Magnetic field dependence of the localization length on the dielectric side of metal–insulator transitions

B. Spivak
Department of Physics, University of Washington, Seattle, Washington 98195

Hui Lin Zhao[1)] and Shechao Feng
Department of Physics, University of California, Los Angeles, California 90024

(Submitted 10 March 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **59**, No. 9, 599–604 (10 May 1994)

Magnetic field dependence of negative magnetoresistance (MR) in variable-range-hopping conduction is discussed. Above four dimensions, the negative MR saturates at large magnetic fields. In dimensions lower than four, the negative MR corresponds to a field-dependent correction to the localization length. A qualitative coarse-graining picture is presented for such behavior.

It is well known that the low-field magnetoresistance (MR) on the dielectric side of a metal–insulator transition in the variable-range-hopping (VRH) regime can be negative (see Ref. 1 for a review).

In three dimensions the weak localization theory,[2] which works well in metals, predicts that the magnetic field induces a correction to the conductivity $\langle\delta\sigma\rangle = \langle\sigma(H)\rangle - \langle\sigma(0)\rangle = e^2/\hbar L_H$. Here L_H is the magnetic length and the brackets $\langle\ \rangle$ correspond to averaging over random realizations of the scattering potential. The expression for this correction does not contain length scales other than L_H itself. This stems from the fact that the main contribution comes from self-crossing diffusion paths with a characteristic size of loops of order L_H.

In Ref. 3 it was assumed that this correction dominates the magnetoresistance in all metallic regions, including the neighborhood of a metal-insulator transition. This leads to $\sigma(H) \sim e^2/\hbar\xi(0) - A(e^2/\hbar L_H)$, where A is a coefficient about which nothing is known. This equation is equivalent to a change of the correlation radius $\delta\xi = \xi(H) - \xi(0) \sim -(\xi_0^2/L_H)$. This behavior was interpreted in Ref. 3 as a shift of the mobility edge due to magnetic fields.

In the spirit of scaling theory of metal–insulator transition, the behavior of the localization length below the transition and the correlation radius above the transition should be the same. It is natural to expect that on the dielectric side of the transition the magnetic field correction to the correlation radius has the same form on both sides of the transition. On the other hand, on the dielectric side of the transition weak localization theory does not work, because at $L_H \gg \xi$ the contribution to the conductivity from self-crossing paths of size L_H is exponentially small and can be ignored.

A qualitative picture and a simple model for MR in the VRH regime, based on the interference of directed tunneling paths, was proposed in Refs. 1 and 4. The numerical simulations of MR for relatively small hopping lengths in the framework of this model yielded a MR on the order of unity.[1,4] Further numerical studies in Refs.

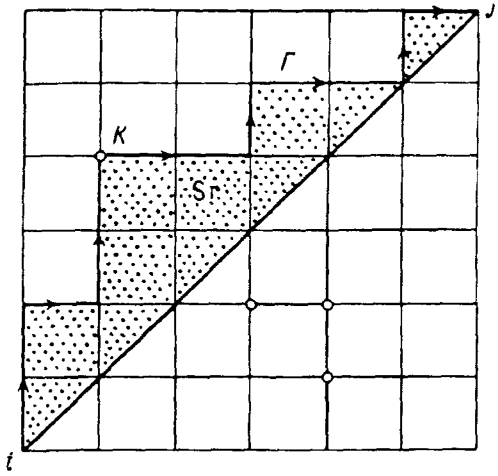

FIG. 1. The region of coherent tunneling. Γ is a typical directed path and S_Γ is the area covered by the path.

5 and 6 for samples much larger than those in Refs. 1 and 4 led to the conclusion that in two dimensions the negative MR corresponds to corrections to the localization length.

In this paper we show that in the framework of the model based on the interference of directed paths the MR is universal and depends only on the dimensionality. We introduce a coarse-grained picture of the negative MR, in which the tunneling area is divided into blocks of the size of the magnetic length, and the global MR is attributed entirely to the magnetic field reduction of the variance of the tunneling transmission probability of each block. Each plaquette in the coarse-grained lattice represents a block of the size of the magnetic length in the original model. The solution of the coarse-graining model shows that for dimensions $d<4$ the MR corresponds to a correction to the localization length of the same form as the H-dependent corrections to the correlation radius in the metallic regime. This means that the results of the model[4] are universal in the sense that they do not contain parameters other than L_H, and that they depend only on the dimensionality of space. In the cases $d\geqslant 4$, MR saturates at large H.

