Entrainment of a vortex structure by a longitudinal ultrasonic wave

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The interaction of a vortex structure with a longitudinal ultrasonic wave results in a motion of the structure along the wave propagation direction, at an average velocity proportional to the wave power. This motion of the vortex structure gives rise to a transverse electric field, which has a well-defined, temperature-dependent maximum.

A basic feature of the high-T_c superconductors is a relatively high vortex mobility, which is seen in an observable giant flux creep.[1] This property is a consequence of (on the one hand) the short coherence length, which leads to a small activation energy for pinned vortices, and (on the other) the high superconducting transition temperature T_c.

The interaction of ultrasound with vortex structures due to friction forces between a vortex and the crystal[2] was apparently first studied in Ref. 3. A similar approach was later taken in Ref. 4 to describe the attenuation of ultrasound in the high-T_c superconductors. It was shown in Ref. 5 that vortices in semiconductors in an external magnetic field interact directly with ultrasonic waves of various polarizations. In this letter we wish to show that this interaction leads to an entrainment of a vortex liquid by a longitudinal ultrasonic wave.

The physical meaning of this effect can be summarized as follows. A longitudinal ultrasonic wave propagating through a superconductor induces, throughout the volume of the superconductor, an alternating supercurrent, which flows perpendicular to the external field and to the wave vector of the ultrasonic wave. As this current interacts with vortices, it gives rise to an additional Lorentz force, which acts along the direction of the wave vector of the ultrasonic wave on each vortex, causing it to oscillate. In a vortex liquid, this motion leads to oscillations of the density of vortices, according to the continuity equation. Since the Lorentz force acting on a vortex liquid is equal to the product of the density of vortices and the current, a constant component of the Lorentz force arises in this product. This component acts on the vortex structure. It drives the vortex structure as a whole into motion, with some average velocity along the wave propagation direction. This effect is analogous to the entrainment of electrons by a longitudinal ultrasonic wave in a piezoelectric material. A motion of vortices along the propagation direction of the ultrasonic wave gives rise to a transverse electric field in the superconductor. In particular, the existence of such a field, induced by a surface acoustic wave, has recently been observed[6] in a tin film on a lithium niobate substrate. Below we discuss a homogeneous and isotropic superconductor in an external magnetic field B_0, directed along the negative z axis. For definiteness we assume that a plane longitudinal ultrasonic wave $\mathbf{U}=\mathbf{U}_0\, e^{iky-i\omega t}$ is propagating along the positive y axis, where $\mathbf{U}$ is the displacement vector of the medium, $\mathbf{k}$ is the wave vector, and ω is the frequency of the oscillations.

To show that an alternating supercurrent is induced throughout the volume of the superconductor under these conditions, we write the first London equation[7] in a local coordinate system moving with the oscillating medium:

$$\frac{\partial}{\partial t}(\Lambda \mathbf{j}_s) = \mathbf{E} + [\mathbf{V} \times \mathbf{B}_0], \tag{1}$$

where $\Lambda = m/n_s e^2$, m and e are, respectively, the mass and charge of an electron, n_s is the density of superconducting electrons, $\mathbf{E}$ is the electric field in the lab coordinate system, and $\mathbf{j}_s$ is the supercurrent. Using Maxwell's equations

$$\text{curl } \mathbf{H} = \mathbf{j}_s, \tag{2}$$

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{h}}{\partial t}, \tag{3}$$

carrying out some simple manipulations, and setting $\mu = \mu_0$, we find the equation

$$\frac{\partial}{\partial t}(\lambda_L^2 \text{curl curl } \mathbf{h} + \mathbf{h}) = \text{curl } [\mathbf{V}\mathbf{B}_0]. \tag{4}$$

For the case under consideration here, this equation becomes

$$\lambda_L^2 \nabla^2 \mathbf{h} + \mathbf{h} = \text{curl } [\mathbf{U}\mathbf{B}_0], \tag{5}$$

where $\lambda_L^2 = \Lambda/\mu_0$ is the London penetration depth.

