Resonant polarization bremsstrahlung in a plasma

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The resonant polarization bremsstrahlung of particles of arbitrary energy in an isotropic plasma is analyzed. Near the plasma frequency the spectra contain a high, narrow peak. A correct calculation of the shape of this peak, allowing for spatial dispersion, shows that the height is greater by a factor of c/v_T than has previously been assumed in the analysis of polarized radiation.

The electromagnetic radiation which results from the conversion of the quasisteady field of a charged particle into propagating waves at equilibrium microscopic inhomogeneities in a medium (e.g., thermal fluctuations of a plasma) is called "polarization bremsstrahlung." Polarization bremsstrahlung in a plasma has been studied by Akopyan and Tsytovich[1] in the two limiting cases of high and low frequencies. At high frequencies, $\omega_p \ll \omega \ll \omega_p v/v_T$, where ω_p is the plasma frequency, v is the velocity of the fast particle, and v_T is the thermal velocity of the plasma electrons; the intensity of this radiation is

$$I_\omega^p = \frac{16}{3} I_o \epsilon^{-3/2} \left(\ln \frac{v\omega_p}{v_T \omega} - \frac{1}{2} \right), \quad I_o = \frac{e^2 q^2 e_i^2 n_i}{v m^2 c^3}. \tag{1}$$

Here q is the charge of the fast particle, e and m are the charge and mass of an electron, and e_i and n_i are the charge and density of the ions in the plasma. This result was derived by the standard methods of the theory of radiation in media (Ref. 2, for example); spatial dispersion in the photon Green's functions was ignored. It is easy to see that expression (1) tends toward infinity if we formally let the frequency approach the plasma frequency: $\omega \to \omega_p$. Accordingly, the low-frequency region, $(\omega-\omega_p)/\omega_p \ll v_T^2/v^2$, in which spatial dispersion in the Green's functions is important, but in which we can assume $\omega=\omega_p$, was analyzed separately in Ref. 1. The following expression was derived for the intensity of the polarization bremsstrahlung:

$$I_\omega^p = \frac{2}{27} I_o \epsilon^{1/2} \left(\frac{v}{v_T} \right)^4. \tag{2}$$

The same result is given in some monographs[2,3] with a numerical coefficient of 2 in place of the 2/27 (this was apparently a misprint). However, it was pointed out in a later monograph[3] that Eq. (1) has a broader range of applicability, $(\omega-\omega_p)/\omega_p \gg v_T^2/v^2$, and that Eqs. (1) and (2) yield identical results, which are correct in order of magnitude, at frequencies $(\omega-\omega_p)/\omega_p \approx v_T^2/v^2$:

$$I_{\max} \sim I_0 (v/v_T)^3. \tag{3}$$

The increase in the intensity of the polarization bremsstrahlung near the plasma frequency was called "resonant polarization bremsstrahlung" in Ref. 3. A quantitative estimate of the magnitude of the effect was made by extrapolating the correct asymptotic expressions beyond their range of applicability. On the other hand, the temporal dispersion and the spatial dispersion in the photon Green's functions were not taken into account jointly and correctly.

In this letter we calculate spectra of the polarization bremsstrahlung which are valid at arbitrary frequencies $\omega_p \leqslant \omega \ll \omega_p v/v_T$. We show that the actual value of the resonant polarization bremsstrahlung at the maximum is larger than estimate (3) (Ref. 3) by a factor of about c/v_T. We show that the asymptotic expression in (1) is valid only at $\omega > 2\omega_p$.

Let us consider radiation which arises near the plasma frequency, where the phase velocities of transverse waves satisfy $v_{\rm ph} \gg c$, where c is the velocity of light. We thus have $v/v_{\rm ph} \ll 1$ for any particles, and in calculating the resonant polarization bremsstrahlung it is sufficient to consider only the longitudinal Green's function (this is the nonrelativistic approximation). The expression for the dielectric constant which appears in this Green's function should incorporate spatial dispersion: $\epsilon(\omega,\mathbf{k}) = \epsilon(\omega) - 3k^2d^2 + i\epsilon''$. The intensity of the resonant polarization bremsstrahlung is then written in the following form (this expression can be derived easily from the standard formulas for the intensity of the radiation in a medium[2,4]):

$$I^p_{\mathbf{n},\omega} = \frac{8\pi z^2 e^4 q^2 \epsilon^{1/2}}{m^2 c^3} \int k'^2 dk' \frac{[\mathbf{n}\cdot\mathbf{k}']^2 \delta\,[\omega - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]\,|\delta N|^2_{\mathbf{k}'}\, d\varphi d\cos\vartheta}{(\mathbf{k}-\mathbf{k}')^4\,[(\epsilon(\omega) - 3(\mathbf{k}-\mathbf{k}')^2 d^2)^2 + \epsilon''^2]}. \tag{4}$$