We start with a brief discussion of the model. In the VRH conduction the electrons which hop between localized states are associated with the absorption or emission of phonons. At low temperatures, the typical electron hopping distance r is much larger than the localization length and the average distance between impurity atoms. As a result, in the course of hopping between localized states, a hopping electron undergoes multiple elastic scatterings with impurities. The lattice model proposed in Ref. 4 (see Fig. 1) takes into account the interference of different tunneling paths between initial and final sites "i" and "j". Tunneling paths containing returns and loops are ignored, because of their exponentially small contribution to the total probability of a hop. A typical path is shown in Fig. 1 for a two-dimensional lattice. Generalization to higher dimensions is straightforward. Let Γ denote a direct path in Fig. 1. The tunneling amplitude associated with Γ is given by

$$A_\Gamma = \exp(i2\pi\phi_\Gamma/\phi_0) \prod_{k\in\Gamma} \alpha_k, \tag{1}$$

where α_k's are independent random variables which are associated with each site and which represent the random scattering amplitudes of impurities encountered by the electron during tunneling. The index k in Eq. (1) runs over sites belonging to the path Γ, ϕ_Γ is the magnetic flux through the shaded area enclosed by Γ and the diagonal, and $\phi_0 = hc/e$ is the flux quantum. The hopping probability is

$$\sigma_{ij}(H) = |A|^2 = |\sum_\Gamma A_\Gamma|^2. \tag{2}$$

Equations (1) and (2) can be derived directly from the Anderson model, using the locator expansions.[1] The macroscopic conductance can be obtained on the basis of percolation theory, by performing a logarithmic average of Eq. (2) over the impurity ensembles.[1,7] We define the magnetoconductance (MC) as follows:

$$L = \left\langle \ln \frac{\sigma_{ij}(H)}{\sigma_{ij}(0)} \right\rangle. \tag{3}$$

The directed path model can also be used to describe the insulating regime near the metal–insulator transition, where the localization length ξ is much larger than the length of the individual impurity state wave functions. The idea is to divide the region of tunneling into "blobs" of size ξ and assume that it is possible for the tunneling paths to take backward steps within a given blob but not between blobs.[1] In this analog each site in Fig. 1 should then be understood as a blob. The results presented here are valid when the magnetic length is larger than the localization length. In Ref. 8 this picture was extended to the superconductor–insulator transitions.

It was shown in Ref. 1 that if the random variable α_k in Eq. (1) has a large enough probability to take negative values, the total tunneling amplitude $A = \Sigma_\Gamma A_\Gamma$ will have a random sign at $H=0$ for large enough tunneling distances. We can introduce a sign persistence length r_s, beyond which the tunneling amplitude A becomes random in sign. Then one can coarse-grain the lattice so that each site of the new lattice represents a block of size r_s. This procedure corresponds to a renormalized α_k with random signs (i.e., $\langle\alpha_k\rangle = 0$) in the new coarse-grained lattice. We will focus below on such coarse-grained lattices.

The qualitative picture of negative MR proposed in Ref. 1 is the following. If α_k's are of random signs, then $\langle |A|^2\rangle = \langle |\Sigma_\Gamma A_\Gamma|^2\rangle$ does not depend on H, while all higher moments decrease with H, because the variance of a sum of real numbers with random sign is *larger* than the variance of a sum of the corresponding complex numbers which are generated from the random real numbers by multiplying the random phases. As a result, the magnetoconductance defined in Eq. (3) is an increasing function of H. It is on the order of unity when the magnetic phases in Eq. 1 for typical A_Γ's become on the order of π. It was suggested in Ref. 1 that this occurs when $H \gg H_c = \phi_0/(r^3\xi)^{1/2}$; here $(r^3\xi)^{1/2}$ is the area covered by typical directed paths.

Below we will show that at larger fields the MR at $d<4$ is large and corresponds to a correction to the localization length.

Consider a two-dimensional lattice, let us first divide the sample of size r into subsquares of size L_H. This guarantees that the phase of the tunneling amplitude A_Γ for a typical path Γ through each subsquare is on the order of π. It is evident that the width of the distribution function of the probability for the transmission through subsquares is reduced by a factor on the order of 1 compared to the corresponding zero field value, while the average of the transmission probability remains the same. For the reason discussed above, this result leads to an increase of the logarithmic average of the transmission probability for each subsquare. However, whether this result leads to an increase of the total $\langle \ln |A|^2 \rangle$ is not clear. This can be checked by a coarse-graining model described below.

