Distribution of neutral and charged pions through the production of a classical pion field

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High-energy reactions may produce a state near the collision point that is best described by a classical pion field. Such a field might be an isospin rotated vacuum of the chiral σ-model or, as discussed in this work, a solution of the equations of motion resulting from the coupling of fields of this model to quarks produced in the collision. In such configurations all directions in isospin space are allowed. This circumstances leads to a sizable probability of events with essentially only charged particles (centauros) or all neutral particles (anti-centauros). (In more common statistical models of multiparticle production, the probability of such events is suppressed exponentially by the total multiplicity.) We find that the isospin violation due to the mass difference of the up and down quarks has a significant effect on these distributions and enhances the production of events consisting predominantly of neutral particles.

1. In recent years several authors[1-9] have suggested that the celebrated centauro events,[10] in which no π^0's versus a large number of charged hadrons have been observed, might be explained by the production at these high energies of a classical pion field. An interesting example is the "disoriented chiral condensate."[6,7] The idea is that such a process, considered event-by-event, would correspond to the field oriented along a given Cartesian isospin direction. In the events where the isospin is oriented (almost) parallel to the third axis, one would expect mainly neutral pions, while in the events where the isospin lies in the perpendicular plane, predominantly charged pions would be produced. Let (π_1, π_2, π_3) be the three Cartesian isotopic amplitudes of the classical pion field. Since all the orientations are equivalent, the distribution in the amplitude π_3 is

$$d\omega \sim d\pi_3, \quad \pi^2 = \pi_1^2 + \pi_2^2 + \pi_3^2 = \text{const.} \tag{1}$$

The number of neutral pions, n_0, is proportional to π_3^2, while the total number of produced pions, $n = n_0 + n_+ + n_-$, is proportional to π^2. With $f = n_0/n$, the fraction of neutral pions, we have from (1) the relation

$$d\omega = \frac{df}{2\sqrt{f}}. \tag{2}$$

This distribution is normalized to unity.

Obviously, (2) predicts many more events, with a small number of neutrals, than do usual statistical mechanisms for pion production. In the latter case one expects $d\omega/df$ to peak at $f=1/3$ ($n_+=n_-=n_0=1/3$ as $n\to\infty$) and to decrease exponentially with n, as f deviates from this value. The distribution (2) corresponds to the limit $n\to\infty$ and gives for the relative number of events with the fraction of neutrals less than f

$$P(f)=\int_0^f \frac{d\omega}{df'}\,df'=\sqrt{f}\,. \tag{3}$$

For a typical centauro event we have $f\sim 1/100$ and $P\sim 10\%$. This seems to be a reasonable number, since the five "classic" centauros represents about 1% of the events with appropriate energies.[10]

At the other end of the spectrum, near $f=1$, the probability of an event having an anomalously large fraction of π^0's is

$$1-P(f)=1-\sqrt{f}\sim\frac{1}{2}(1-f). \tag{4}$$

We do not have the square root enhancement exhibited in (3), instead we find a linear dependence at the end of the spectrum; however, there is still a finite probability of finding events with a large number of π^0's. It is possible that such "anti-centauro" events have been observed.[11] We shall propose a mechanism for enhancing their probability over that of (4).

The distribution (2) results from an exact isospin symmetry. At the quark level this symmetry is rather strongly violated due to the up-down quark mass difference, $m_u\neq m_d$. In this letter we shall demonstrate that this mass inequality can enhance the probability of anti-centauros.

2. A class of solutions of the pion field, whose dynamics are governed by a nonlinear chiral Lagrangian, was presented in Ref. 3. The results of that work may be understood in the following simple way. The Lagrangian is

$$\mathscr{L}=\frac{f_\pi^2}{2}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger), \tag{5}$$

where $f_\pi=93$ MeV, and the unitary matrix U is related to the pion fields by

$$U=\exp\left(\frac{i\tau\cdot\pi}{f_\pi}\right). \tag{6}$$

For the particular form

$$U=\exp[i\tau_3\theta(\mathbf{r},t)] \tag{7}$$

the Lagrangian (5) leads to the free equation of motion

$$\partial^2\theta=0. \tag{8}$$

For constant unitary matrices V_L and V_R a generalization of (7) is

$$U=V_L^\dagger\exp(i\tau_3\theta)V_R. \tag{9}$$

This is a general class of solutions which has been studied in Ref. 3. All other known solutions[5,12] are particular cases of (9).

