Structural phase transitions in lattices of magnetic bubbles

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It is shown theoretically that a close-packed hexagonal lattice of magnetic bubbles of identical diameter can transform into a lattice of magnetic bubbles with different diameters as the temperature is lowered in a zero external field. The conditions for the existence of the latter lattice and for its transformation into a lattice of magnetic bubbles similar to the crystal lattice of graphite are determined.

A set of cylindrical magnetic domains (magnetic bubbles) can form, because of the repulsive dipole forces of these bubbles, an ordered periodic structure of a 2D-lattice type.[1–3] It is assumed here that the magnetic bubbles which form a lattice of this sort (usually a close-packed hexagonal lattice) all have the same diameter. On the other hand, there is evidence[4] that bubbles of two different diameters can coexist in a lattice of magnetic bubbles in a certain temperature interval in a zero external field. The bubbles of the smaller diameter are at the center of the hexagonal cell, and those of the larger diameter are at the sites of this cell. As a result, the bubbles of the smaller diameter form their own hexagonal lattice. As the temperature is lowered, the lattice of bubbles of the smaller diameter collapses, and only the lattice of bubbles of the larger diameter persists. The latter has a structure like the crystal lattice of graphite and is stable in a certain temperature interval.[4]

A selective collapse of bubbles in a lattice of identical magnetic bubbles has been seen previously.[3] On the other hand, there has been no previous study or discussion of the possible existence of a bubble lattice like the graphite crystal lattice or of the stability boundary for such a lattice.

Let us consider a case approximating the experiments of Ref. 4. A film of a bubble-containing material, unbounded along the x and y directions, has a thickness h and an easy-magnetization axis perpendicular to the plane of the film. We choose an elementary rectangular cell with periods ap^2 and ap along the x and y axes, respectively ($p=\sqrt{3}$ is a numerical parameter of the hexagonal lattice). The unit cell contains six magnetic bubbles: four of diameter d_1 and two of diameter d_2. As a result, the composite hexagonal lattice of magnetic bubbles in which we are interested here is formed from two lattices which are nested in each other. The first of these lattices, with the bubbles of diameter d_1, is similar to the graphite crystal lattice with a period

a. The second, with the bubbles of diameter d_2, is hexagonal and has a period ap. The bubbles of diameter d_2 are thus at the center of the composite hexagonal lattice, while those of diameter d_1 are at the sites of this lattice. The total energy of this lattice of bubbles includes the surface energy of the bubble walls, the magnetostatic energy of the lattices of bubbles of diameters d_1 and d_2, and the energy of the magnetic-dipole interaction among all bubbles:

$$E=2\pi M^2-(2\pi M)^2\frac{2}{3p}\left(\frac{h}{a}\right)^2 V+(4\pi M)^2\frac{k^3}{3p}\left(\frac{h}{a}\right)^5 Q, \tag{1}$$

$$V=[2I(x_1)+I(x_2)]-\frac{1}{h}(2x_1+x_2), \tag{2}$$

$$Q=(1+\alpha)x_1^4+2(1-\alpha)x_1^2x_2^2+\alpha x_2^4. \tag{3}$$

Here $\alpha=(3p)^{-1}$, $x_i=d_i/h$, $k_3\simeq 0.172$ is the constant of the dipole approximation in the theory of lattices of magnetic bubbles, $\mathbf{M}$ is the saturation magnetization, l is a length scale of the magnetic material, and $I(x)$ is the Thiele stability function.[5]

We wish to find the conditions for equilibrium and stability of the composite lattice. From the condition

$$\frac{\partial E}{\partial x_i}=0 \quad (i=1,2)$$

we find the system of equations

$$\frac{l}{h}-F(x_1)+\frac{q}{2}[(1+\alpha)x_1^3+(1-\alpha)x_1x_2^2]=0, \tag{4}$$

$$\frac{l}{h}-F(x_2)+q[\alpha x_2^3+(1-\alpha)x_2x_1^2]=0, \tag{5}$$

where $q=8k_3(h/a)^3$, and $F(x)$ is a stability function.[5] The solutions of this system of equations must satisfy the stability conditions

$$\left(\frac{\partial^2 E}{\partial x_1^2}\right)\left(\frac{\partial^2 E}{\partial x_2^2}\right)-\left(\frac{\partial^2 E}{\partial x_1\partial x_2}\right)^2>0,\quad \frac{\partial^2 E}{\partial x_1^2}>0. \tag{6}$$