Here $|\delta N|^2_{\mathbf{k}'} \approx n_i/(2\pi)^3$ is the spectrum of thermal fluctuations in the plasma at $k' \ll d^{-1}$, $d = v_T/\omega_p$ is the Debye length, $z = e_i/e$, and the imaginary part of the dielectric constant, ϵ'', is taken into account in order to eliminate a divergence during the integration of (4). At the frequencies under consideration here we have $\epsilon(\omega) \ll 1$ and $k \ll k'_{\min} = \omega_p/v < k'$. We can thus ignore $\mathbf{k}$ in comparison with $\mathbf{k}'$ everywhere except in the resonant denominator. It is then convenient to integrate (4) over the angles of the vector $\mathbf{n}$, i.e., to find the energy which is radiated into the total solid angle (the directional pattern in this case corresponds to a dipole[3]):

$$I^p_{\omega} = \frac{32\pi^3 z^2 e^4 q^2 \epsilon^{1/2}}{v m^2 c^3} \int_{\omega/v}^{d^{-1}} \frac{dk'}{k'}\,|\,\delta N|^2_{\mathbf{k}'} \int_{-1}^{1} \frac{\sin^2\vartheta\, d\cos\vartheta}{[\epsilon(\omega) + 6kk'd^2\cos\vartheta - 3k'^2 d^2]^2 + \epsilon''^2}. \tag{5}$$

Here we have also carried out a trivial integration over the azimuthal angle ($\int d\varphi ... = 2\pi$), and ϑ is the angle between the vector $\mathbf{k}'$ and the particle velocity $\mathbf{v}$. Breaking this expression up into elementary integrals, and integrating over angle, we find

$$I^p_{\omega} = \frac{32\pi^3 z^2 e^4 q^2 \epsilon^{1/2}}{v m^2 c^3} \int_{\omega/v}^{d^{-1}} \frac{dk'}{k'}\,|\,\delta N|^2_{\mathbf{k}'} \frac{J_\vartheta}{36 k^2 k'^2 d^4}, \tag{6}$$

where

$$J_\vartheta = a \ln\frac{(a+1)^2+b^2}{(a-1)^2+b^2} - 2 + \frac{1+b^2-a^2}{b}\pi\theta(1-a^2) + \arctan\frac{2b}{a^2+b^2-1}, \quad (7)$$

$a=[3k'^2d^2-\epsilon(\omega)]/6kk'd^2$, and $b=\epsilon''/6kk'd^2$. Let us take a more detailed look at expression (7) for J_ϑ. The case of a nonabsorbing medium corresponds to the limit $b\to 0$. For $a^2\leqslant 1$, we thus have $J_\vartheta \to \infty$ as π/b. This divergence has a simple physical origin: Under the condition $a^2\leqslant 1$, the condition for (Vavilov–) Čerenkov radiation is satisfied for longitudinal waves (plasma waves). The field of the particle for the corresponding values of ω, $\mathbf{k}$, and $\mathbf{k}'$ is thus not a quasisteady field but a propagating field. Its interaction with plasma inhomogeneities corresponds to a scattering of quanta which have already been emitted, not the generation of new quanta. The range of Čerenkov plasmons in an infinite absorbing medium is infinite; this circumstance is the reason for the divergence. To calculate the intensity of the resonant polarization bremsstrahlung, understood as the result of the conversion of the quasisteady field of the particle into propagating waves,[5] we should eliminate the value $a^2\leqslant 1$ from the range of integration over dk'. In this case the function J_ϑ can be simplified substantially. Discarding the term $\pi\theta(1-a^2)$, and expanding arctan x in a series, as is legitimate because of its small argument $(a^2>1)$, we find

$$J_\vartheta = \left\{ a\ln\frac{(a+1)^2+b^2}{(a-1)^2+b^2} - 4 \right\}\theta(a^2-1). \quad (8)$$

The quantity J_ϑ has a singularity as $b\to 0$, $a^2\to 1$, but this singularity is integrable. That this is true can be seen by expanding J_ϑ in a power series in $1/a$; this series converges in the circle $1/|a|<1$. Retaining the first nonvanishing term of this expansion,

$$J_\vartheta \simeq \frac{4}{3a^2}\theta(a^2-1), \quad (9)$$

we can keep the error to at most 30%. After we substitute (9) into (6) and switch to the dimensionless variable $\mu = k'v/\omega$, we can write the spectrum of the resonant polarization bremsstrahlung as follows:

$$I_\omega^p = \frac{16}{3} I_o F(\alpha), \quad F(\alpha) = \frac{\epsilon^{1/2}}{9}\left(\frac{v}{wd}\right)^4 \int_1^{\mu_{\max}} \frac{d\mu}{\mu}\frac{\theta(a^2-1)}{(\mu^2-\alpha)^2}, \quad (10)$$

where $\alpha=\epsilon/3(v/\omega d)^2 \approx \epsilon/3(v/v_T)^2$ and $\mu_{\max}\approx v\omega_p/v_T\omega$. The integration in (10) is carried out in terms of elementary functions; it yields