At large distances from the collision point, we use the normal structure of the vacuum, i.e., $U=1$. Likewise, we choose solutions in which $\theta(\mathbf{r},t)\to 0$ as $r\to\infty$. This forces $V_L=V_R$, and solutions (9) reduce to isotopic rotations of (7). In other words, (9) takes the form

$$U=\exp[i\tau\cdot n\theta(x)] \tag{10}$$

for a certain direction n in the isotopic spin space. A possible scenario for the production of a classical pion field, discussed in Refs. 6 and 7, is that inside a certain volume around the collision point a state corresponding to a constant (in the volume) θ is produced. This state is degenerate with a normal vacuum (in the limit $m_\pi=0$), but it is rotated with respect to it in the isotopic spin space. In Refs. 6 and 7 this situation is referred to as "disoriented chiral condensate." It follows from (10) that any solution of (8) describes chiral dynamics.

We now introduce interactions of pions with quarks, keeping in mind that the pion field is the chiral phase of the quark field.[13,14] In the presence of pion fields the quark fields should be modified

$$\begin{aligned} q_L(x) &\to \exp\left[\frac{i}{2}\tau\cdot n\theta(x)\right]q_L, \\ q_R(x) &\to \exp\left[\frac{-i}{2}\tau\cdot n\theta(x)\right]q_R. \end{aligned} \tag{11}$$

The quark mass terms give rise to the quark–pion interaction Hamiltonian

$$\mathscr{H}=m_u\bar{u}u=m_d\bar{d}d\to\bar{q}\exp\left(\frac{i}{2}\tau\cdot n\theta\right)(m_+ +m_-\tau_3)\exp\left(\frac{i}{2}\tau\cdot n\theta\right)q+\text{H.c.}, \tag{12}$$

where $m_\pm=\frac{1}{2}(m_u\pm m_d)$. For the solution (10) we have

$$\begin{aligned}\mathscr{H}=\bar{q}(m_+ +m_-\tau_3)q-(1-\cos\theta)\bar{q}(m_+ +m_-n_3\tau\cdot n)q+\sin\theta\bar{q}i\gamma_5(m_+\tau\cdot n \\ +m_-n_3)q.\end{aligned} \tag{13}$$

Because of the quark masses, the energy depends on θ and the probability for the creation of different θ's will be different. This circumstance, however, will not lead, by itself, to the deviation from the inverse-square-root law since the three-dimensional isotopic space remains isotropic. Quark mass differences do lead to an anisotropic distribution. We would like to know qualitatively the form of this deviation. In the normal vacuum Eq. (13) accounts for the pion mass term in terms of the existence of the chiral condensate[14] $\langle\bar{q}q\rangle\neq 0$. From (13) we see that $m_\pi^2=-m_+\langle\bar{q}q\rangle/f_\pi^2$, where $\pi=f_\pi\theta$. In the presence of hadronic matter and at finite temperatures the changes in the condensate change the effective pion mass. We should note that use of current quark masses in the above interactions does not imply that there are light quark excitations in the hadronic matter. The strength of their coupling to pions is nevertheless given by the current quark masses. The constituent quark mass, again, depends on the particular features of the chiral symmetry breaking.[14] Finite values of the other bilinear quark averages, in addition to providing us with an effective pion mass that depends on the surroundings, make it possible that θ is nonzero at the minimum of the potential.

The distributions, in the parameters θ and n, of a classical pion field produced in a high-energy collision are expected to depend on the production temperature T. They have the form

$$d\omega \sim \int \prod_x d\theta(x) \exp\left(-\frac{1}{T}\int d^3x\mathscr{H}\right) dn\,\delta(n^2-1). \tag{14}$$

A question arises as to what temperature to use and even the extent to which we can consider thermal equilibrium.[8] We assume that in the region of a very high energy collision a disoriented chiral vacuum, which is in contact with thermalized quark matter,[9] is produced. The temperature of this matter is higher than T_c, the temperature of chiral symmetry is restored.[6,7] The average of θ over the whole collision region vanishes; however, smaller regions may have nonzero values for this average. As time elapses, this region grows and the temperature decreases. In Refs. 6, 7, and 9, the system is assumed to stay in thermal equilibrium as it cools down through T_c, and to hadronize at some $T<T_c$. In Ref. 8, this transition is assumed to take the system out of equilibrium by quenching the high-temperature configuration and then letting it evolve by the equations of motion at a fixed energy (micro-canonical ensemble). For the present discussion, it is not important to know whether the quark matter does or does not stay in equilibrium, as long as it is correlated over large regions; this assumption will be used below. The probability of finding a distribution of $\theta(x)$ is assumed to be thermal and determined by the Hamiltonian in (13) and by some temperature $T<T_c$. In the rest of this paper we assume $T=T_c$ and note that this assumption underestimates the size of the effects we are considering.