Solving Eq. (8), we find the following expression for the supercurrent induced by the ultrasonic wave:

$$\mathbf{j}_s = \frac{k^2 [\mathbf{U} \times \mathbf{B}_0]}{\mu_0(1 + \lambda_L^2 k^2)} e^{iky - i\omega t}. \tag{6}$$

This expression shows that the ultrasonic wave induces a supercurrent throughout the volume of the superconductor. As this current interacts with vortices, it gives rise to a Lorentz force, which acts on each vortex throughout the volume of the superconductor.

The elastic-theory equation describing the propagation of the ultrasonic wave takes the following form in our case:

$$\rho \frac{\partial^2 U}{\partial t^2} = E \frac{\partial^2 U}{\partial x^2} + f_{\text{fr}}, \tag{7}$$

where ρ is the density of the superconductor, $E = \rho c^2$, c is the velocity of the ultrasonic wave, and f_{fr} is the friction force exerted by the vortices on the lattice as they move with respect to the crystal. (In the TAFF regime, this force is equal to the average pinning force acting on the vortex structure as it moves with respect to the superconducting medium. In the FF regime, this force actually means the friction force exerted by the vortex structure on the crystal lattice.) This force can be written

$$f_{\mathrm{fr}}=\eta(\dot{W}-\dot{U}), \tag{8}$$

where $\dot{W}$ is the local velocity of the vortex structure in the lab coordinate system, η is the viscosity coefficient per unit volume of the vortex lattice, and $\dot{U}$ is the local velocity of the superconductor.

In our case the equation of motion of the vortex liquid is found from the balance equation for the forces acting on a unit volume of the vortex lattice. In the continuum approximation this equation is

$$f_{\mathrm{fr}}=C_{11}\frac{\partial^2 W}{\partial x^2}+[\mathbf{j}_s\times\mathbf{B}_z]_y, \tag{9}$$

where $\mathbf{j}_s$ is the superconducting current induced by the ultrasonic wave, $\mathbf{B}_z=\Phi_0 n\mathbf{Z}$, the unit vector $\mathbf{Z}$ is directed along the negative z axis, $\Phi_0=2\pi\hbar c/2e$ is the quantum of magnetic flux, n is the number of vortices which cross a unit surface area, and C_{11} is the bulk modulus of the vortex lattice. The surface density of vortices must also satisfy the continuity equation

$$\frac{\partial n}{\partial t}-\frac{\partial(\dot{W}-\dot{U})}{\partial y}=0. \tag{10}$$

Equations (6)–(10) constitute a nonlinear system of equations which can be solved by perturbation theory. We write the surface density n and the velocity $\dot{W}$ of the vortices in the form $n=n_0+n_1$, $\dot{W}=\dot{W}_1+\dot{W}_2$, where the subscript 1 specifies the small terms of first order, and 2 the small terms of second order. Substituting these expressions into Eqs. (6)–(10), and retaining only the first-order terms, we find a linear system of equations. The solutions of this system show that incorporating the interaction of the ultrasonic wave with the vortices leads, in first order, to the appearance of an additional attenuation and a change in the velocity of the wave.

Now retaining the second-order terms in Eq. (9), and ignoring the spatial nonuniformity of the wave due to the attenuation, we find the expression

$$\dot{W}_2=\frac{\Phi_0 n_1}{\eta}[\mathbf{j}_s\times\mathbf{Z}]_y. \tag{11}$$

Taking the time average of this expression, we find an expression for the average velocity of the vortex liquid with respect to the superconductor:

$$\langle\dot{W}_2\rangle=k\omega\frac{X^2}{1+X^2}U_0^2. \tag{12}$$

where $X=k^2C_{11}/(\eta\omega)$. The motion of the vortices with respect to the superconductor caused by the ultrasonic wave leads to a transverse electric field[8]

$$E=\Phi_0 n(\dot{W}-\dot{U}). \tag{13}$$

In this expression we are interested in only the second-order terms in the amplitude of the wave, since only these terms make a nonvanishing contribution when an average is taken over the time:

