Nuclear spin relaxation via dipolar interaction in a two-dimensional electron gas

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The nuclear spin–lattice relaxation in a two-dimensional electron gas under quantizing magnetic fields is considered. It is shown that in contrast with the case of three-dimensional conductors and two-dimensional systems with sufficient degree of disorder, where the hyperfine contact interaction is operative in the relaxation of the nuclear spins, the dipole–dipole interaction between the nuclear and the conduction electron spins in sufficiently pure two-dimensional systems can be the leading mechanism for the nuclear spin relaxation.

Nuclear spin–lattice relaxation in three-dimensional conductors is usually governed by the contact (Fermi) hyperfine interaction between the nuclear spins and the spins of conduction electrons.[1] This interaction conserves the total spin of the interacting particles. Recently there has been a growing activity in experimental[2,3] and theoretical[4–6] studies of the nuclear spin relaxation in heterojunctions in strong magnetic fields, i.e., under the condition of the quantum Hall effect. It was found that the existence of energy gaps in the spectrum of a two-dimensional electron gas (2DEG) under strong magnetic fields greatly reduces the effectiveness of the hyperfine contact interaction. This can be explained in the following way. In the flip-flop process (simultaneous reversal of the nuclear spin and the electron spin) the energy needed for crossing of the electron Zeeman gap is larger, by a factor of M_i/m_0 (where M_i is the nuclear mass and m_0 is the free electron mass), than the energy which can be provided by a nuclear spin. As a result, the alternative relaxation channels may begin to operate.

We will study the magnetic electron–nucleus interactions (dipolar) which do not conserve the total spin and are therefore not sensitive to the existence of the Zeeman gap in the electron spectrum. In this process the spin angular momentum of nuclei is converted, as a result of the interaction, to the orbital angular momentum of electron gas. As we will see below, a good agreement with the experiment[2] can be achieved without the introduction of any adjusting parameter.

Let us consider the model of a two-dimensional electron gas in a quantizing magnetic field which interacts with the nuclear spins and phonons. The Hamiltonian $\hat{H}$ of such a system can be written in the form

$$\hat{H}=\hat{H}_e^0+\hat{H}_n^0+\hat{H}_{\text{int}}, \tag{1}$$

where $H_{e,n}^0$ are Hamiltonians of the electron and nuclear system in a magnetic field, and $\hat{H}_{\text{int}}$ is the dipolar interaction between the nuclear and conduction electron spins[1]

$$\hat{H}_{\text{int}}=\mu_n\mu_e\int d^3r_e\int d^3r_n\frac{\Psi_e^+\Psi_n^+\hat{S}_n\hat{S}_e\Psi_n\Psi_e}{|\mathbf{r}_e-\mathbf{r}_n|^3}$$
$$-\frac{3\Psi_e^+\Psi_n^+(\hat{S}_n,r_n-r_e)(\hat{S}_e,r_n-r_e)\Psi_n\Psi_e}{|r_e-r_n|^5}-\frac{8\pi}{3}\delta(r_e-r_n)\Psi_e^+\Psi_n\hat{S}_n\hat{S}_e\Psi_n\Psi_e. \tag{2}$$

Here Ψ_e, $\hat{S}_e$, $\mathbf{r}$ and Ψ_n, $\mathbf{S_n}$, $\mathbf{r_n}$ are the wave functions, the spin operators, and the position vectors for the interacting electrons and nuclei, respectively.

Interaction with phonons is included in the Hamiltonians $H_{e,n}^0$. The magnetic field is assumed to be directed along the z axis. In the Landau gauge, $\mathbf{A}=H_0(0,x,0)$, the single particle wave function is

$$\Psi_e(\mathbf{r})=\frac{1}{\sqrt{L_y}}\exp^{iky}\cdot\psi^n(x)\cdot\chi(z), \tag{3}$$

where L_y is the size of the sample in the y direction, and

$$\psi^n(x)=\left(\frac{|eH|}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^n\Gamma_{(n+1)}}}H_n\left(\frac{x-x_0}{a_H}\right)\exp\left\{\frac{-(x-x_0)^2}{2a_H^2}\right\}. \tag{4}$$

Here $x_0=-a_H^2k_y$ is the center of the electronic orbit, $a_H=\sqrt{\hbar c/eH}$ is the magnetic length, H_n are the Hermite polynomials, and $\chi(z)$ is the Airy function:

$$\chi_{(z)}=C_1A_i\left[\left(\frac{2\alpha m_e}{\hbar^2}\right)^{1/3}\left(z-\frac{\tilde{E}_{n'}}{\alpha}\right)\right]. \tag{5}$$