If the quark density in the collision is not too high, $\langle\bar{q}q\rangle$ should be set equal to its usual vacuum value. Expanding around $\theta=0$, we can write (14) in the form

$$d\omega \sim \int \prod_x d\theta(x) \exp\left(-\frac{m_+|\langle\bar{q}q\rangle|}{2T}\int d^3x\theta^2\right) dn\,\delta(n^2-1). \tag{15}$$

At $T=T_c\sim 140$ MeV (Ref. 15) and a volume $V\sim 100$ fm^3, the above expression is $\exp[-4\langle\theta^2\rangle]$; large values of θ are not excited. However, after the functional θ integration, the distribution in isospin directions remains uniform, leading immediately to (2).

3. Our critical assumption is that in the high-density medium created by such collisions the quark density and other bilinears in q, $\bar{q}$ acquire classical values which may be comparable to or larger than the vacuum chiral condensate, $\langle\bar{q}q\rangle \simeq -(250\ \mathrm{MeV})^3$. From the explicit dependence of (13) on n^3 we see that isospin rotation symmetry is broken. We consider two possibilities: either $I(x)=\langle\langle\bar{q}\tau_3 q\rangle\rangle\neq 0$ or $P(x)=\langle\langle\bar{q}i\gamma_5 q\rangle\rangle\neq 0$, in addition to $S(x)=\langle\langle\bar{q}q\rangle\rangle\neq 0$, which are sizable values. The symbol $\langle\langle\cdots\rangle\rangle$ denotes the averaging over quantum fluctuations and we allow for a smooth (on the microscopic scale) position dependence. The value of $S(x)$ may differ significantly from the vacuum value of $\langle\langle\bar{q}q\rangle\rangle$.

We first consider the first case, $I(x)\neq 0$. Although this case has less interesting consequences, it is simpler to analyze. The functional integration over $\theta(x)$ in (14) (in the quadratic approximation) yields

$$d\omega \sim \frac{1}{\sqrt{|m_+ S(x) + m_- I(x) n_3^2|}} dn_3 . \tag{16}$$

For its dependence on n_3 to be significant, it is necessary for the second term in the square root to be comparable in magnitude to the first term. This is, however, unlikely since their ratio is (even for $f=n_3^2=1$)

$$\frac{m_- I(x)}{m_+ S(x)} = \frac{m_u - m_d}{m_u + m_d} \frac{\langle\langle u\bar{u} - d\bar{d}\rangle\rangle}{\langle\langle u\bar{u} + d\bar{d}\rangle\rangle} . \tag{17}$$

With $m_u - m_d/m_u + m_d \sim -0.3$ and the second factor smaller than unity, the n_3 dependence will be insignificant. We reach the same conclusion if we allow other components of $\bar{q}\tau q$ to acquire a classical value.

The situation is significantly different if we assume that $P(x)$ has a sizable value. We will consider below whether this is feasible, but let us first discuss the consequences of this assumption. Let us now evaluate

$$d\omega \sim \int \prod_x d\theta(x) \exp\left\{\frac{1}{T} \int d^3 \exp[m_+ S(x)(1-\cos\theta) - m_- n_3 P(x) \sin\theta]\right\} dn \delta(n^2 - 1). \tag{18}$$

The exponent has a minimum for a nonzero θ which is obtained from $\tan\theta = m_- n_3 P(x)/m_+ S(x)$. The functional integral can be written (again in a quadratic approximation). Aside from a prefactor, it yields

$$d\omega \sim \exp \frac{1}{T} \int d^3x [+ \sqrt{m_+^2 S^2(x) + m_-^2 P^2(x) n_3^2} + m_+ S(x)] dn_3 . \tag{19}$$