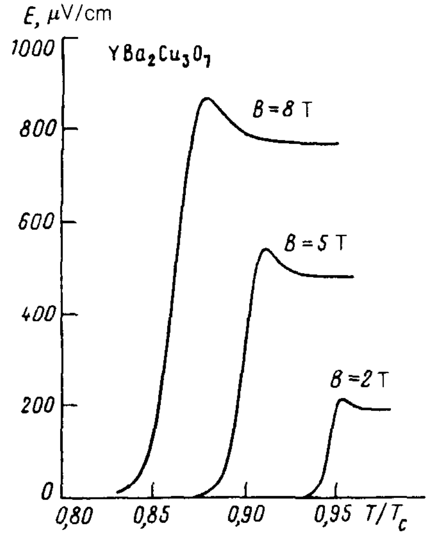

FIG. 1. Electric field induced by an ultrasonic wave in $YBa_2Cu_3O_7$ versus the temperature for various values of the external magnetic field in the region of the TAFF regime.

$$E_x=\Phi_0\, n_0\langle \dot{W}_2\rangle+\Phi_0\langle n_1(\dot{W}_1-\dot{U})\rangle. \tag{14}$$

The angle brackets here mean a time average. The first term contains the average velocity of the vortices with respect to the superconductor, which we derived above, and the external magnetic induction. The second term is the product of two quantities which are linear in the wave amplitude. Here is the final expression for the electric field induced by the ultrasonic wave:

$$E=k\omega B_0U_0^2\left[\frac{X^2}{1+X^2}+2\frac{X^2}{(1+X^2)^2}\right]. \tag{15}$$

Expressions (12) and (15) hold at temperatures below T_c, in the region in which a mobile vortex structure exists, in particular, the region of the TAFF and FF regimes. In the TAFF region, the electric field has a well-expressed maximum as a function of the temperature. As an example we will derive the electric field induced by a longitudinal ultrasonic wave in a $YBa_2Cu_3O_7$ crystal in a magnetic field directed perpendicular to the ab plane. We will derive this field as a function of the temperature in the TAFF regime. The velocity of a longitudinal ultrasonic wave in this case is[9] 4.1×10^3 m/s; we assume the frequency of the wave to be 100 MHz and its amplitude $U_0=10^{-9}$ m. The viscosity coefficient in the TAFF regime is[10] $\eta=B^2/r$. To calculate the resistance, we use the Tinkham formula[11] $r=r_0I_0^{-2}(\gamma_0/2)$, where $\gamma_0=1.2\times10^3(1-T/T_c)^{3/2}\ \mathrm{V}^{-1}$. Figure 1 shows results calculated for the electric field as a function of the temperature for three values of the external magnetic field.[1)] As the magnetic field is increased, the maximum of the field as a function of the temperature shifts to a lower temperature. This temperature dependence might be utilized experimentally to determine the viscosity coefficient and elastic modulus of a vortex liquid. The maximum velocity of a vortex liquid for the wave parameters which we selected is $\simeq 10$ mm/s.

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[1)]Figure 1 does not show the behavior of the field as a function of the temperature near the transition point, since this region is not described within the framework of the TAFF regime. A study of the behavior of the induced field in this region goes beyond the scope of the present study.

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[1]Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).
[2]J. Bardeen and M. J. Stephen, Phys. Rev. A **140**, 1197 (1965).
[3]V. P. Galaĭko and N. I. Fal'ko, Zh. Eksp. Teor. Fiz. **52**, 977 (1967) [*sic*].
[4]J. Pankert, Physica C **168**, 335 (1990).
[5]E. D. Gutlyanskiĭ, Fiz. Nizk. Temp. **18**, 428 (1992) [Sov. J. Low Temp. Phys. **18**, 290 (1992)].
[6]N. V. Zavaritskiĭ, JETP Lett. **57**, 707 (1993).
[7]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
[8]B. D. Josephson, Phys. Lett. **16**, 242 (1960).
[9]S. Bhattacharya *et al.*, Phys. Rev. Lett. **60**, 1181 (1988).
[10]E. H. Brand, Int. J. Modern Phys. B **5**, 751 (1991).
[11]M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988).

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