Here the coefficient C_1 is the normalization factor, the coefficient α is defined by the electric field in the heterojunction which restricts the motion of the electron gas in two dimensions:

$$V(z>0)=\alpha z. \tag{6}$$

The energy levels $\tilde{E}_{n'}$ in this field can be found from the boundary condition

$$\chi(z=0)=0. \tag{7}$$

Since in most practical cases the distance between neighboring energy levels, $\tilde{E}_{n'}$, is much larger than the temperature and the distance between the Landau levels, we assume below that only the quantum-size ground level, $n^1=1$, is occupied. In sufficiently clean systems at low temperatures: Γ, $k_BT<\hbar\omega_z<\hbar\omega_c;\Gamma$, where $\hbar\omega_z$ and $\hbar\omega_c$ are the electron Zeeman splitting and the distance between the Landau levels, respectively, and Γ is the broadening of a Landau level, the nuclear spin relaxation is connected with the dipolar interaction between the nuclear and the electron spin, as is shown in the inset in Fig. 1.

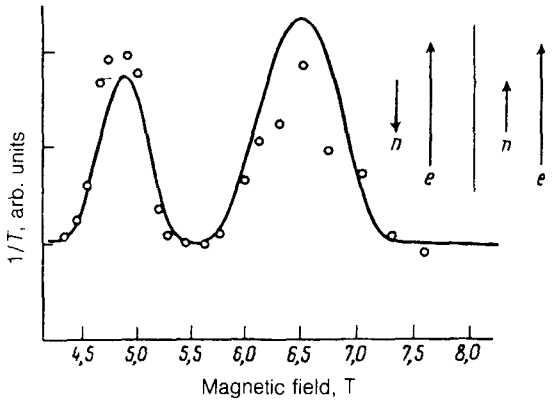

FIG. 1. Magnetic field dependence of the nuclear spin relaxation rate, T_1^{-1}. The curve represents our calculation based on the Eq. (25), with the sample parameters corresponding to the experimental data from Ref. 2. The dots are the experimental data from Ref. 2. Inset—relaxation of the nuclear spin via the dipole interaction with the electron spin.

In the first order of perturbation theory the transition probability, the expression for W (the inverse relaxation time) is

$$W=2\frac{(\mu_n\mu_e)^2}{L^2}\,\mathrm{Im}\sum_{k_i,k_f}\frac{n^i_{e\uparrow}\,(1-n^f_{e\uparrow})}{2\mu_n H+\epsilon_{if}-i(\tilde{\gamma}_i+\tilde{\gamma}_f)}$$

$$\times\left|\int d^3r_e\left|\chi(z)\right|^2\exp^{i(k_i-k_f)y}\psi_{k_i}\psi_{k_f}\frac{(r_e-r_n)_z[(r_e-r_n)_x+i(r_e-r_n)_y]}{|r_e-r_n|^5}\right|^2. \quad (8)$$

Here $\gamma_{i,f}$ is the half-width of the levels in the initial state (i), and the final state (f). It is equal to one-half of the sum of the transition probabilities in all other states. In Eq. (8), ϵ_{if} is the difference in energies of an electron in the initial state and the final state. At low temperatures only the transitions inside one spin-split Landau level account for the relaxation of the nuclear spins. In this case ϵ_{if} is connected with the nonuniform distribution of the impurities. It will be estimated below.

The transition probability W [Eq. (8)] can be presented in the form

$$W_N=A_N(\tilde{z}_n^2)\frac{(\mu_n\mu_e)^2}{\hbar}\,a_H^{-6}n(1-n)^2\,\frac{(\tilde{\gamma}_i+\tilde{\gamma}_f)}{(2H\mu_n+\epsilon_{if})^2+4(1-n)^2(\tilde{\gamma}_i+\tilde{\gamma}_f)^2}$$

$$\times\left(\int_0^\infty dz|\chi(z)|^2\,\mathrm{sign}(z-z_n)\right)^2. \quad (9)$$

