abc}_{4ij}(-p,0,p)=igf^{abc}\left\{\delta_{ij}\left[1+\frac{F(p)}{p^2}\right]+p_j\frac{\partial}{\partial p_i}\left[\frac{F(p)}{p^2}\right]\right\}p_4. \tag{6}$$

It depends on one function which determines the usual representation for the $\mathscr{D}^{-1}_{4i}(p)$ propagator

$$\mathscr{D}^{-1}_{4j}(p)=-\left(1+\frac{F(p)}{p^2}\right)p_jp_4. \tag{7}$$

Unfortunately, the limit (6) is not our case, but we shall return to Eq. (6) when the nonperturbative expression for the infrared $\Gamma^{abc}_{4ij}(0,p,-p)$ limit is discussed.

The exact graph representation for the gluon polarization tensor is well known (see, e.g., Refs. 6 and 8). It contains (in the axial gauge) the standard four nonperturbative graphs. However, if one considers the Π_{44} components, only two one-loop nonperturbative graphs are essential, since the other graphs (the two very complicated graphs) are exactly equal to zero. The analytical expression for the first two graphs has a rather simple form. After using some algebra (taking into account that $\Gamma^{abc}_{ij4}=-igf^{abc}\Gamma_{ij4}$) we obtain the equation for m_E^2 [where $m_E^2=\Pi_{44}(0)$]

$$m_E^2=\frac{g^2N}{\beta}\sum_{p_4}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mathscr{D}_{ii}(p)$$

$$-\frac{g^2N}{2\beta}\sum_{p_{p_4}}\int\frac{d^3\mathbf{p}}{(2\pi)^3}2p_4[\mathscr{D}_{li}(p)\Gamma_{ij4}(p,-p,0)\mathscr{D}_{jl}(p)]. \tag{8}$$

We are going to solve it by keeping the gauge covariance at each step of the calculations. Here all the functions (including the vertex function) are exact and our main problem is to find the nonperturbative expression for the infrared $\Gamma_{ij4}(p,-p,0)$ limit.

Unfortunately the exact expression for the infrared $\Gamma_{ij4}(p,-p,0)$ limit [which is not Eq. (6)] lies beyond our possibilities and therefore only its nonperturbative ansatz will be represented here by the following more general formalism developed in Ref. 7. This formula is

$$\Gamma_{4ij}^{abc}(q,r,p)=-igf^{abc}\Bigg\{\delta_{ij}(r_4-p_4)-\frac{1}{r^2-p^2}\Bigg[\left(\frac{G(r)}{\mathbf{r}^2}-\frac{F(r)}{r^2}\frac{r_4^2}{\mathbf{r}^2}\right)$$
$$-\left(\frac{G(p)}{\mathbf{p}^2}-\frac{F(p)}{p^2}\frac{p_4^2}{\mathbf{p}^2}\right)\Bigg][(\mathbf{pr})\delta_{ij}-p_ir_j](r_4-p_4)$$
$$+\delta_{ij}\left(r_4\frac{F(r)}{r^2}-\frac{F(p)}{p^2}p_4\right)+\frac{1}{q^2-r^2}\left(\frac{F(q)}{q^2}-\frac{F(r)}{r^2}\right)q_ir_4(q-r)_j$$
$$+\frac{1}{p^2-q^2}\left(\frac{F(p)}{p^2}-\frac{F(q)}{q^2}\right)(p-q)_ip_4q_j$$
$$-\frac{1}{r^2-p^2}\left(\frac{F(r)}{r^2}-\frac{F(p)}{p^2}\right)r_4p_4(r-p)_4\delta_{ij}\Bigg\}. \tag{9}$$

It is valid for any momentum, including the soft domain. To find Eq. (9), we use the standard inverse gluon propagator

$$\mathscr{D}_{ij}^{-1}(p)=\left(\delta_{il}-\frac{p_ip_j}{p^2}\right)(p^2+G(p))+\left(1+\frac{F(p)}{p^2}\right)p_4^2\frac{p_ip_j}{\mathbf{p}^2} \tag{10}$$

and the exact Slavnov–Taylor identities (see Ref. 7 for details). The transverse part of the $\Gamma_{4ij}^{abc}(q,r,p)$ function is omitted in Eq. (9), since it is not essential for what follows. Of course, it should be kept in mind that all singularities should be excluded from Eq. (9) for any momentum going to zero.