According to the argument advanced above, after dividing a sample into blocks of size L_H, the only effect of magnetic fields is to reduce the variance of the transmission probability of each block. If we coarse-grain the lattice in such a way that each block is reduced by an elementary plaquette (as in Fig. 1), an interesting situation arises: If we disregard the magnetic field and consider only the change in $Var\{|\alpha_k|^2\}$, while keeping $\langle \alpha_k \rangle = 0$ and $\langle |\alpha_k|^2 \rangle$ the same, how will the change of $\langle \ln |A|^2 \rangle$ depend on r? In other words, we can mimic the effect of applying a magnetic field by a change in the variance of $|\alpha_k|^2$. We will consider below a quantity analogous to L in Eq. (3) for a bond model,

$$L_1 = \left\langle \ln \left| \frac{A\{\alpha_k^{(1)}\}}{A\{\alpha_k^{(2)}\}} \right|^2 \right\rangle, \tag{4}$$

where $\{\alpha_k^{(1)}\}$ and $\{\alpha_k^{(2)}\}$ have different distribution functions which satisfy the constraints stated above.

For $d=1$ the answer is obvious: $L_1 = \beta r$, where β depends on the difference

$$\delta = Var\{|\alpha_k^{(2)}|^2\} - Var\{|\alpha_k^{(1)}|^2\}.$$

In the case of more than four dimensions, L_1 does not depend on r asymptotically. This assertion can be proved exactly, because the distribution function of A can be calculated from its moments. In the calculation of the moments $\langle A^m \rangle = \langle \Sigma_{\Gamma,\Gamma',\Gamma''} A_\Gamma A_{\Gamma'} A_{\Gamma''} \ldots \rangle$, because of the sign randomness of α_k, only the even moments survive the average, and the paths in the summation should be "paired," i.e., any site on the lattice should be visited by an even number of paths. The crucial point is that in the case of more than four dimensions any two typical paired paths do not, statistically speaking, intersect. In other words, the paired paths propagate independently. In the absence of magnetic fields, a simple combinatorial counting gives $\langle A^{2m} \rangle = (2m-1)!!(z\langle \alpha^2 \rangle)^{nm}$, where z is the number of neighbors in the forward direction, and n is the number of steps on the lattice between site i and j, which is proportional to the hopping length. This leads to a Gaussian distribution of A with a zero average and a variance equal to $(z\langle \alpha^2 \rangle)^n$. Under the conditions imposed on calculating L_1, it should vanish. Correspondingly, in the framework of the model of Ref. 4, there are no corrections to the localization length due to the magnetic fields and MC saturates to a number on the order of unity above four dimensions.

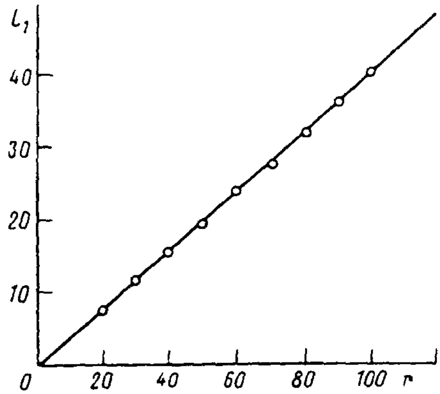

FIG. 2. In two dimensions, magnetoconductance L_1, defined by Eq. (4), versus the mean hopping distance r.

This behavior is different from $d=1$ and $d=2,3$ (discussed below), where MC increases linearly with the hopping length, which corresponds to a correction to the localization length. The relation between the saturation of MR and the assumption about the independence of the tunneling paths was discussed in Ref. 9 in two dimensions, where the approximation is not valid. Only in the case of more than four dimensions, where two diffusive lines do not intersect, the independent-path approximation is valid.

Figures 2 and 3 show the results of numerical simulations of L_1 for $d=2,3$. We assumed $\delta\approx 2$. The results in these figures clearly show that $L_1 \propto r$, which means, according to our coarse-graining model, that the MR scales with the number of blocks of size L_H along the length r,

$$L=\left\langle \ln \frac{\sigma_{ij}(H)}{\sigma_{ij}(0)} \right\rangle \sim \frac{r}{L_H}. \tag{5}$$

This result corresponds to a field correction to the localization length

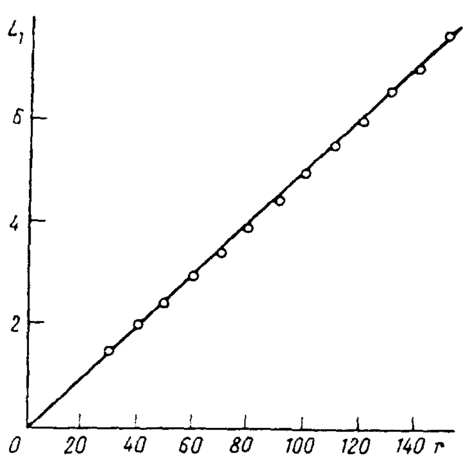

FIG. 3. In three dimensions, the magnetoconductance L_1, defined by Eq. (4), versus the mean hopping distance r.

$$\delta\xi=\xi(H)-\xi(0)\sim\xi^2/L_H. \quad (6)$$

Here we deliberately use the same symbol for the localization length on the dielectric side of the transition as we have used for the correlation radius on the metallic side.