Here $\tilde{z}_n=z_n/a_H$ is the dimensionless z coordinate of the interacting nuclei, and the function A_N depends on the number N of filled Landau levels. In Eq. (9) we separate the factor $(1-n)$ from the level width and denote

$$\gamma_{i,f}=(1-n)\tilde{\gamma}_{i,f}. \quad (10)$$

The expression for the functions $A_{1,2}$ (the first two Landau levels) are

$$A_1(\tilde{z}_n)=\begin{cases}\dfrac{4}{9}, & \text{if } \tilde{z}_n \ll 1\\ \dfrac{1}{12\tilde{z}_n^2}, & \text{if } \tilde{z}_n \gg 1\end{cases},$$

$$A_2(\tilde{z}_n)=\begin{cases}\dfrac{4}{3}, & \text{if } \tilde{z}_n \ll 1\\ \dfrac{1}{12\tilde{z}_n^2}, & \text{if } \tilde{z}_n \gg 1\end{cases}. \tag{11}$$

In order to estimate the value of $\tilde{\gamma}_{if}$, which is attributable to the interaction of electrons with phonons and impurities, we will calculate the minimal effective Landau level width which is required to satisfy the energy conservation in the dipole–dipole interaction between the nuclear and the electron spins. We assume a model with a Gaussian distribution of the electron energy levels within a single Landau level, caused by the elastic scattering with a random impurity potential. The excess electron energy (on the order of the nuclear Zeeman energy) is removed by the long-wave phonons.

Assuming the following inequality:

$$\mathcal{N}_0(da_H^2) \ll 1, \tag{12}$$

where $\mathcal{N}_0$ is the impurity density and $d = 1/\sqrt{\alpha m_e}$ is the localization depth in the z direction, we obtain the dispersion ϵ for the Gaussian distribution of the electron energy levels in the Zeeman branch of a given Landau level

$$\epsilon = V_a \frac{a^3 \mathcal{N}_0}{a_H \sqrt{d}}. \tag{13}$$

Here a is the interatomic distance in the sample, and V_a is the potential of the electron–impurity interaction. Real transitions can take place only between $\{i,f\}$ states which are spatially separated only at distances on the order of the magnetic length

$$R \propto a_H.$$

Under these conditions the energy differences ϵ_{if} are much smaller than ϵ. In what follows we assume that the following two conditions are satisfied:

$$\epsilon_{if} \leqslant \mu_n H \quad \text{and} \quad T > \epsilon. \tag{14}$$

As was mentioned above, the value of $\gamma_{i,f}$ is connected with different processes. One of them is the transition to localized impurity states. Such a process is accompanied by the emission or absorption of a phonon with an energy on the order of T. The electron transitions inside the Zeeman branch of a given Landau level are accompanied by emission of a real phonon with an energy $2\mu_n H$. Since the nuclear Zeeman energy is very small, the wave vectors of the corresponding phonons are large, and the deformation potential is not effective.

Let us now consider two different, two-dimensional electron systems: heterojunctions and silicon MOSFET's. At low temperatures, in GaAs the main contribution to the electron–phonon interaction is the piezoelectric component.[7] In a silicon MOSFET, where the quantum Hall effect is initially observed, the two-dimensional electron gas is subjected to an external electrical field of the gate voltage, which may strongly modify the electron–phonon interaction. In each case the electron–phonon interaction is a wave vector which is independent, as can be seen, from the Hamiltonian

$$\hat{H}_{\text{int}}=\frac{g_1}{\sqrt{V\rho\omega_k}}\sum_k\sum_\alpha\int d^3r\psi^\dagger\psi f^\alpha(\theta_k)(b_{k,\alpha}e^{ikr}+b^\dagger_{k,\alpha}e^{-ikr}), \tag{15}$$

where θ_k is the angle between $\mathbf{k}$ and the normal n to the surface. The function depends on the type of phonon and is equal to ($\alpha=1,2,3$),

$$f(\theta)=\begin{cases}\cos\ \theta_k, & \text{for LP,}\\ \sin\ \theta_k, & \text{for TP with the polarization in the } \mathbf{k},\mathbf{n} \text{ plane,}\\ 0 & \text{for polarization in the } \mathbf{k},\mathbf{n} \text{ direction.}\end{cases} \tag{16}$$

The coupling constant g_1 is

$$g_1=\frac{eE}{\sqrt{2}}+\text{ piezoelectric term,} \tag{17}$$

where eE is estimated from Eq. (6) to be $eE=\alpha$. The piezoelectric contribution to g_1 can be found in the literature (see Ref. 7).