The vertex function (9) easily reproduces the exact formula (6) if one momentum goes to zero (at first $r_4=0$ and then $|\mathbf{r}|\to 0$), but it is more important that this representation can be used in a more general case to find the infrared limit $\Gamma_{ij4}^{abc}(p,-p,0)$. The final result has the form

$$\Gamma_{ij4}^{abc}(p,-p,0)=-igf^{abc}\left\{2\delta_{ij}\left(1+\frac{F(p)}{p^2}\right)+2\frac{p_4^2}{\mathbf{p}^2}\left(\frac{F(p)}{p^2}\right)\left(\delta_{ij}-\frac{p_ip_j}{\mathbf{p}^2}\right)\right.$$

$$+\frac{1}{|\mathbf{p}|}\left[\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{G(p)}{\mathbf{p}^2}\right)\right][\mathbf{p}^2\delta_{ij}-p_ip_j]$$

$$\left.+p_4^2\left[\frac{1}{|\mathbf{p}|}\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{F(p)}{p^2}\right)\right]\frac{p_ip_j}{\mathbf{p}^2}\right\}p_4. \tag{11}$$

We apply this representation to Eq. (8). The vertex which we found is qualitatively different from the bare vertex and we hope that Eq. (11) will be useful for many application.

All algebra in Eq. (8) is very simple and therefore is omitted. The equation for m_E^2 is found as follows:

$$m_E^2=-\frac{g^2N}{\beta}\sum_{p_4}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left\{\frac{1}{p_4^2}\frac{1}{1+F(p)/p^2}-\frac{2}{p^2+G(p)}+\frac{4p_4^2}{[p^2+G(p)]^2}\right.$$

$$+\frac{1}{|\mathbf{p}|}\left[\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{F(p)}{p^2}\right)\right]\left[1+\frac{F(p)}{p^2}\right]^{-2}+\left[\frac{2p_4^2}{|\mathbf{p}|}\left(\frac{\partial G(p)}{\partial|p|}\right)\right.$$

$$\left.\left.-4p_4^2\left(\frac{G(p)}{\mathbf{p}^2}-\frac{F(p)}{\mathbf{p}^2}\right)\right]\frac{1}{[p^2+G(p)]^2}\right\}. \tag{12}$$

Equation (12) is the main subject of the discussion below. It correctly summarizes the ring graphs and basically exploits the dressed three-gluon vertex in a nonperturbative manner. Furthermore, it is very likely that this equation is exact for the g^3 term. Of course, Eq. (12) correctly reproduces the leading order for m_E^2 if the functions $G(p)$ and $F(p)$ are omitted:

$$m_E^2=\frac{g^2N}{\beta}\sum_{p_4}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\partial}{\partial p_4}\left[\frac{2p_4}{p^2}+\frac{1}{p_4}\right]. \tag{13}$$

We use the standard regularization for the temporal axial gauge,

$$\frac{1}{\beta}\sum_{p_4}\frac{1}{p_4^2}=0. \tag{14}$$

No other terms should be taken into account in Eq. (12), since the remaining graphs (which usually determine the $\Pi_{\mu\nu}$ tensor) are exactly equal to zero if Π_{44} is the only quantity taken into account. This is attributable to a simple Lorentz tensor structure of the bare Γ_4 vertex function and to the specific feature of the temporal axial gauge, in which the $\mathscr{D}_{44}$ function is eliminated from the formalism. The g^3 term (and all other terms) should be completely determined by Eq. (12). We found that this equation is free from any divergences (if the full sum over p_4 is taken into account).