We thus can draw the following conclusion: On the basis of the model [Eq. (3)] there are magnetic field corrections to the localization length in all dimensions less than four. This conclusion stems from the fact that in considering MR we can divide a sample into blocks of size L_H such that a magnetic field will reduce the variance of the transmission probability of each block by a factor on the order of 1. The field effect on each block will be magnified into a correction to the localization length [Eqs. (5) and (6)] in dimensions lower than four, in agreement with the concept of a shift of the mobility edge by magnetic fields.

A completely different situation arises in the presence of a spin–orbit scattering. In this case the magnetic field correction to the localization length, which originates from the model of Ref. 4, is positive and analytic in H when $L_H \gg \xi$. On the metallic side of the transition, however, the correction to the correlation radius has the same nonanalytical form as before but with the opposite sign, i.e., $\delta\xi\sim\xi_0/L_H$.

Many experimental groups have been measuring the magnetoresistance in hopping conductivity for almost twenty years. Not all of them (especially in three-dimensional samples) have observed negative magnetoresistance. The reason for this fact is not clear at present. One of the explanations is that the existence of paramagnetic spins suppresses the orbital interference effect (see corresponding discussions in Refs. 1 and 6). Following this point of view, the experiments which show a negative magnetoresistance may correspond to the absence of an unpaired spin. In most experiments in which the negative magnetoresistance in VRH was observed it was less or on the order of unity.[10] These experiments correspond to relatively small hopping lengths, for which the calculations performed in Ref. 1 are relevant. More recent experiments have shown, however, that the measured magnetoresistance can be much larger than unity.[11,12] In those cases the model which we presented here is more suitable for the explanation of the experimental data.

We are grateful to B. Shklovskii for useful discussions. The work in UCLA is supported, in part, by the ONR under grant number N00014-90-J-1829, by the DOE under grant number DE-FG03-88ER45378, and by the Alfred P. Sloan Foundation.

[1)]Present address: Department of Physics, University of Florida, Gainesville FL 32611.

[1] B. I. Shklovskii and B. Z. Spivak, in *Hopping Transport in Solids*, edit. by M. Pollak and B. I. Shklovskii (Elsevier, 1991).
[2] B. L. Altshuler, D. E. Khmel'nitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
[3] D. E. Khmel'nitskii and A. I. Larkin, Solid State Commun. **39**, 1069 (1981).
[4] V. L. Nguyen, B. Z. Spivak, and B. I. Shklovskii, JETP Lett. **41**, 42 (1985); Sov. Phys. JETP **62**, 1021 (1985); JETP Lett. **43**, 44 (1986).
[5] E. Medina, M. Kardar, Y. Shapir, and X. R. Wang, Phys. Rev. Lett. **64**, 1816 (1990).
[6] Hui Lin Zhao, B. Z. Spivak, M. P. Gelfand, and S. Feng, Phys. Rev. B **44**, 10760 (1991).
[7] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, 1984).
[8] S. Kivelson and B. Z. Spivak, Phys. Rev. B **45**, 10490 (1992).

[9]U. Sivan, Entin-Wolman, and Y. Imry, Phys. Rev. Lett. **60**, 1566 (1988).
[10]A. O. Orlov and A. K. Savchenko, JETP Lett. **44**, 41 (1986); Z. Ovadyahu, Phys. Rev. B **33**, 6552 (1986); O. Ye, Qiu-Yi, A. Zreniver, F. Koch, and K. Ploog, Semicond. Sci. and Tech. **4**, 500 (1989); F. Tremblay, M. Pepper, R. Newbury *et al.* Phys. Rev. B **40**, 10052 (1989-I).
[11]F. P. Milliken and Z. Ovadyahu, Phys. Rev. Lett. **65**, 911 (1990).
[12]H. W. Jiang, C. E. Johnson, and K. L. Wang, Phys. Rev. B **46**, 12830 (1992).

Published in English in the original Russian journal. Reproduced here with stylistic changes by the Translations Editor.