Consider, first, the width $\gamma^{\text{ph}}_{\text{imp}}$, which is connected with the transitions to the impurity states. At low temperatures

$$T\ll 2\mu_e H,\ \hbar\omega_c \tag{18}$$

the filling of impurity states is

$$n_{\text{imp}}=\frac{n}{n+(1-n)\exp(\epsilon/T)}, \tag{19}$$

where energy ϵ is counted from the position of the Landau level in a filled zone, and n is the electron filling factor in this zone. We introduce now the density $\tilde{\mathcal{N}}$ of the impurity states in the unit volume and the energy by means of the relation

$$\tilde{\mathcal{N}}(\epsilon)=\frac{1}{V}\sum_{\text{imp}}\left|\int\psi_{e,\text{imp}}\,d^3r\right|^2\theta(\epsilon_{e,\text{imp}}<\epsilon),\quad \mathcal{N}(\epsilon)=\frac{\partial\tilde{\mathcal{N}}(\epsilon)}{\partial\epsilon}. \tag{20}$$

The sum in Eq. (20) is taken over the impurity states in the volume V with energies smaller than E. In the second order of perturbation theory we obtain

$$2\gamma^{\text{ph}}_{\text{imp}}=2\pi g_1^2\frac{\mathcal{N}}{\rho}(1-n)\int_{-\infty}^{\infty}d\epsilon\int\frac{d^3k}{(2\pi)^3}\sum_\alpha\frac{1}{\omega_k^\alpha}f_\alpha^2(\theta)\times\left[\delta(\epsilon-\omega_k^\alpha)\frac{1}{e^{\omega_k^\alpha/T}-1}\right.$$

$$\left.+\delta(\epsilon+\omega_k^\alpha)\frac{1}{1-e^{-\omega_k^\alpha/T}}\right]\frac{\exp(\epsilon/T)}{n+(1-n)\exp(\epsilon/T)}. \tag{21}$$

Finally, after simple calculations we obtain

$$2\gamma_{\text{imp}}^{\text{ph}}=\frac{g_1^2\mathcal{N}(1-n)T^2}{3\pi\rho}\left(\frac{2}{s_\perp^3}+\frac{1}{s_\parallel^3}\right)\int_0^{(\mu-H)/T}\frac{x}{e^x-1}\left[\frac{1}{(1-n)+ne^{-x}}\right.$$

$$\left.+\frac{1}{n+(1-n)e^{-x}}\right]dx, \tag{22}$$

where $s_{\perp,\parallel}$ are the velocities of the transverse and the longitudinal sound, respectively. We note that the contribution of this process to γ is proportional to T^2 and is independent of the value of the splitting energy, $2\mu_n H$, in the nuclear spin system.

Let us now consider the contribution to γ due to the electron transitions inside a given spin-split Landau level. In the first order of perturbation theory, using the Hamiltonian [Eq. (15)], we obtain

$$2\gamma_i^{\text{ph}}=\frac{2\pi g_1^2}{\rho}\frac{1}{L}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_f}{2\pi}\left|\int dxdye^{i\mathbf{kr}}\psi_{k_i}\psi_{k_f}\exp\{i(k_i-k_f)y\}\right|^2$$

$$\times\sum_\alpha\frac{1}{\omega_k^\alpha}f_\alpha^2(\theta_k)\delta(2\mu_n H-\omega_k)\frac{1-n}{1-e^{-\omega_k^\alpha/T}}. \tag{23}$$

Since $T\gg 2\mu_n H$, and correspondingly $k\ll\sqrt{eH}$, Eq. (23) reduces to a simple expression

$$2\gamma_i^{\text{ph}}=\frac{g_1^2kT(1-n)}{3\pi\hbar\rho}\left(\frac{2}{s_\perp^3}+\frac{1}{s_\parallel^3}\right). \tag{24}$$

Note that the width γ_i^{ph} is proportional to the first power of the temperature, and the coupling constant g_1 in Eqs. (17) depends on the applied electrical field E.

Since both γ's are small, and since they differ only by the temperature dependence, we will consider only one of them and assume that the second mechanism is essential. We will then obtain a simple temperature and a magnetic field dependence of the relaxation rate

$$W=\beta^N\frac{Tn(1-n)^2H}{1+[\alpha T^2(1-n)^2/H^2]}, \tag{25}$$

where the coefficient α is independent of N, the Landau level number, and β depends on N. The relationship between them is given for the first two Landau levels by Eq. (11). In Fig. 1 we compare the experimental data from Ref. 2, after substraction of the background values, with the theoretical results for α and β.