Although we could analyze this result, it is simpler to consider the case in which $|m_- P/m_+ S| < 1$. Keeping only the first term in the square-root expansion, we obtain [ignoring in the case in which $S(x)$ is positive the terms which do not depend on n_3]

$$d\omega \sim \exp\left[\frac{1}{2T} \frac{m_-^2}{m_+} \int d^3x \frac{P^2(x)}{|S(x)|} n_3^2\right] dn_3 . \tag{20}$$

Recalling that $f=n_3^2$, we find

$$d\omega = N(A) e^{Af} \frac{df}{2\sqrt{f}}, \tag{21}$$

where

$$A = \frac{1}{2T} \frac{m_-^2}{m_+} \int d^3x \frac{P^2(x)}{|S(x)|}, \tag{22}$$

and the normalization factor

$$N^{-1}(A) = \int_0^1 dx e^{Ax^2}. \tag{23}$$

We assume that the probability for the creation of a given θ is proportional to the Boltzmann factor, (14). At equilibrium or not quite at equilibrium, we believe that, on the average, this assumption is not far from the truth. We then immediately obtain (21) as a parametrization. Evidently the change in the distribution is important only if A is large enough. However, as we shall see, A is proportional to the collision volume and we will argue that it cannot be small. In this case the distribution (21) has a minimum at $f = 1/2A$ and, contrary to the case described by (2), it increases as f approaches 1. For $A \gg 1$, an approximate evaluation of (23) yields

$$d\omega \simeq A e^{-(1-f)A} \frac{df}{\sqrt{f}}. \tag{24}$$

This distribution has a peak at $f = 1$ and is enhanced near that value by a factor $2A$ over that of (2), making anti-centauros more probable.

We shall now attempt to estimate possible values of A. Note that (22) depends on the absolute value of $S(x)$ and on the square of $P(x)$. Thus, all spatial regions will add to the value of A. In a region of significant nuclear density $S(x)$ will differ from its vacuum value and will be related to the sum of the quark and anti-quark densities. We would like to compare it to $\rho(x) = \langle \bar{q} \gamma^0 q \rangle$, which is the difference between quark and anti-quark densities. Although the collision region of interest contains an equal number of quarks and anti-quarks, $\rho(x)$ may be either positive or negative and large over sizable regions [see the discussion following (14)]. For $|\rho(x)| \geqslant (250 \text{ MeV})^3$, $S(x)$ will coincide with $\rho(x)$ rather than with its vacuum value. $P(x)$ can be represented as

$$P(x) = \xi R \sigma(x) \cdot \nabla \rho(x). \tag{25}$$

Here $\sigma(x)$ is a spin density, R is a characteristic linear size of the effective volume (or characteristic time before hadronization), and ξ is a constant, which is probably smaller than one.

Integrating (22), we obtain

$$\int d^3x \frac{P^2(x)}{|S(x)|} = 4\pi R^2 r \frac{\xi^2 \rho^2 R^2}{r^2} \frac{1}{\rho} = \frac{4}{3} \pi R^3 \frac{3R}{r} \xi^2 \rho. \tag{26}$$

We use r as a characteristic length for the gradient; this variation in density is likely to be confined to the surface of the quark matter produced in the collision. We assume that the volume over which $P(x)$ does not vanish is $4\pi R^2 r$. The spin densities are averaged approximately to unity. Thus, for the parameter A we have

$$A = \frac{\xi^2}{2T} \frac{m_-^2}{m_+} \frac{3R}{r} N \simeq \frac{1}{70} \frac{R}{r} \xi^2 N. \tag{27}$$

Here $N = 4\pi R^3 \rho/3$ is the number of quarks produced. We believe that one could expect $N \geqslant 200$ in a sphere of $R \simeq 3$ fm (note that for the vacuum $\rho = \langle \bar{q}q \rangle = 2\ \mathrm{fm}^{-3}$, so that $N \simeq 200$). For $R/r \simeq 5$ we find $A \sim 15\xi^2$ and for $\xi \geqslant 0.25$ A is large enough to enhance the probability of anti-centauros. Note that $\xi \leqslant 0.8$ is required for the approximation to go from (19) to (20). We are well aware of the crudeness of these estimates. The purpose of this study was to show that values of $A \geqslant 1$ are not excluded. Since A is proportional to the volume, it is unlikely to be very small. We have presented almost a dimensional estimate.

The total change in the distribution of neutrals is due to the violation of isotopic spin invariance; the parameter A in (21) is proportional to $(m_u - m_d)^2$. Can we claim that the anti-centauro events are caused by the mass difference of light quarks?

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