Damping of ion acoustic waves in a plasma with rare collisions in connection with nonlocal hydrodynamics

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An expression for the damping rate of ion acoustic waves is derived from a solution of the electron kinetic equation corresponding to a fractional-asympototic expansion in reciprocal values of the Knudsen number. Some consequences of the hydrodynamics of Grad moments in a model of a nonlocal electron thermal conductivity are discussed.

In the rare-collision limit, in which the electron mean free path is long in comparison with the wavelength, the dissipation of ion acoustic waves due to electrons is usually linked with Landau damping alone (see, for example, Ref. 1). Still, one can point out a region of wavelengths in which electron collisions, although fairly rare in the usual sense of the term, are nevertheless more important than Landau damping. One can see this by making use of the result found by the approach of Refs. 2 and 3, in which a fractional-asymptotic expansion in reciprocal powers of the electron mean-free path was proposed. Taking that approach, we find the following expression for a perturbation (δn_e) of a Fourier component of the electron density due to a nonequilibrium electric potential $\delta\varphi\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$:

$$\delta n_e=-\frac{|e|\delta\varphi}{\kappa_B T_e}\left[1+\frac{i\omega}{kv_{Te}}\left(\sqrt{\frac{\pi}{2}}+\frac{3Z^{2/7}}{2k^{3/7}l_e^{3/7}}\right)\right]. \tag{1}$$

Here we are assuming $kl_e\gg 1$, $kv_{\mathrm{Te}}\gg\omega$, and a degree of ionization $Z=|e_i/e|\gg 1$. We are also using the following notation: n_e is the equilibrium, spatially uniform electron density; e is the electron charge; T_e is the electron temperature; κ_B is the Boltzmann constant; $v_{Te}=\sqrt{\kappa_B T_e/m_e}$ is the electron thermal velocity; k is the absolute value of the wave vector; $l_e=(3/4\sqrt{2\pi})(\kappa_B T_e)^2/(e^4 Z n_e\Lambda)$ is the mean free path of the electrons with respect to collisions with ions; and Λ is the Coulomb logarithm. Expression (1) allows us to write the following expression for the electron component of the complex longitudinal dielectric constant of the plasma:

$$\delta\epsilon_e(\omega,k)=\frac{4\pi e^2 n_e}{k_B T_e k^2}\left[1+\frac{i\omega}{kv_{Te}}\left(\sqrt{\frac{\pi}{2}}+\frac{3Z^{2/7}}{2k^{3/7}l_e^{3/7}}\right)\right]. \tag{2}$$

This expression yields, in particular, the following expression for the rate at which ion acoustic waves are damped by electrons:

$$\gamma=\frac{\omega^4\omega_{Le}^2}{\omega_{Li}^2 k^3 v_{Te}^3}\left\{\left(\frac{\pi}{8}\right)^{1/2}+\frac{3}{4}\left(\frac{Z^2}{k^3 l_e^3}\right)^{1/7}\right\}. \tag{3}$$

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Collisional effects are obviously dominant at wavelengths which are not too short, under the condition

$$Z^2 \gg k^3 l_e^3 > 1. \quad (4)$$

Aside from being of practical importance, Eq. (3) now supports a completely different and more general assertion regarding the use of the now-popular approach of the hydrodynamics of Grad moments with nonlocal spatial couplings of fluxes and thermodynamic forces (see, for example, Refs. 4–7). In particular, the wave damping is linked with a nonlocal electron thermal conductivity, in which the Fourier transforms of a perturbation of the electron heat flux and a perturbation of the temperature is related by

$$\delta\mathbf{q} = -i\mathbf{k}\delta T_e \kappa(\mathbf{k}), \quad (5)$$

where the popular expression[7-10]

$$\kappa(k) = \kappa_{\rm SH}[1 + (\alpha k \lambda_e)^\beta]^{-1} \quad (6)$$

is being used for the nonlocal thermal conductivity. Here $\kappa_{\rm SH} = C_{\rm SH} n_e v_{Te} \kappa_B l_e$ is the electron thermal conductivity of a highly collisional, fully ionized plasma. We have the value $C_{\rm SH} = (128/3\pi)$ for $Z \gg 1$ and $\lambda_e = l_e (2Z/9\pi)^{1/2}$. The value $\beta = 1$ was used in Refs. 4 and 5 to describe Landau damping with the help of a nonlocal thermal conductivity in the limit $\alpha k \lambda_e \gg 1$. The same value of the exponent β was given in Ref. 8 as the result of a numerical solution of the Boltzmann equation for the problem of electron heat transfer. Other numerical studies have yielded different values of the exponent β. For example, the value $\beta = 2$ was used in Ref. 9, while two values, $\beta = 4/3$ and 1.44, were reported in Ref. 8. Berger *et al.*[7] cite the value $\beta = 1.148$. Because of the large scatter in the values of β reported by different investigators, and because the corresponding papers provide nothing in the way of a solid basis for these values, we are obliged to give preference to an analytic theory which leads to an asymptotic expansion in fractional negative powers of the electron mean free path. That theory[2,3] yields $\beta = 10/7 = 1.42857...$ (see also Ref. 10).

Accordingly, if we use the equations of the hydrodynamics of Grad moments and expression (5) for the heat flux with the nonlocal thermal conductivity in (6), we find the following expression for the electron component of the dielectric constant:

$$\delta\epsilon_e(\omega,k) = \frac{4\pi e^2 n_e}{\kappa_B T_e k^2}\left[1 + \frac{i\omega}{k v_{Te}}\left(\frac{n_e \kappa_B v_{Te}}{\kappa(k) k}\right)\right]. \quad (7)$$

The value $\beta = 10/7$ evidently leads to a dependence on the wave number which is qualitatively different from (2). It can thus be asserted that our analytic theory[3] demonstrates that the hydrodynamics of Grad moments with a nonlocal heat flux, (5), (6), is not valid for describing a weakly collisional damping of ion acoustic waves. We can offer the even more general suggestion that a plasma hydrodynamics incorporating a finite number of Grad moments does not have even a remotely approaching universal application method under conditions of nonlocal transport.

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