$$F(\alpha) = \frac{\epsilon^{1/2}}{18}\left(\frac{v}{wd}\right)^4 \left\{ \left[\frac{1}{\alpha(1-\alpha)} + \frac{1}{\alpha^2}\ln(1-\alpha)\right]\theta(\omega_1-\omega) \right.$$
$$+ \frac{1}{\alpha^2}\frac{c}{2\times 3^{0.5}v_T}\left[1 - \frac{2\times 3^{0.5}v_T}{c}\ln\frac{c}{2\times 3^{0.5}v_T}\right]\theta(\omega-\omega_1)\theta(\omega_2-\omega)$$
$$\left. + \left[\frac{1}{\alpha^2}\frac{c}{3^{0.5}v_T} + \frac{1}{\alpha}\left[\frac{1}{1-\alpha} - \frac{1}{\mu_{\max}^2-\alpha} + \frac{1}{\alpha}\ln\frac{(\alpha-1)\mu_{\max}^2}{\mu_{\max}^2-\alpha}\right]\right]\right.$$

$$\times\theta(\omega-\omega_2)\theta(\omega_3-\omega)+\left[\frac{1}{\alpha^2}\frac{c}{2\times 3^{0.5}v_T}-\frac{1}{\alpha(\alpha-1)}+\frac{1}{\alpha^2}\ln\frac{(\alpha-1)c}{2\times 3^{0.5}v_T}\right]$$

$$\times\theta(\omega-\omega_3)\theta(\omega_4-\omega)+\frac{1}{\alpha}\left[\frac{1}{\alpha-\mu_{\max}^2}-\frac{1}{\alpha-1}+\frac{1}{\alpha}\ln\frac{(\alpha-1)\mu_{\max}^2}{\alpha-\mu_{\max}^2}\right]\theta(\omega-\omega_4)\Bigg\}, \tag{11}$$

where

$$\omega_{1,2}=\omega_p\left[1+\frac{3}{2}\left(\frac{v_T}{v}\right)^2\left(1\mp\frac{2\times 3^{0.5}v_T}{c}\right)\right], \tag{12}$$

$$\omega_{3,4}=2\omega_p\left(1\mp\frac{3\times 3^{0.5}v_T}{4c}\right). \tag{13}$$

The changes in slope at the frequencies $\omega_{1,2}$ in the spectrum stem from the physics of the process under consideration here (the properties of the radiation change abruptly when the parameters of the system cross the Čerenkov threshold), while the changes in slope at the frequencies $\omega_{3,4}$ correspond to the approximation used for $|\delta N|^2_{\mathbf{k}'}$. However, that point is unimportant for the resonant polarization bremsstrahlung with which we are concerned here.

Let us analyze the results. At high frequencies, $\omega_p \ll \omega \ll \omega_p v/v_T$, the correctly calculated spectrum of the resonant polarization bremsstrahlung, (10), (11), becomes asymptotic expression (1). Expression (1), however, turns out to be valid only at $\omega > 2\omega_p$, not at $(\omega-\omega_p)/\omega_p > v_T^2/v^2$, as was assumed previously.[3] The reason is the shape of the spectrum of thermal fluctuations in the plasma, which remains flat down to small length scales on the order of d [so the contribution of the upper limit in integral (10) is important]. At low frequencies, $\omega \to \omega_p$, we have

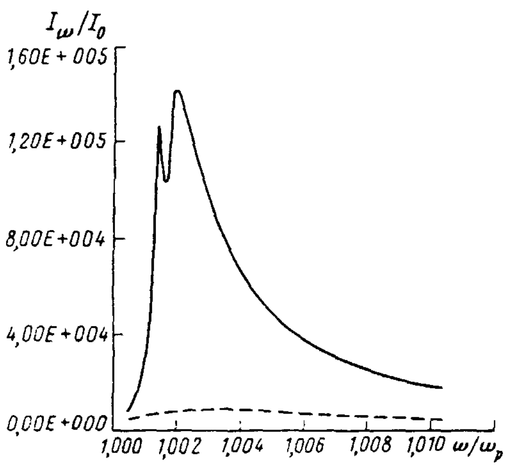

FIG. 1. Spectrum of resonant polarization bremsstrahlung near the plasma frequency. Solid curve—Calculated from Eqs. (10) and (11); dashed curve—calculated from asymptotic expressions (1) and (2) (Ref. 3). The parameter values here are $p/(mc)=10$ (this is the momentum of the particle) and $v_T/c=0.032$.

$$I^p_\omega \approx \frac{4}{27} I_o \epsilon^{1/2} \left(\frac{v}{v_T}\right)^4, \tag{14}$$

in accordance with (2) [the difference between Eq. (2) (Ref. 1) and (14), a factor of 2, apparently stems from the misprint in Ref. 1]. At the maximum of the spectrum, $\alpha \approx 1$, however, we have

$$I^p_{\max} \sim I_0 \frac{v^3 c}{v_T^4}, \tag{15}$$

which is at odds with estimate (3). Figure 1 shows the shape of the peak of resonant polarization bremsstrahlung calculated from (11).

The total energy of the resonant polarization bremsstrahlung, radiated at all frequencies, is

$$I^p_{\text{tot}} = \frac{8}{27} I_o \, \omega_p \frac{vc}{v_T^2}. \tag{16}$$

It is larger by a factor of vc/v_T^2 than the power of ordinary polarization bremsstrahlung (without the peak). Consequently, the magnitude of the resonant polarization bremsstrahlung has previously been substantially underestimated.

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