Our next task is to find a suitable equation for calculating the g^3 term on the basis of Eq. (12). This is a pure nonperturbative term which arises in Eq. (12) in the soft momentum region. Because of the different infrared behavior of the functions $G(\mathbf{p})$ and $F(\mathbf{p})$, only the latter contributes appropriately to reproduce the g^3 term and all other terms, of g^4 order, can be omitted. The final equation for the g^3 term (here the δm_E^2 term) has a rather simple form

$$\delta m_E^2 = -\frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left\{ \frac{1}{p_4^2} \frac{1}{1+F(p)/p^2} + \frac{1}{|\mathbf{p}|} \left[\frac{\partial}{\partial |\mathbf{p}|} \left(\frac{F(p)}{p^2} \right) \right] \left[1 + \frac{F(p)}{p^2} \right]^{-2} \right\}. \tag{15}$$

This equation can be solved independently from Eq. (12). However, the analytical behavior of the first term in Eq. (15), which contains the specific singularity of the temporal axial gauge, is not clear. Nevertheless, we insist that this term is equal to zero if only the static $\Pi_{44}(0)$ limit is used

$$\Pi_{44}^{(2)}(p_4=0, |\mathbf{p}| \to 0) = \frac{g^2 N}{3\beta^2}, \tag{16}$$

and all other calculations in (15) should be performed in the standard infrared manner (when the sum over p_4 is replaced by one term with $p_4=0$). Finally, the above equation is found to be

$$\delta m_E^2 = -\frac{g^2 N}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{|\mathbf{p}|} \left[\frac{\partial}{\partial |\mathbf{p}|} \left(\frac{\Pi_{44}^{(2)}(0)}{\mathbf{p}^2} \right) \right] \left[1 + \frac{\Pi_{44}^{(2)}(0)}{\mathbf{p}^2} \right]^{-2}, \tag{17}$$

where all functions are known and the integral can be calculated as usual. Our result has a rather simple form

$$m_E^2 = \left[\frac{g^2 N}{3} + \frac{3}{4\pi} \left(\frac{g^2 N}{3} \right)^{3/2} \right] T^2, \tag{18}$$

which is in an agreement with the results of Refs. 1, 2, and 5 if they are considered in the Feynman gauge only. The other results (Refs. 3 and 4) should be checked to solve reliably the problem of the gauge dependence of the g^3 term.

In conclusion we note that the result found for the infrared $\Pi_{44}(0)$ limit is not reproduced through the effective action calculated in a constant background field (see, e.g., Refs. 9 and 10). This case applies only to the leading term, but the correspondence, in general, seems to be more complicated. This fact needs to be further investigated in order to conclusively establish the status of the g^3 term and its connection with the Debye screening. However, the latter problem should be solved independently from the scenario with the magnetic mass (which is essentially considered in Refs. 1 and 2) and the magnetic screening seems to be separated (at least in the temporal axis gauge) into the outstanding problem. There is also a nonperturbative equation[11]

$$m_M^2 = \frac{3N^2 g^4}{4\beta^2} \sum_{p_4, q_4, r_4} (2\pi)^3 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{r}}{(2\pi)^3} \delta^{(4)}(p+q+r)$$
$$\times \mathscr{D}_{jn}(q) \mathscr{D}_{it}(r) \frac{\partial}{\partial p_i} [\mathscr{D}_{nm}(p) \Gamma_{mjt}(-p, -q, -r)]. \tag{19}$$

The solution of this equation (see, e.g., Ref. 12), which involves the infrared divergences, is still unconfirmed. It is important that Eq. (19) is also accessible only in the temporal axial gauge. This accessibility results from the two remaining nonperturbative graphs which are equal to zero in the above case.

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