Relations (4)–(6) serve as a generalization of the corresponding conditions for a lattice with identical magnetic bubbles. To demonstrate the point, we set $x_1=x_2=x$; we find that this system reduces to the equation

$$\frac{l}{h}-F(x)+qx^3=0, \tag{7}$$

which is the same as the corresponding equation of Ref. 3 with $\mathbf{H}=0$. Inequalities (6) become

$$(F'(x)-3\alpha q x^2)(F'(x)-3q x^2)>0, \tag{8}$$

$$F'(x)-(2+\alpha)qx^2>0,$$

where

$$F'(x)=\frac{d}{dx}F(x).$$

From the condition for a loss of stability we find the critical values of the bubble diameters. Approximating the function $F(x)$ by[6]

$$F(x)=\frac{x}{1+0.726x},$$

we find the solutions of the equations:

$$F'(x_j)_c=3\alpha_j q(x_j)_c^2,\ \alpha_j=(1,\alpha). \tag{9}$$

The first of them, $(x_0)_c$, is associated with a collapse of a lattice of magnetic bubbles of identical diameter.[1-3] That term will be of interest below only as a step toward the second solution. The second solution, $(x_1)_c$, determines the point at which the lattice becomes unstable, i.e., the critical bubble diameter, at which a hexagonal lattice with bubbles of identical diameter transforms into a lattice with bubbles with different diameters:

$$(x_1)_c=b\left[\left(1+\frac{2}{b}(3\alpha q)^{-1/2}\right)^{1/2}-1\right],\quad (x_1)_c>(x_0)_c, \tag{10}$$

where $b=\frac{1}{2}\left(\frac{1}{0.726}\right)$.

Assuming $a=20.7\ \mu$m and $h=8.7\ \mu$m, we find the critical dimensions of the bubbles:

$$(x_0)_c\simeq 1.03,\quad (l_0)_c\simeq 0.48h,$$

$$(x_1)_c\simeq 1.79,\quad (l_1)_c\simeq 0.19h.$$

Figure 1 shows the results of a numerical solution of system (4), (5), which describes the dependence $x=x(l/h)$ with stability conditions (6). The solid curves describe the behavior of the bubble lattice as the temperature changes; these curves can serve as a justification of the experimental results of[1)] Ref. 4. Specifically, in the region $l<(l_1)_c$ there exists a standard hexagonal lattice with bubbles of identical diameter. At $l=(l_1)_c$, there is a structural phase transition to a lattice of bubbles of different diameters. Analysis of the solution of the system of equations shows that the dimensions of the bubbles of diameter d_2, at the center of the hexagonal cell, decrease with a further increase in l (with a further decrease in the temperature). In contrast, the bubbles of diameter d_1, at the sites of the cell, increase. When a critical value $l=(l_2)_c\simeq 0.22h$ is reached, the bubbles of diameter d_2 collapse, while those of diameter d_1 abruptly increase in size. We are then left with only the lattice of bubbles of diameter d_1:

$$\frac{l}{h}-F(x_1)+\frac{q}{2}(1+\alpha)x_1^3=0,$$

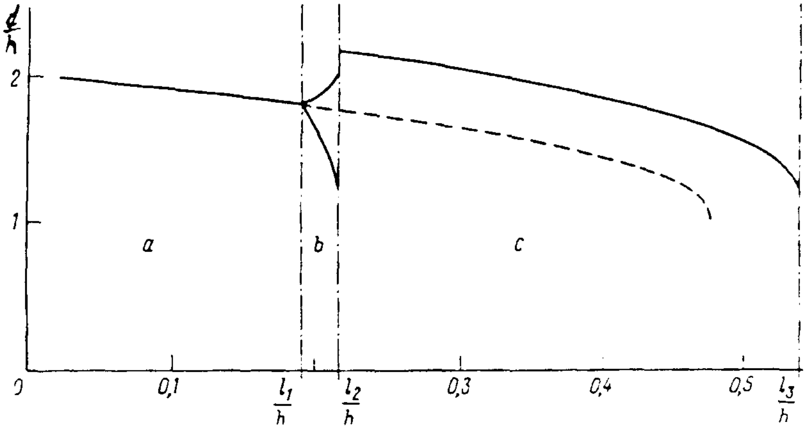

FIG. 1. Intervals of stability of a lattice of magnetic bubbles with respect to collapse as a function of the length scale of the material. a—With magnetic bubbles of identical diameter; b—with bubbles of different diameters; c—a lattice like the crystal lattice of graphite.

$$F'(x_1) - \frac{3}{2}(1+\alpha)qx_1^2 > 0, \tag{11}$$

which is similar to the crystal lattice of graphite. Finally, at a certain $l = (l_3)_c \simeq 0.54h$, this lattice, too, collapses. The collapse diameter of the bubbles in this lattice is larger than that of the bubbles in a hexagonal lattice with identical bubbles.

The dashed curve in Fig. 1 illustrates the difference between the lattice which we are discussing here—a composite lattice with different bubbles—and the standard lattice, with identical bubbles. The existing theory[1-3] describes specifically this curve. The dependence $x = x(l/h)$ shown in this figure persists as the lattice period a is changed. Large bubble dimensions and, correspondingly, large critical bubble diameters correspond to large values of a.

In comparing these results with experimental results, one should bear in mind that the experiments of Ref. 4 did not determine the critical points for the beginning of the transformation of the standard lattice of magnetic bubbles into a composite lattice, with bubbles of different diameters, or for the transformation of a composite lattice into a lattice of bubbles of the graphite type. For example, the point at which a lattice of magnetic bubbles of identical diameter becomes unstable, $(x_1)_c$, is identified with the case in which a lattice of bubbles with different diameters occupies the greater part of the volume of an initial standard lattice of magnetic bubbles.

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[1)]The length scale l as defined in Ref. 4 is greater than the customary value[1-3] by a factor of 4π.

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