To complete our analysis, we present here the magnetic field dependence of the filling factor n. Assuming that $\tilde{\mathcal{N}}_0$, the number of impurity states per unit area and per unit energy, is constant in the energy region under consideration and small compared to $e/\hbar c\mu_e$, we can write the equation for the chemical potential μ in the form[8]

$$n_0=\frac{H}{\Phi_0}\sum_{n=0}^{\infty}\left[\frac{1}{1+\exp[(\epsilon_n^+-\mu)/T]}+\frac{1}{1+\exp[(\epsilon_n^--\mu)/T]}\right]$$

$$+\tilde{\mathcal{N}}_0\int_0^{\infty}\frac{d\epsilon}{1+\exp[(\epsilon-\mu)/T]}, \tag{26}$$

where $\Phi_0=2\pi\hbar c/e$ is the elementary magnetic flux and

$$\epsilon_n^{\pm}=\pm\mu_e H+\tilde{E}_1+\hbar\omega_c\left(n+\frac{1}{2}\right). \tag{27}$$

In the temperature region [Eq. (18)] Eq. (26) can be simplified:

$$n_0=\tilde{\mathcal{N}}_0\mu+\frac{H}{\Phi_0}(N+n), \tag{28}$$

where N is the number of completely filled Landau levels (including the spin splitting). The chemical potential coincides with the energy of the highest filled energy level (if n is different from to zero). It is easy to see that there exist finite regions in the magnetic field, where $n=0$. Assume that N_{2k+1} is odd and H is the boundary point. Equation (28) can then be written

$$n_0=\frac{H_{2k+1}}{\Phi_0}(2k+1)+\tilde{\mathcal{N}}_0\epsilon_k^-. \tag{29}$$

When the magnetic field decreases: $H=H_{2k+1}-\delta H$, the chemical potential increases and the filling factor n on the $2k+2$th level equals zero up to the point

$$\delta H\leqslant\delta H_{2k+1}^{cr}, \tag{30}$$

where

$$\delta H_{2k+1}^{cr}=\frac{2\mu_e H_{2k+1}}{\mu_e+(e\hbar/mc)(k+\frac{1}{2})+(2k+1)/\Phi_0\tilde{N}_0}. \tag{31}$$

The filling factor subsequently begins to grow until it reaches unity at the magnetic field H_{2k+2}, which can be found from the equation

$$n_0=\frac{H_{2k+1}}{\Phi_0}(2k+1)+\tilde{N}_0\epsilon_k^+. \tag{32}$$

In the region

$$H_{2k+2}-\delta H_{2k+2}^{cr}<H<H_{2k+2}, \tag{33}$$

$$\delta H_{2k+2}^{cr}=\frac{[(e/mc)-2\mu_e]H_{2k+2}}{-\mu_e+(e\hbar/mc)(k+\frac{3}{2})+(2k+2)/\Phi_0\tilde{\mathcal{N}}_0} \tag{34}$$

the filling factor of the $2k+2$th Landau level is equal to unity and the filling factor of the next Landau level is zero.

The factor $n(1-n)$ in Eq. (25) results therefore in the windows in a magnetic field, where the dipole–dipole interaction between the nuclear and the electron spins in a two-dimensional electron gas in strong magnetic fields is suppressed (see Fig. 1). These are the regions in which the Landau levels are either completely occupied or empty; i.e., the chemical potential lies in the localized states between the Landau levels. Finite temperature leads to smearing of this picture. The width of this window depends on the parity of N and decreases with k as k^{-2}. Note that Eqs. (9) and (25) are in good agreement with the experimental data of Ref. 2, even without adjusting the parameters.

In summary, we have proposed a new mechanism for the nuclear spin relaxation under the conditions of the quantum Hall effect, based on the dipole–dipole interaction between the nuclear and the electronic spins. This mechanism is contrasted with he previous theoretical work on two-dimensional electron systems with a large amount of disorder,[4,5] in which the hyperfine contact interaction was considered. We show that the principal physical difference between these two mechanisms is that in the contact interaction the total spin is conserved and the electron must flip its spin in order to relax the nuclear spin. Since the nuclear Zeeman splitting is several orders of magnitude smaller than the electron Zeeman energy, sufficient amount of disorder is needed in order generate finite nuclear relaxation times. This mechanism is operative under the integer quantum Hall effect conditions.

On the other hand, the conserved quantity in the dipole–dipole interaction is the total (spin-plus-orbital) angular momentum. Therefore, an electron can relax a nuclear spin, without changing its own spin state, just by shifting its center of orbit. This mechanism should therefore be the prevailing mechanism in clean samples, i.e., under the conditions of the fractional quantum Hall effect.

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